A Computational Corpus Study of Harmony in the Music of Anton Webern

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Abstract

This thesis sets out to apply digital analysis to the music of Anton Webern, with the aim of quantifying elements of his harmonic style, and tracing their change across his practice. It is a corpus study, taking Webern's 31 works with Opus numbers as its subject, and uses music21 for the data collection. Analytical subjects include distributions of pitch classes, intervals, and pitch-class sets; techniques for assessing these include clustering, regression analysis, and the Discrete Fourier Transform. Along the way, the thesis interrogates commonly-held assumptions about Webern's music for which there is often little empirical evidence, and provides a multi-level perspective on the corpus, from the wide angle of whole-movement macroharmonies to the close-up detail of individual pitch-class sets. Chronologically, the results suggest that the advent of dodecaphony in Webern's music had a limited effect on the surface features of the music he presented; rather that the major shift in his practice happened in his mid-period Lieder. *Contra* Allen Forte, I downplay the importance of octatonicism in his harmonic language in favour of a quartal quality, but in the process I explore the differing features between the various harmonic levels of Webern's music.

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Glossary

Anomaly . . . . A datum that has an unusual value, typically very high or very low, compared to other members of its dataset.

Ausfälle . . . . A type of row intersection in which there is a sense of omission. Rather than being shared, a note is missing from one dodecaphonic row because it is being sounded in another.

Clustering . . . . A statistical technique that calculates the distance between the items of a group and then separates them into sub-groups based on their proximity.

DFT . . . . Discrete Fourier Transform. An analytical technique that describes a pitch class distribution in terms of a set of magnitude and phase values. These values can be read to describe harmonic features of the original distribution.

IQR . . . . Inter-Quartile Range. A measure of spread that is defined as the numerical disparity between the upper and lower quartiles of a set of numbers.

Intersection . . . . The shared use of one or more notes by multiple dodecaphonic rows unfolding concurrently.

Interval Class . . . . The most basic description of an interval which assumes not only octave but inversional equivalence.

Interval Distribution . . . . A set of twelve numbers that describes the frequencies of the twelve intervals in a given extract.

Optical Music Recognition . . . The conversion by a computer of a sheet music image into a digital file.

Pitch Class . . . . A set of all pitches that are exactly an octave apart.

Pitch Class Distribution . . . . . A set of twelve numbers that describes the frequencies of the different pitch classes in a given extract.

Principal Component Analysis . A statistical technique that reduces high-dimensional data into a few dimensions that display as much of the information as possible.

Range . . . . A measure of spread that is defined as the numerical disparity between the maximum and minimum values of a set of numbers.

Row Chain . . . . A row chain is created when two adjacent row forms are linked by a common segment of any size that ends the first row and begins the second, thus overlapping them as an elision.

Row Chain Cycle . . . . A succession of rows linked by chains created from the same transformation that returns to the original row form.

Simple Interval . . . . A description of an interval which assumes octave equivalence but not inversional equivalence.

Chapter 1

Introduction

'Epigrammatic' (Colter Walls 2022); 'jewel-like' (Clements 2022); 'a bracing new style of crystalline compression' (Platt 2017). The stereotypical adjectives associated with Webern's style are clear. Ever since Igor Stravinsky described Webern's 'dazzling diamonds' (Stravinsky 1959, vii) the music has been identified, at least in the popular press, as characterised primarily by tight construction and a fierce beauty, alongside comments about its temporal concentration.[1] This is perhaps the usual description of Webern's music, focussed on its supposedly cerebral nature, a reputation surely encouraged in part by its treatment at the hands of the Darmstadt coterie.

'Lyrical' is the other word that is often connected with Webern, deployed by authors including Paul Griffiths (2000), Andrew Clements (2009), and Tom Service (2013). If it was Stravinsky who inaugurated the connection to diamonds, it seems to have been Erwin Stein (1946) who first made the English-language case for Webern as a lyrical composer in his obituary. This was preceded twenty years earlier, however, by none other than Theodor Adorno (1984), in a review of Op. 10. As Sebastian Wedler (2023, 89–91) has pointed out, although Webern was initially sceptical of this description, he quickly came to embrace it and described himself accordingly. 'Lyrical' might appear as something of a

1. Whether Stravinsky actually made this comment, or it was invented by his assistant, Robert Craft, is unclear, but either way its effect has been significant.

provocation, signalling the connoisseur's discernment, but for Stein at least the choice of word carried deliberate and precise meaning. This has been unpacked both by Christopher Wintle (1996) and more recently by Wedler, and I shall return to both authors at the close of this discussion. It is the question of style that inspires my own research: what is particular to Webern, and when it changes. My research considers his corpus of 31 works with Opus numbers and uses statistical analysis of computationally derived data to assess this, with particular regard to his harmonic characteristics. Before considering Webern in any detail, I offer a brief outline of some thoughts on style that will help set up the ensuing work.

This research falls into that wide domain typically called 'style analysis', a field with a long and much-discussed history. A comprehensive survey of the aesthetic literature on musical style is thus far beyond the scope of this mere introduction; however, it will prove useful to consider some of the major contributions to the field in order to set out how I think about 'style', particularly in the light of the novel technologies deployed in this thesis. Unsurprisingly, Charles Rosen features heavily, whose *The Classical Style* (1971) remains one of the most significant interventions on this subject. In particular, Rosen prompts us to consider what repertoire might constitute a style, and what is achieved by an understanding of style. Leonard Meyer's *Style and Music* (1997) is another key contribution to the discussion. His hierarchical approach to style, conceived primarily in terms of choices, encourages a structured way of thinking about style in terms of limitations and options, not dissimilar from the way in which I will conceptualise style as, metaphorically, the bounds on a set of imagined variables. I will then use Glenn McDonald's *Every Noise at Once* (McDonald 2022b) as a case study to assess this empirically inspired model of style. After discussing style in the abstract, I conclude with a brief discussion of stylistic commentary about Webern's music itself. Wintle's (1996) work is useful here at proposing a discussion of style that is much closer to the philosophical than the empirical. Indeed, it is a helpful reminder at the close of this survey that an empirical approach to style is inevitably partial, although it is notable that Wintle deploys formal analysis in support of his broader argument, in an attempt to contextualise and make concrete an otherwise abstract position.

Before examining Rosen more closely, however, some helpful starting points are set out in Robert Pascall's (2001) Grove entry on the subject. In colloquial terms, style is usually seen as a surface matter: the 'manner, mode of expression, type of presentation' (Pascall 2001, para. 2), as Pascall puts it. This assumes a discrete topic that can be clothed in various styles. While this is generally true in literature and arguably so in representational visual art, it is more contentious in the case of music (and indeed Nelson Goodman (1975) has argued that this dichotomy cannot even be assumed in these genres). A basic example of the subject-style division can be given by the reuse of themes, whether musical or extramusical, in different pieces (most obviously in sets of variations on pre-existing themes, but also in programmatic depictions of extramusical topics) but to universalise this distinction to all music is more difficult.

One confident proponent of this distinction is Arnold Schoenberg, who argued forcefully in *Style and Idea* (1950, Ch. 3) that the difference implied by his title is significant. Schoenberg is typically vague about a precise definition of his terms: although at one point he admits that in common practice 'the term idea is used as a synonym for theme, melody, phrase or motive', in fact he considers 'the totality of a piece as the idea' (Schoenberg 1950, 49). In true organicist tradition everything composed comes from and is inherent in the idea, a theme is merely a manifestation of this metaphysical idea.[2] Severine Neff (2021) has outlined the various stages of the idea, from a flash of inspiration 'intuitively perceived but not rationally comprehended' (Neff 2021, 123) into a more concrete guise of pitches and rhythms. Schoenberg goes on to describe the idea as imbued with the 'problem' that the piece first sets out and then aims to solve. As such, the heart of the composition is in the idea: it is this problem that not only inspires but

2. Schoenberg's analytical perspectives, expressed loosely across a handful of articles, change throughout his life and, busy as he was with composing and teaching, are never fully articulated. What follows is a reasonable summary that draws together the various different uses of these terms.

generates and justifies the ensuing composition (for more on this see Neff 1993). Jack Boss (2014) has discussed the musical manifestations of the idea, showing how in tonal music the problem is usually some question about the correct harmonic and or/metric place for elements of the theme, while in the dodecaphonic music Schoenberg sought different problems, often concerning symmetries. Contrasted to the idea, style is instead 'the quality of a work' which he views as being governed by the particular qualities of the individual artist: 'out of such subjectivity grow the traits which comprise the style of the finished product' (Schoenberg 1950, 47). Inevitably these are governed by the cultural context in which the composer is working, but Schoenberg also suggests that these stylistic traits are controlled by the inherent properties of the idea, giving the example of the simple treatment of 'Thème Russe' from Beethoven's second Razumovsky quartet. In Schoenberg's view, in this melody 'there is no problem which suggests development into a theme' (Schoenberg 1950, 197), this therefore dictates the options available to Beethoven, and thus the style. Implicit in his description is a value hierarchy between style and idea: the idea is the most important part of the composition, with the style merely a subsidiary result, of little inspiration to the composer. Despite Schoenberg's confidence, however, the surety of this distinction is not so clear: if subject (idea) governs style, surely the opposite question must be asked, whether style can govern subject? Implicit in the notion of this dichotomy is the assumption of synonymy: that two musical extracts in differing styles can nonetheless express the same subject. Without sidestepping into a discussion of aesthetics, let it merely stand that it is by no means safe to assume the operation of true synonymy in language (Hornischer 2020; Leitgeb 2008; Murphy 2003), and Julian Johnson (1999) has likewise convincingly argued that even when the same topical idea (in his case, 'nature') returns, it not only does so in a different guise, but indeed *as a different idea*. To at least some extent, the presentation of a subject through style will shape its very meaning, and thus the subject. Furthermore, the subject itself, perhaps contrary to Schoenberg's view, is not some abstract notion divinely inspired, but rather the product of that same composer who then shapes it through style. Thus, it seems highly likely that the subject itself will be shaped by those very same subjective traits that Schoenberg relegates to the subsidiary and secondary realm of style. Immanent in the subject, therefore, is the style: the two cannot be so easily separated. What, then, do I mean by 'style'? I will propose an answer towards the end of this chapter that, in simple terms, is not that dissimilar from Schoenberg's definition of style as those traits typical of a body of music. Crucially, however, I view anything Schoenberg might think of as an idea *as part of the style*: themes, melodies, or motives are all in a symbiotic relationship with the composer's style, both governed by it, and contributing to it.

In the introduction to *The Classical Style*, Rosen sets up a spectrum ranging from, at one end, the 'anonymous' style of an era (e.g. 'nineteenth-century French painting'), to, at the other, the individual work. For Rosen, the first of these is so vague as to be unhelpful, and the latter clearly leaves the history of music as simply a list of individual works, with no grouping possible. So far, this presents a reasonable way of considering style. Proceeding, he argues for settling on a compromise of a 'group-style', which he defines as follows: 'a construction that enables us to interpret the change in the musical language without being totally bewildered by the mass of minor composers, many of them very fine, who understood only imperfectly the direction in which they were going, holding on to habits of the past which no longer made complete sense in the new context, experimenting with ideas they had not quite the power to render coherent' (Rosen 1971, 22). Clearly, there is much to assess here, particularly given the very specific ideas about what renders a useful group-style. To consider this more closely, Rosen's basic premise is that analysts need some middle grouping between the extremes of his spectrum. Again, this is certainly correct: the notion of style is useful precisely because it allows us to consider multiple works without being overwhelmed; it is a tool for the analyst to generalise so as to avoid endless restriction to individual works. Indeed, it is important to note that the construction of the group-style may well be devoid of strictly biographical features: from an historical perspective Rosen sees it as a 'fiction' (Rosen 1971, 22), a view endorsed by Ted Underwood, who proposes that 'Genres are clearly historical constructions' (The NovelTM Research Group and Underwood 2016, para. 11). Underwood makes this claim in order to critique a historicist basis for genre classification, a view that would construct groupings according to the descriptions of contemporary commentators in an effort to identify a *Zeitgeist*; however, as genres remain fluid and relations are often tenuous, this historicist approach can often miss important groupings. Further, contemporary commentators often disagree, unsurprisingly, and there are few criteria with which to weigh their assertions. Underwood's response is a quantitative one, Rosen's is certainly not, but both scholars correctly identify that analytically helpful stylistic designations are usually *post-hoc* affairs. The thrust of Rosen's definition, however, goes elsewhere, homing in on the relative importance of composers and their works in the group-style. For him, the crucial contribution of the group-style concept is that it allows the analyst to consider the generalised motion of compositional change, what the broad, longer-term effects were, without being 'bewildered' by the plethora of composers and pieces involved. This is a topic, however, that needs much further discussion: should style be characterised through the most famous composers of a given chronological-aesthetic period, or should it be conceived of as the broader accumulation of the various possibilities being explored by the plethora of composers working in that period?

The very posing of this question implies my answer, that despite Rosen's remarkable knowledge of the repertoire, not to mention his supreme confidence in his own ability to discern the significance of a composer, there are major issues with discounting the relevance of the majority of composers to the musical period in which they composed. There are two domains in which this is problematic, the ethical and the historical. Both stem from a perhaps obvious issue which is the vague process of reception. Indeed, Rosen acknowledges as much when he argues that 'composers are not equal in the sight of posterity' (Rosen 1971, 32).

He fails, however, to take account of this as a process: the path from composition to celebration does not run smoothly in line with musical value or musico-historical significance. From an ethical perspective, reception is clearly shaped by the prevailing discriminatory practices of society, which devalues works by composers from backgrounds marginalised due to gender, sexuality, race, class, or other characteristics. From an historical perspective, we are presented with a deceptive fiction. An instructive comparison is with the work of Justin London, who has developed a corpus (or at least a model for a corpus) of that music 'that we are most likely to hear' (London 2013, 68). London's intention is specifically to develop an understanding of what the twenty-first-century listener considers 'Classical Music', rather than what this music might actually be. For this, he draws on a variety of sources derived from textbook examples and performance and recording data, and discusses the uneven representation of certain composers at length. By contrast, *The Classical Style* presents us with a sleight of hand, describing a twentieth-century conception of 'The Classical Style', not the style as it was instantiated in the late eighteenth century.

Furthermore, Rosen explicitly argues that the relevance of a composer to a style, and indeed the point of the style itself, is to help the analyst locate 'the direction in which they were going' (Rosen 1971, 22). This presents an image bordering on the farcical of the pantheon of composers all taking part in some totalising pilgrimage in which a Walpurgis might initially move with the pack in the correct direction, but then—alas!—unfortunately she puts a foot wrong and ends up separated from the group in some abandoned cul-de-sac, consigned to the irrelevance of suburbia. Nevertheless, no need to panic! Fortunately, a Beethoven or a Schubert was more careful, and has guided the party to the intended location. The foundational issue with this approach is the totalising idea of a single musico-historical progression, which relegates alternatives to a status of mere digression. If we follow this, we are left with a situation in which we analyse the music of the 'major' composers because they shape the direction of music against the broader mass of minor composers, but in so doing we fail to comprehend the situation of this mass, and so we are left with a context-less direction, defined only in terms of the passage of these individuals. To extend the geographic metaphor, it is as if we are given a compass direction but no coordinates. This is obviously a caricature, but a revealing one. Rosen is probably right to argue that it is 'that which is most exceptional, not what is most usual, has often the greatest claim on our interest' (Rosen 1971, 22)—with the appropriate caveats about the vagaries of reception—but it is not these exceptions which define a style, even though they might help us clarify it by their very difference. Indeed, the double-meaning of 'exceptional' is interesting here: strictly meaning an exception to the norm, it has come to imply value merely by dint of atypicality. Nonetheless, this *does not* guarantee historical value, and constructing our history of music on such partial information is unrepresentative and inaccurate. Whilst I appreciate Rosen's abstraction of the 'Classical' style, what his book really hinges on is three composers. *The Style of Mozart, Haydn, and Beethoven*, as his subtitle suggests, might be a bit less catchy, but it would be far more accurate. He suggests that 'the style of any age is determined not only by what is done but by the prestige and influence of what is done' (Rosen 1971, 32); I suggest that without a strong empirical understanding of the period in question, we fail to comprehend even what was done, rendering questions of prestige and influence far beyond the knowable scope.

In the case of Webern, this means developing a conception of his style based on *all* the music rather than on a selection of works—the 'Webern canon', as Anne C. Shreffler (1994a, 4) has put it. Past analytical scrutiny has been very unevenly focussed. Shreffler notes this with regard to Zoltan Roman's Webern bibliography, but Darin Hoskisson's (2017) bibliography of post-1975 publications is helpful here too as a crude measure of scholarly interest in different works. The key players are, inevitably, obvious: the *Fünf Stücke*, Op. 5 (25 publications concerned with this piece or part of it); the *Symphonie*, Op. 21 (33 publications); the *Konzert*, Op. 24 (34); the *Bagatellen*, Op. 9 (44); and the winner, the *Variationen*, Op. 27 (62). By contrast, none of the Lieder make it into double digits, and indeed the only vocal music with more than 5 publications is Op. 3. The central run of songs from Op. 12–Op. 18, those works that Alexander Goehr professes to 'love particularly' (Goehr and Wintle 1992, 167), only inspire 21 publications between them. This is a simplistic measure, but it reveals an important truth: that we have a concept of Webern's style that is constructed from a small subset of his works, rather than his actual output *in toto*. In part this reflects the classic hierarchy of genres, privileging the instrumental above the vocal, though it is also influenced by particulars of Webern's reception history. Martin Iddon (Iddon 2013), for example, has charted the fortuitous route by which Karlheinz Stockhausen ended up presenting on the *Konzert* at Darmstadt in 1953, an evening that clearly had a profound impact on many of the composers of that generation. As for the Lieder themselves, they are hardly helped by what Wintle describes as their 'sheer difficulty' (Wintle 1996, 263). As he points out, with few recordings and even fewer concert performances, it is unsurprising that this music has remained little-known, especially compared to works like Op. 5 and Op. 9 which have almost become standard repertoire at this stage. Again, Rosen (himself one of the few interpreters of these songs!) might argue that their limited reception precisely makes the case for their exclusion from a stylistic study of Webern: if nobody knows the pieces, then necessarily they can have had little influence. Nonetheless, this simply fails to provide an adequate picture: not only do we have an inaccurate sense of Webern's music in its actuality, but even those works we consider to be exceptional are proclaimed as such with no justifiable basis for that position.

Rosen's work provocatively encourages us to consider the repertoire that might constitute a style and what that style might tell us; Meyer's *Style and Music* is more valuable for how we think in practical terms about what constitutes a style. He commences with a definition. An unceremonious start it may be, but, like much of his ensuing discussion, its clarity is welcome.

> *Style is a replication of patterning, whether in human behavior or in the artifacts produced by human behavior, that results from a series of choices made within some set of constraints* (Meyer 1997, 3, italics his).

The crux of Meyer's approach is in that last phrase: 'a series of choices'. Indeed, he goes on to establish a carefully delineated hierarchy of levels at which different choices are made, ranging from transcultural laws at the broadest level through rules down to intraopus strategies at the most restricted. For Meyer, the compositional process is therefore, in some sense, a series of choices across these levels. Of course, the choices a composer makes are often not actively conscious, but may instead, taking a structuralist perspective, be the product of various external or culturally conditioned influences. Nonetheless Meyer holds that a composer has responsibility even for choices made in a manner that may be considered almost unconscious. The implication of his focus on choice is 'the existence of alternatives' (Meyer 1997, 6 n. 4). Each choice in the compositional process is made in the face of alternative possibilities, and crucially the range of these possibilities is governed by the style. Or to reverse it, the style is understood as the extent of these possibilities. If a composer deviates too far from the possible set of alternatives, what Meyer terms the 'standard practice' (Meyer 1997, 7), then they create a new alternative (typically a strategy but sometimes a rule). Typically, however, the listener or analyst is able, due to an affinity with the style, to imagine those alternatives: with the inevitable Frost reference, Meyer proposes that 'the road actually taken is invariably understood partly in terms of those not taken' (Meyer 1997, 32). This is a similar conception of the composer as is proposed by James Hepokoski and Warren Darcy (2006), who suggest that 'a composer was faced with an array of common types of continuation-choices established by the limits of "expected" architecture' (Hepokoski and Darcy 2005, 9): again we have the idea of the composer navigating various possible paths within a larger space laid out by stylistic constraints. As with Meyer, these authors are careful to note that 'This is not to say that any skilled composer soberly pondered these choices' (Hepokoski and Darcy 2006, 9) which is, I think, an important caveat. The proposal in both cases, instead, is that the composers were enculturated, particularly through statistical learning (Daikoku 2019; White 2022, 25–28), and so learnt the options available to them. Indeed, even Schoenberg shares a similar view, suggesting that 'The positive and negative rules', i.e. the stylistic constraints, 'may be deduced from a finished work as constituents of its style' (Schoenberg 1950, 47).

I propose, therefore, a conceptualisation of style inspired by empirics and modelled on the interaction of variables. A piece can be described in terms of a set of n variables, potentially infinite in number. These can then be plotted in n-dimensional space, i.e. a dimension for each variable, (inevitably this must be to some extent imaginary as the number of dimensions will quickly outstrip our capacity for visualisation). Each piece is therefore represented by a point in this n-dimensional space, and the similarity between pieces is therefore described by the difference between these points. A style can thus be defined in terms of a group of these pieces that are located in close proximity to each other, and indeed would encompass the spaces between them (even if no piece, to our knowledge, exists in that space). To maintain an empirical approach, this grouping should be carried out according to some clustering algorithm (Müllensiefen and Frieler 2022, paras. 40–48). This may take account of all dimensions or may be restricted to a subset of dimensions (though this restriction must be made explicit *ex ante*), and the analyst may choose to weight the different variables to prioritise certain features. Following from this, we can describe Meyer's set of possible choices as the range of values a piece might adopt in a given dimension whilst remaining in the bounds of that style. This is a model of style reached also by Christopher William White (2022, 61–70), who introduces two descriptive variables to characterise these different styles: uniqueness and coherence. Uniqueness is an inter-style property which describes how different the various styles are from each other: whether the different styles all use, for example, the same harmonic vocabulary. Coherence is an intra-style property which describes how different the constituent parts of a style are from each other: whether the different pieces in a stylistic group all use the same harmonic vocabulary.

To take a trivial example, we can consider the use of singers in Mahler's ten symphonies. Figure 1.1 organises these according to two variables: the proportion of movements using a vocal soloist, and the proportion of movements using a chorus. An informal clustering approach discerns three clusters, which we might think of as stylistic groups: the instrumental symphonies (1, 5, 6, 7, 9, 10); a single truly vocal symphony (8); and a hybrid group (2, 3, 4). To use Meyer's terminology, we might therefore think of Mahler as having three strategic possibilities that constitute different aspects of his symphonic style, each bounded by the set of possibilities encompassed by the red rings of Figure 1.1. In White's terms, the instrumental symphonies and vocal symphony styles are wholly coherent while the hybrid group is not; meanwhile, all three groups are unique. Of course, using only these two very basic variables provides a simplistic perspective. It could well be argued that this is a subsidiary matter of genre rather than style; certainly, this is only a very crude approach. However, even this simple example reveals that an utterly crucial, and likely infinite, task is the choice of these variables. Different analysts might focus on radically different features, and might ascribe to them different weights (i.e. significance). That seems to me not to be a problem: there always has been and always will be a degree of subjectivity in any analysis, and this is merely another iteration of the same. What for one analyst might be determinative would, for another, be irrelevant. Indeed, Timothy Burke (2016) specifically makes the case that a humanistic perspective is eternally incomplete because its epistemology is dependent on the analyst's interpretation. While the presence of numbers can suggest a patina of objective fact, the reality is that not only do the ensuing interpretations remain incomplete, but in fact the very process of measurement and data collection will always be partial. As an example, a significant question here is whether extrinsic variables are permitted. Chronology would be an obvious and easily quantifiable variable. In another important book on eighteenth-century style, Leonard Ratner (1980) uses nationality. While national identity is fraught with complexity in this period, place of composition is a more feasible variable (though not without its own problems as composers often move around during the compositional process). These facts are not, in some sense, intrinsic to the music (this does not claim Bartók's *Concerto for Orchestra*, say, to be American based on some aesthetic quality), and yet may well have stylistic relevance. It is also true that this model can allow for a structural hierarchy of styles: Pascall points out that 'A style may be seen as a synthesis of other styles' (Pascall 2001, para. 2), and indeed it is easy to imagine nested and overlapping shapes which would communicate this phenomenon. Paul Kincaid (2003) has discussed a model of genre inspired by Wittgenstein's idea of family resemblance: that rather than single determining features, membership of a generic group is instantiated by the interaction of a combination of variables. One of the advantages of this is that it eschews setting boundaries based on categorical rules, but instead, as a 'bottom-up' approach, remains flexible to the changing contributions of new items. In the context of my example, a clustering algorithm does not pre-define the boundaries of each group and then count those items within it, but rather it starts from the items themselves and then makes groupings dependent on their similarity. Indeed, in his analysis of novelistic genres Underwood (2016) has persuasively argued that quantitative analysis, and in his case specifically predictive modelling, is particularly attuned to this model of genre (or in my case, style) grouping as it so easily takes account of multiple variables, rather than dogmatically prioritising individual features.

At this stage I should acknowledge that to some extent this is clearly hypothetical. We will never be able to codify the total set of variables required to define style, and even if we were there is no objective way to mediate the significance ascribed to the competing variables. Nonetheless, I think this is a helpful model for introducing the work that follows. In a sense, my research seeks to identify variables, apply them to Webern's music, and ask what they tell us about his style. I make no claims, therefore, that the following chapters provide a comprehensive account; rather, they provide several different lenses through which we can

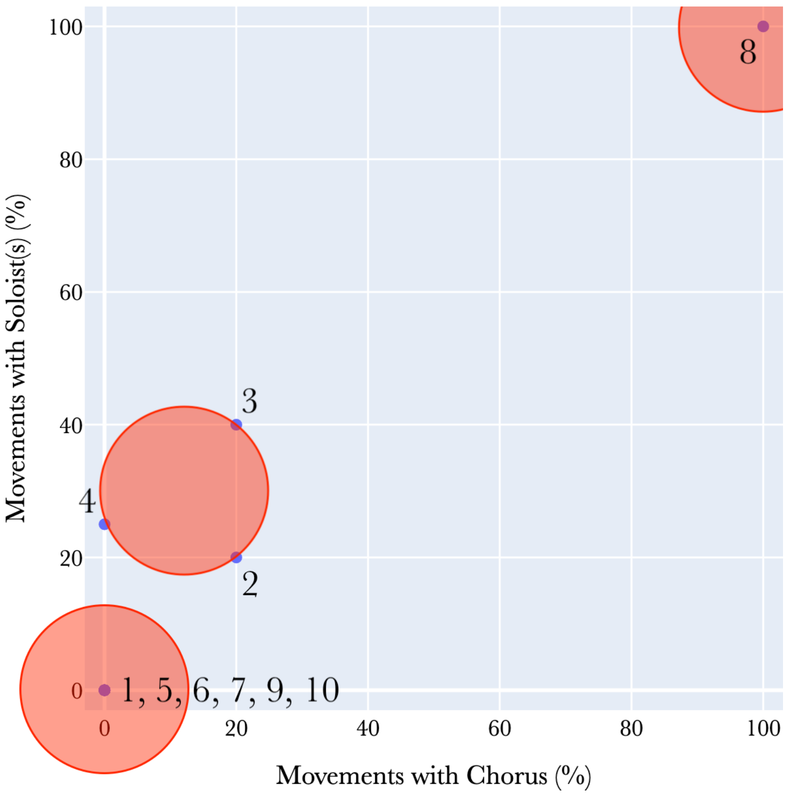

Figure 1.1: Mahler symphonies organised by use of voices.

consider the corpus. My focus is on deploying intra-musical variables, though these are certainly complemented by extra-musical ones (chronology being the most significant).[3] Crucial for my work is that they are all strictly quantifiable, and so the similarity between different pieces can be measured numerically. This is not wholly necessary for this model of style, however. It is merely required that pieces can be placed on an axis which, while most obviously easy with quantitative data, can also be achieved for categorical data, although it will affect the types of similarity measure used (Müllensiefen and Frieler 2022, para. 43).

In the example given above, a piece's stylistic designation is binary: either a piece sits within a stylistic group, or it does not. This therefore does not allow for any ambiguous or transitory designations with regard to a stylistic group howsoever

3. Drawing a line between intra-musical and extra-musical features is traditionally a thorny issue, but for the sake of this discussion and my own research I take a pragmatic approach: any variable derived directly from the source material (in my case, the musical scores) is treated as intra-musical; any variable derived from the metadata, or some other source of secondary material is treated as extra-musical.

defined, which may be seen as a major weakness: after all, Pascall rightly suggests that styles are typically 'in a constant state of flux' (Pascall 2001, para. 17). There are two brief counters to this: firstly, that stylistic groups could be defined so as to take account precisely of whatever features are in transition and thus locate a distinct 'transitional' group; and secondly that different analysts might use different levels of granular detail to highlight these transitions by their difference. To reuse the Mahler example from above, whilst one analyst might simply find two groups, vocal (2, 3, 4, 8) and instrumental (1, 5, 6, 7, 8, 10), another might distinguish 4 from 2 and 3 on account of the lack of chorus: comparing the fine-grained from the coarse-grained precisely reveals those middling works. Further, providing a measure of similarity (point-to-point proximity) might be able to give a more sophisticated indication of those works that are stylistically ambiguous as the analyst can quantify precisely which works are most unusual in a stylistic grouping (in simple terms, these are the pieces furthest from the graphical centre of the group). Neither of these arguments is particularly convincing, but that is no matter because the example given above is slightly misleading. This approach to style consists of two parts: the plotting of pieces as coordinates in space, and the grouping of these coordinates to designate stylistic groups.

Above, I clustered the works informally in a manner loosely modelled on the well-known K-Means Clustering approach. This is an intuitive clustering approach which gives an easily interpretable result, and so provides a helpful first example. Other clustering techniques could be used, however, which would give a much clearer sense of gradual change and therefore the transition between different styles. Hierarchical Agglomerative Clustering is probably the most obvious of these, the results of which are shown in Figure 1.2.[4] This dendrogram has the advantage of showing a comprehensive hierarchy of stylistic groups, and thus the sub-styles. Rather than deciding for the analyst that Symphonies 2, 3, and 4 have to be grouped together, this visualisation shows that even if they are

4. Average Linkage was used to calculate the clusters.

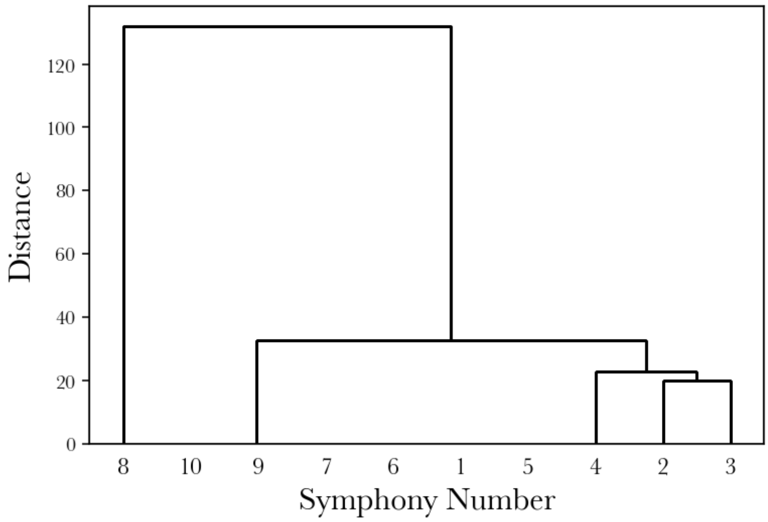

Figure 1.2: Mahler symphonies grouped with agglomerative hierarchical clustering.

grouped as one unit, this comprises two sub-units, 2 and 3, and 4. Similarly, this clustering approach suggests that the two principal larger units are not vocal and instrumental, but rather Symphony 8 and the rest, supporting an interpretation of the *Symphony of a Thousand* as closer to a cantata than a symphony. Crucially, the interpretative choices are left up to the analyst, and in this method they are able to decide for themselves how many clusters are appropriate and explore the multiple structural levels of the groupings. Thus, this rejects the binarism of the example offered above and fully embraces the transitions between different styles.

On a more abstract level, Underwood and Richard Jean So (2021) have explored the very metaphor of space as an analogue for cultural similarity: they make the point that imagining cultural items as points in space (not dissimilar to the Bourdieuan *champ*) implies certain relationships between the items, perhaps most crucially that of distance as indicative of similarity. They question whether this is a viable analogy for cultural items, but although they outline the limitations imposed by this model, when they compare distance to other non-spatial measures of difference, they find that it performs no worse than alternatives. Furthermore, while some might argue that it imposes an inaccurate way of thinking about cultural relationships, as the authors point out 'all models are simplifications' (Underwood and So 2021, 49) and so to suggest that this approach is problematically reductive is unfair. Indeed, spatial models have a long history in music analysis prior to the onset of the digital humanities from the circle of fifths to the *Tonnetz* (for extensive treatment of this idea see Tymoczko 2011). While any analyst has to be alive to the limitations of such a representation, it remains a useful way of thinking.

A pre-existing tool from the commercial world that serves as a helpful illustration of a similar idea is McDonald's Every Noise at Once (McDonald 2022b).[5] He describes the tool as:

> an ongoing attempt at an algorithmically-generated, readability-adjusted scatter-plot of the musical genre-space, based on data tracked and analyzed for 5,963 genre-shaped distinctions by Spotify (McDonald 2022c).

In short, the result is a two-dimensional scatterplot that attempts to chart the similarities between different genres based on ongoing algorithmic analysis.[6] McDonald is relatively opaque about precisely how the algorithms work or indeed what data inputs the algorithms are using to make these genre classifications (the details are presumably proprietary). Tom Vanderbilt suggests that decision are made 'not by what the artists sound like, but how they are described in the world: on music blogs; in record-company catalogues; on your cousin's Facebook page – anything The Echo Nest's semantic trawl pulls up' (Vanderbilt 2013, para. 5). The article includes quotations from McDonald and various high-level employees at The Echo Nest (the company where McDonald

5. A Github repository that enables analysis of the genre information can be found at https://github.com/AyrtonB/EveryNoise-Watch.

6. McDonald uses the colloquial 'genre' rather than 'style'. Although 'genre' has other connotations, such as the importance of the genre title and the communicative wish of the composer, in this context 'genre' and 'style' are similar, with the primary exception that 'genre' seems to imply that the lowest possible unit of organisation is the artist, rather than the work. I will therefore use 'genre' when referring to McDonald's work. In a similar discussion, White (2022, 56) has distinguished genre from style by characterising the former as concerned with types of work (string quartet, symphony, sonata) whereas the latter involves other types of contextual critera (e.g. date, gender, nationality).

originally developed this map), which suggests accurate sourcing, and indeed much of their explanation of their work is concerned with the difficulties of grouping genres merely based on acoustic phenomena, which lends support to Vanderbilt's description. Elsewhere, however, McDonald has suggested that the map uses everything 'from parsing the contents of scraped web pages to analyzing the psycho-acoustic properties of the actual audio files' (Schroeder 2014, para. 3), which implies that intra-musical features have at least some role to play. This appears to provide a good example, then, of a classification tool that uses mixed data inputs for generic sorting.

McDonald claimed in 2013 that the tool uses ten internal dimensions and two independent measures of genre similarity (2013, para. 30), though the map only operates in two dimensions, with an implied, though undiscussed, third dimension communicated by text-colour, so presumably some form of Feature Aggregation (Müllensiefen and Frieler 2022, paras. 19–24) is used to compress these dimensions. The result is that the implied x-axis of the scatter-plot is organised along a spectrum from 'left is denser and more atmospheric, right is spikier and bouncier' while on the y-axis 'down is more organic, up is more mechanical and electric' (McDonald 2022c). Quite what McDonald means by 'organic' is unclear. In 2013 he described the lower y-axis values as sounding 'more acoustic' (2013, para. 29), so whether 'organic' is meant to be synonymous with 'acoustic', or to connote a more romantic feeling of humanity and/or 'natural-ness', in contrast to the artificial mechanical sounds otherwise described, is unclear. Certainly the bottom of the scatter-plot does seem to feature a preponderance of genres based on acoustic instrumentation, from 'classical piano' genres ascribed to certain nations or national groups[7] to 'jewish cantorial' and merely 'guqin' and 'koto'. As a small descriptive critique, using colour for this third genre relegates it to a subsidiary status, as it is harder to link genres precisely based only on their colour match. The tool would probably be

7. In ascending order the lowest seven are: polish, baltic, czech, russian, belgian, swiss, romanian (genre names are all given in lower-case).

improved by a re-mapping in three visual dimensions, unless the third dimension is intended to be of lesser importance.

One can find oddities (I am cautious to describe these as downright errors) in the classification, as two small observations show (correct at time of writing). The choir 'Polyphony' appears in the 'cambridge choir' genre (McDonald 2022a). While this is to some extent understandable—the choir is directed by Stephen Layton, also Director of Music at Trinity College Cambridge, and sing similar repertoire to many choirs based in Cambridge—they are not a choir from Cambridge in the manner of the other ensembles listed in the genre, all of which are based in the city (whether affiliated with the University or not).[8] Although this reveals something interesting about the identity of this ensemble (again, quite *what* it reveals is impossible to know without transparent access to the data prompting this classification) it perhaps leaves a question mark hanging over what should be included in this genre. A further odd result follows from McDonald's description that, roughly speaking, 'down is organic, up is more mechanical and electric' (McDonald 2022c), and yet 'musique concrete [sic.]' appears in the lowest 20% of the map. Assuming a discussion-based sorting method, this can perhaps be explained by assuming that much discussion of musique concrète takes place in the context of mid-twentieth-century Avant Garde music, which was still a predominantly acoustic genre. Frankly more odd is the cluster of proximate genres: 'shehnai', 'rabindra sangeet', and perhaps most strangely, 'salon music', although when the genre-space is re-sorted with 'musique concrete' at its centre it is surrounded by 'electroacoustic composition', 'acousmatic', 'free improvisation', and 'austrian contemporary classical', a rather more convincing list.

At its heart, *Every Noise at Once* is intended to be a recommendation service and coming out of a commercial context it unsurprisingly lacks much of the transparency in explanation and documentation that academics desire (by way of comparison, Born and Haworth 2017, is a much more conscientious, though

8. The date given here is 20th September 2022 as this was the most recent (published) update to the map when I consulted it.

technologically less sophisticated and ultimately less impressive genre study that uses the Internet Crawler to trace similar networks; Seaver 2022, is an anthropological study of the people and the companies that create these sorts of recommendation services, focussed on the other side of the coin from style, taste). More importantly, although a by-product of using the tool is to notice features that genres have in common, it is designed to encourage exploratory listening rather than knowledge about genres. The poor definition of the axes is just one example of this, which outsiders could improve were it not for the undefined data inputs. More widely, there are ethical quandaries surrounding the website: its universalising title belies its dependence on Spotify data, which means not only that it will emphasise the listening habits of Spotify users (an impressive 433 million people, but hardly a representative sample of global music habits (A. and Vogel 2022, 29)), but also that it channels users towards a company widely seen as profiting from the exploitation of musicians and other content-producers. Furthermore, while Spotify has a sizeable collection of music, it is by no means complete or representative. Nonetheless, it provides a thought-provoking and impressive example of the way that multivariate analysis can be used to group music.

I began with reference to Stein's description of Webern as a lyrical composer, and I will conclude with a brief discussion of the topic. As I mentioned above, in the popular press I suspect that this is sometimes meant as a provocation, showing off the specialist's affinity for music that might appear to defy the easy melodies typically associated with the word. More seriously, however, Wintle has explored Stein's terminology, drawing on Hegel and Dahlhaus to contextualise the lyrical in terms of its dialectical partner, the dramatic. While lyrical music is concerned with itself, 'unmodified by awareness of an attentive other', dramatic works 'address the problems posed by the presence of "another" (the psychoanalytic "object")' (Wintle 1996, 232). Wintle traces this description of Webern as a composer of the lyrical character through various other commentators including Hans Keller (1974, 13) and Alexander Goehr (1992, 167), concluding that this feature of Webern's personality was in fact 'the heart of his creative enterprise' (Wintle 1996, 235). Shreffler (1994a) makes a similar point (after all, her book is titled *Webern and the Lyric impulse*), strangely unacknowledged by Wintle. In her case the term lyric seems more to relate to Webern's interest in writing vocal music, although she does make some reference to lyric poetry and indeed to Webern's reference to his songs as *Gedichte* which identifies the music with poetry. Adorno, likewise unacknowledged by Wintle, also describes Webern in these terms, arguing that his music is defined by 'absolute lyricism: the attempt to resolve all musical materiality, all the objective elements of musical form, into the pure sonority of the subject, without an alien remainder that refuses to be assimilated' (Adorno 1999, 92–93). In Adorno's telling, Webern focussed on this through the ability of music to express not only extremes of experience, but to move towards 'a mysterious dimension of an endlessly questioning contemplativeness' (Adorno 1999, 93). In concrete terms, or as close as Adorno gets to them, this is represented by the fixation on ever increasing concentration, generating ever greater intensification. Thus, the avoidance of any risk that development might weaken a moment of pure expression, defined fundamentally by its immediacy. As is typical of Adorno's approach there is little detailed analytical writing about the music, and as ever his more specific comments about the music focus in on a smattering of selected pieces, which make his arguments difficult to adjudicate from an empirical perspective.

Returning to Wintle, he does make some attempt to pin this discussion on something tangible: following Stein he associates the lyrical, at least as it is manifested in Webern, with wide melodic intervals, extremity, and a fascination with especially Austrian landscape and nature. He goes on to explore the implications of this perspective in an analysis of Op. 25/iii, identifying in Webern's personality an interest in synthesising clearly defined opposites. In this song, Wintle describes these as '*developmental lyricism*' and '*declamatory espressivo*' (Wintle 1996, 261), but he extends this to the fusion of symphonism and polyphony in Op. 21, and homophonic Lied with contrapuntal accompaniment in the mid-period songs. Rather than accept Keller's description of Webern as a 'stater and varier rather than a contraster and developer' (Keller 1974, 13), Wintle proposes that Webern is more interested in synthesising variation and development than choosing one or the other.

This is an interesting and rich discussion of this facet of Webern's style, though it is typically guilty of the representation charge laid out above. On his own, Wintle considers only one movement by Webern. Though he makes glancing reference to Goehr's comments about Op. 21 and Arnold Whittall's (1996) analysis of Op. 30, this is a universal proposal about Webern's style founded on unsubstantiated aesthetic convictions, and the analysis of 78 bars of music. On the whole, Wintle makes a convincing case that this movement displays the features he ascribes to Webern generally. His analysis of the vocal line makes the point about wide intervals and excess, while the naturalistic text fulfils that requirement, though I might suggest that the theoretical position of the lyric that he lays out concerns more than big leaps and text about nature, hardly determinative in early-twentieth-century Austria. Whether this song is representative, or merely an unusually good example, we simply cannot adjudicate. Nonetheless, there are important points here, and both Adorno and Wintle's analyses serve as useful reminders that empirical analysis does not hold all the answers (more on this later), although it certainly has more to contribute than it has thus far.

I also mentioned Wedler's (2023) study of Webern and lyricism, and before proceeding further this certainly merits discussion. Wedler's contention is that there is a plethora of aesthetic features in Webern's music that all reflect his stance as a lyricist, but further, that this operates on a higher structural plane than merely the formal content of his music. Thus, Wedler considers Webern's analysis, of his own works and others', his politics, and his self-description, as well as more traditionally formal choices: the importance of Lieder, the new status given to silence, and even the interval vector content of his music. All of these, in Wedler's reading, display Webern's lyricism. As for the lyric itself, although

Wedler describes it as 'notoriously difficult to define' (2023, 87), frequent gestures to Adorno and indeed to Hegel suggest that this is the underlying model to which Wedler subscribes. From an historical perspective, one of Wedler's most interesting contributions is to trace the chronological coincidence of Webern's self-description as a lyrical composer with his new embrace of the dodecaphonic technique, itself coinciding with the 1921 publication deal with Universal Edition that motivated Webern to organise many of his earlier works for publication. Meanwhile, when Wedler actually digs in to Webern's theoretical and philosophical writings, though the lyric is not featured by name, he identifies a perspectival focus that sets up an understanding of lyricism as 'variations of the same' (2023, 93). The variation form was a famous focus in Webern's dodecaphonic period: as Kathryn Bailey (1991) has charted, a third of the instrumental dodecaphonic works are cast in an explicit variation form. For Wedler, this is not restricted to the macro formal level, but it also permeates more local concerns. In a sketch for a string trio, Wedler charts how over the opening two bars Webern introduces all twelve pitch classes (pc), in the process setting up a dialectic between two set-classes, 4-1 and 4-6, which maximise opposite interval classes (ic) (respectively the semitone and the tritone). This thus establishes an ic landscape through which Webern can then move in later harmonic variations.

An alternative hermeneutical perspective on Webern's corpus comes from Johnson (1999), who argues that practically Webern's entire output is animated by an 'idea of nature', although what this idea constitutes and how it is manifested in his music changes across the course of Webern's life. Johnson does an impressive job of covering a wide swathe of the music, with glancing comments on all the works with Opus numbers, and some of the uncategorised works too, complemented by the usual close analysis of selected examples. As such, these are not comprehensive analyses of any of the works, or indeed of the corpus in its entirety; rather, they focus on highlighting a particular facet of the pieces. Subdivisions of the corpus are therefore also animated by this very partial perspective, which, in short, changes 'from representation to abstraction' (Johnson 1999, 152) across the corpus. Inevitably these often line up with otherwise-justified groupings, but similarity or difference is always framed in terms of the analytical method of the moment and its implications for the manifestation of nature.

A truly 'comprehensive' analysis is probably impossible, and I would certainly not claim such status for my own research; rather the principal difference between Johnson's work and my own therefore concerns analytical methodology, and indeed method. His approach is inspired by a hermeneutical concept, the idea of nature, which he then seeks to trace across the body of work, deploying whichever analytical method is most convincing at a given time. With regard to the early music (*Im Sommerwind*, the String Quartet, and the early songs), for example, he identifies four relevant musical fingerprints in the manner of an Agawu-esque topical analysis and outlines their relevance to the repertoire; by contrast, the discussion of the middle-period Lieder, and especially Opp. 12–14 is almost entirely a discussion of text. My own research, by contrast, very consciously retains a consistency of method across the entire corpus in order to trace changes in formal style, leaving interpretation as a principally secondary matter (chronologically, not in terms of significance). It is certainly arguable that there is a weakness here, that different musics demand different analytical methods and it is inappropriate to apply the same set of tools to such a changeable repertoire as Webern's. Indeed, there are plenty of examples of the unsuitable application of analytical tools to certain repertoire (and not just the endless attempts to 'Schenker-ise' everything!). That said, there is clearly a common creative thread that runs through Webern's music, unified by his creative personality. Though this was clearly a changeable entity, this creative consistency justifies the re-application of the same analytical tools, which of course are advantageous in seeking a diachronic perspective on the repertoire. The obvious criticism of Johnson's approach is that the end is not so much justified by the means as dictating them: rather than the evidence of the analytical method proving or disproving a hypothesis, the method is cherry-picked in order to back up the hypothesis most convincingly.

To conclude, then, I offer a brief survey of the ensuing research. My own work applies my model of style to the music of Anton Webern in the manner of a computational corpus study focussing on harmony. The primary research question that lies at the heart of this project is as follows: What are the long-term tendencies and practices that define Webern's music? As a crucially related question: How did his compositional practice change? What trends can we observe? By considering the entire corpus, this will also identify <u>anomalous</u> pieces, allowing the interrogation of what is typical or unusual in Webern's practice. The corpus will be primarily partitioned by chronology, but other variables are relevant too, such as genre and instrumentation. Bailey, for example, segments the instrumental from the vocal music (1991). Drawing attention to anomalies also shifts the focus from the quantitative to the qualitative: whilst the data-based side of this project is able to generate a baseline from which we can identify outliers, understanding them requires a more qualitative approach, informed but not dominated by the data. This is often described this as 'last-mile analysis': the data tells us where the story is, it's up to us to figure out what's happening on the ground.

Across the course of the following chapters I use various approaches to consider different aspects of Webern's music, and in so doing I seek to provide a multi-faceted perspective on this repertoire. Each time, I apply the analytical tool in question to all 31 works with opus numbers, the corpus in this project, thus providing a contextualising and long-term view on the entire repertoire. The analytical tools I deploy focus exclusively on harmony and matters of pitch. Whether this is the most defining feature of Webern's music is certainly debateable: I imagine many would make claims either for texture or duration, or perhaps rhythm, and I certainly do not wish to imply that this is necessarily at the top of the hierarchy. Indeed, if the discussion above says anything, it is that style is crucially dependent on the pluralistic interaction of multiple variables, rather than reducible to singular characteristics. The decision to focus on harmony thus comes from several motivations. If not necessarily the most important, it nonetheless is crucially important: pitch has typically won out in the hierarchy of parameters in the classical tradition, and developments in harmony were evidently of great importance to Webern and his contemporaries. What is more, even from a superficial perspective there is clearly a huge variety in harmonic matters across Webern's corpus, and with this a correspondingly large amount of scholarly material with which to converse. Finally, from a computational perspective, it is also comparatively stable and quantifiable. My research uses Webern's scores as the basis for the analysis and deriving data about harmonic patterns is viable in a way that, say, textural analysis would not be.

In my research, I use these different analytical approaches like a series of lenses, gradually changing perspective on the same body of work. I open with a birds-eye view, using pitch-class distributions to assess the overall harmonic character of the works in the corpus, and deploying the Discrete Fourier Transform (DFT) to quantify the harmonic colour that these distributions generate. Next, I take a step closer in to consider interval distributions, a common topic of discussion in the analysis of Webern's music. I question whether his music reveals a preference for a particular ic, explore the structural implications of this, and interrogate the 'Frozen Interval' hypothesis across the corpus as a whole. Finally, I turn to the small-scale harmonies Webern employs and again ask what his preferences were, and how local harmonies were used structurally. I take up the Viennese Trichord as a particular harmony to focus on, and again use the Discrete Fourier Transform to assess the harmonic colour that Webern's choices generate. Throughout all this I adopt a diachronic attitude: evidently Webern's style was not a static entity, but in fact one of the major points of interest is how and when it changed, and a major part of this research is tracking that process, particularly in comparison to the conventional tripartite early-middle-late paradigm that influences so much historical narrative.

A final important disclaimer to offer surrounds the nature of this analysis as strictly formalist. Analysts have argued vociferously over the implications and value of strict formal analysis, an argument I will touch on in the following chapter. At this stage, it is merely worth stating that I make no totalising claims about the possible perception of these effects. While I will at times speculate on the significance to the listener of various of the features I discuss, I do not claim that a listener, however ideal, could necessarily track, for example, a pitch-class distribution across the course of a ten-minute work. As it happens, I suspect that the brevity of Webern's music actually makes many of these features more readily perceptible than in the longer pieces of his contemporaries, but this remains a suspicion. My work seeks primarily to identify what is present in the music.

Chapter 2

Literature Review

2.1 Introduction

The music of Anton Webern has received extensive analytical scrutiny: as a favourite of the Darmstadt generation he was quickly received into the canon and has inspired a quantity of discussion that dwarfs the brevity of his music. The purpose of this chapter is twofold: to summarise some of those well-trodden paths, and in so doing, shine a light upon those regions that remain unexplored, the subject of my own research. As such, this research draws on traditional formal analysis and the digital humanities. A comprehensive survey of either is clearly unfeasible; rather, the following chapter will draw attention to some important debates and situate this project in relation to other authors, in the process teasing open the gaps from which my research emerges. It is notable that little formalised work already exists in this particular intersection. Allen Forte's analysis was aided by computer, but computational elements are not explicitly discussed (Forte 1973; for a broader discussion of the impact of the computer see Schuijer 2008, 236–50). Conversely, although Anthony Pople's Tonalities project would have operated in approximately this region, his untimely death largely ended it (though the software is made accessible by Huddersfield University, little seems to have been done with it, beyond these introductory articles: Cross and Russ 2004;

Pople 2004, 2002; Russ 2004). Other authors have carried out limited research, but the scope of their analyses are very constrained (Eimert 1959; Fuller 1970).

2.2 Webern Studies

Naturally, the principal body of literature relevant to my research is the analysis of Webern's music, though approaches applied to Second Viennese School music are also crucial. Few authors have considered large swathes of Webern's music from an analytical perspective, as this project will. The three principal exceptions are Forte's book on Webern, which deals with Opp. 3–16 (1998); Bailey's (1991) study of Opp. 17–31; and Johnson's (1999) hermeneutical analysis which, as outlined in the previous chapter, tracks his thesis across the full corpus. The other major contribution is Shreffler's (1994a) book which considers in detail only Opp. 12–14, though has implications for the surrounding works. These books notwithstanding, much of the literature exists as isolated articles considering particular movements or pieces. Given the concentration of Webern's music, this is hardly surprising, but it makes for a noticeable dearth of discussion of his long-term tendencies. Crucially, hardly any work bridges the divide between the pre-serial and serial music. This will be a particular focus of my research, as I seek to consider Webern's output as one corpus. The work that does exist is largely from the last century; formal analysis of this music seems to have been somewhat out of fashion in recent years. This is a great shame: in reviewing Bailey's book in 1994, Douglas Jarman suggested that Webern studies were 'poised on the threshold of what promises to be a particularly productive period' (Jarman 1994, 314). Sadly, this forecast has not been borne out, and with the exception of some doctoral theses, there has been little sustained work bridging the divides between the conventional periodisations of Webern's corpus (Wedler 2017).

There is one other author not yet mentioned that deserves some coverage for his attempts in this area: Wallace McKenzie, whose doctoral thesis (1960), like my own research, covers each of Webern's published works and seeks to identify some large-scale trends. As an endeavour of sheer perseverance, McKenzie's work is impressive. He works his way through each of the pieces in publication order, seeking, as he writes in his abstract, 'to probe the music to find the basic elements of the compositional technique and the ways that Webern put these elements together, and to present the changes or developments in Webern's style' (McKenzie 1960, iii). This is a major undertaking, and it reveals some of the issues inevitably thrown up by pursuing such a project through manual analysis. Each of the works is treated to a short and relatively superficial judgement, detailed analysis tends to focus on only small subsections of individual movements, and there is little engagement with what analytical literature already existed, despite significant coverage of the historical literature surrounding 'Expressionism'. It is also worth pointing out that the coverage is deeply uneven, with much greater weight, and many more pages, placed on the first half of the corpus, Opp. 1–16 (313 pages) than the second, Opp. 17–31 (81 pages). McKenzie is explicit about this: he sees his work as a corrective to a previous scholarly focus on the dodecaphonic music, but it inevitably makes for even less detailed coverage. As one extreme example, Op. 18/i is dispatched with a single sentence. I will return to this critique in the discussion of distant reading at the end of this chapter, but for now it is sufficient to leave McKenzie. The work was pioneering in 1960 and includes some interesting comments about isolated works that will figure in my research, but as a truly comprehensive study of this large body of work it is intrinsically flawed by its manual approach.

Across the field of post-tonal analysis there are various overlapping analytical areas of interest. At the heart of the discussion is segmentation; following from this, similarity, and prolongation. The following discussion will summarise and critique the work in these areas, before discussing some of the conceptual apparatus surrounding analysis. Prior to this, however, a brief note on pitch-class set theory. The legitimacy of this theory has been the subject of one of the most vituperative debates in music analysis (Forte 1986; Taruskin 1979, 1986), although in recent years it has calmed down somewhat into a general acceptance of its nomenclature and pedagogical value in particular repertorial contexts. It is often advanced in the context of Schoenberg, and whether he would have perceived and been interested in pitch-class sets (for the defence, see Boss 2019, 36–40; for critique, see Haimo 1996). It is worth pointing out that although these authors often sidestep questions of intentionality, their language suggests, as Lee Tsang says in his review of Forte's book on Webern, that 'intention and perceptibility are implied' (Tsang 2002, 417); indeed, the historical and contextual preconditions for Forte's analysis have been successfully outlined by Michael Schuijer (2008). This remains common, as Zachary Bernstein points out in a review (Bernstein 2016, 276) of Boss's monograph on Schoenberg's serial music (2014). Whilst the history is interesting, and is crucial for matters of compositional intention, it is less significant for post-hoc positivist analysis like my research, and so adjudicating between the evangelists seems unnecessary; more pointed critiques will occur in due course.

2.2.1 Segmentation

The question of segmentation lies at the heart of any analysis: what constitutes a meaningful harmonic unit? Christopher Hasty sets out the basic problem with regard to this repertoire as follows: 'any interval is capable of being heard as self-sufficient; thus, in principle, any pitch may be associated with any other pitch and any number of pitches may conceivably be heard … as a comprehensible harmonic unit' (Hasty 1981, 55). This therefore allows for as many possible analyses as there are analysts willing to offer them, and a rich and endlessly changing set of perspectives on the pieces under consideration. I will employ various segmentation approaches in the different areas of my research, so this discussion aims not to outline them all, but rather to provide a background picture of the debate and introduce some of the principal criteria in developing a technique of segmentation.

On an epistemological level, Jean-Jacques Nattiez (2003) has proposed that set theory, and the segmentation approaches associated with it, is in fact strictly scientific. He argues that 'its explicitness allows any other musicologist, given a particular analysis, to follow the steps taken, and if necessary contest the results and suggest new solutions'. What is more, 'it can be shown to be false', which is often a rare condition in the humanities (Nattiez 2003). This is true of the subsidiary steps in a pc set analysis but it simply does not apply to the process of segmentation. Whilst one can observe the segments that an analyst has chosen, given only the piece of music and their method, a different analyst would likely reach a different conclusion, through no fault of their own, as, in fact, Nattiez goes on to discuss. Nattiez appears to be suggesting that the process of segmentation is some sort of pre-analytical step, before the real analysis gets going with the identification of set classes. This simply does not hold: segmentation is an intrinsic part of the analysis. As Nicholas Cook puts it (though note again the implied differentiation between segmentation and analysis): 'everything in the analysis depends on the segmentation' (Cook 1994, 146).

For Bailey, vertical harmony is hardly of concern in Webern's music. Throughout her book there are disparaging asides concerning vertical features: with regard to Op. 21, for example, she writes that simultaneities are 'the momentary rhythmic coincidence of two linear voices, something quite different from the articulation of a chord as a distinct entity' (Bailey 1991, 45). In the conclusion she asserts: 'I see Webern's interests as almost entirely linear' (Bailey 1991, 334). Indeed, it is worth citing her in full for her justification:

> it is difficult to determine – beyond a few basic decisions concerning consonant intervals and chords that had to be avoided – whether the vertical effect of the coincidence of parts was a matter of much concern. The sketches prove the fact of linear conception: parts are written in open score, and in subsequent revisions individual parts are often shifted horizontally or varied rhythmically so that they coincide differently. The method seems to be trial and choice; vertical collections do not appear to be a determining factor (Bailey 1991, 334).

Bailey's interest is in Webern's conceptions, and her work has been followed up by Shreffler's enquiry into the sketches. Shreffler, however, clearly demonstrates Webern thinking vertically in his earliest serial experiments (1994a, 293–294). Indeed, she proposes that his adoption of canons precisely achieved the 'horizontal and vertical control of pitch' (Shreffler 1994a, 302) that characterised serialism. Likewise, her excavation with Felix Meyer of Webern's revisions of his pre-1915 works shows a clear interest in clarifying vertical pitch relationships as much as horizontal ones (Meyer and Shreffler 1993a, 1993b). In any case, whilst Bailey may be accurate about Webern's interests, for any aural perception or post-hoc analysis a purely linear focus is hugely limiting. The linear is also the focus of Herbert Eimert's (1959) study of Op. 28/i. He applies a statistical approach to the intervallic content of the linear exposition of motives across the contrapuntal voices of the quartet. To some extent, this general perspective is shared by Edward Pearsall, who proposes that in post-tonal music 'chords do not necessarily represent harmonic units' (Pearsall 1991, 348). In his view, legitimate units are often larger than a single chord (a common approach in traditional pitch-class set analysis) and require further criteria to successfully isolate them.

According to Forte, there are a number of possible criteria. In some cases, segments are easily identifiable 'by conventional means' (Forte 1973, 83) or 'contextual criteria': 'references to the local context of the candidate segment or [...] non-local sections of the music' (Forte 1973, 91). This is similar to Hasty's 'domains': the properties (pitch, timbre, dynamic, etc.) of a given musical element (Hasty 1981, 57). According to Hasty, a new musical element is articulated by a 'discontinuity' in at least one domain. Segmentation is thus carried out by considering the cumulative effect of discontinuities in different domains. Boss has largely adopted this rather intuitive approach, writing, 'I am more concerned with using the set class (and the unordered pitch-class set as well) as a kind of equivalence that contributes to the manifestation of a musical idea. Or, … to describe successive sonorities as … a process of developing variation' (Boss 2019, 40). His methods of segmentation therefore tend to follow motivic or chordal divisions. These scholars seem largely, therefore, to be in agreement: segmentation should be carried out according to the surface presentation of material.

Forte has suggested a rather more controversial approach, however which goes against the usual process of segmenting based on a 'neutral' surface hearing (Nattiez 2003, 4) (of course, this so-called 'neutral' hearing is inevitably shaped by the hearer). Instead, Forte proposes to define segments according to their pitch-class relations with other sets, according to four criteria:

> (1) the set occurs consistently throughout—it is not merely "local"; (2) the complement of the set occurs consistently throughout; (3) if the set is a member of a Z-pair, the other member also occurs; (4) the set is an "atonal" set, not a set that would occur in a tonal work (Forte 1972, 45).

The issue with this approach is easily apparent: using the pitch-class content of the segments to justify their own segmentation is vulnerable to accusations that the segments have been chosen to suit the analysis. In his detailed critique of Forte, Ethan Haimo goes so far as to show that he often prioritises these pitch-class set criteria: in one case when a phrase returns 'he extracts a completely different pattern of notes' (Haimo 1996, 186). As damning as this might seem, Haimo's principal frustration with Forte concerns the conceptual underpinning of his analysis, discussed further below. Indeed, he happily admits that 'pitch-class set analysis could be profitably employed for the precise description of a certain category of relationships' (Haimo 1996, 191). Whether there can be such a thing as a 'wrong' segmentation feels, instinctively, to be a difficult matter to adjudicate (I would suggest that, assuming good faith on the part of the analyst, scholars should be prepared to accept all possible segmentations); however, there are certainly some that make more general instinctive sense than others. Whether democracy should be invoked here I rather doubt, but it would seem to be a correct proposition that some segmentations can be 'better' than others. This leaves, therefore, with a situation in which a crucial step in the analytical method hinges on unique subjective decisions, contingent not only on an individual analyst, but on their ability to assess passages of music manually, at a highly detailed level.

For an analysis of the scale undertaken by my research, segmentation requires an algorithmic approach that does not rely on individual judgements. This has some precedent in the analytical literature, although the scope is limited. Forte suggests a procedure of 'imbrication': the 'systematic (sequential) extraction of subcomponents of some configuration … an elementary way of determining the subsegments of a primary segment' (Forte 1973, 83–84). As his language suggests, however, for Forte this is a subsidiary technique that is applied to a pre-ordained segment, rather than a first-order segmentation technique. Thus, it reveals the universe of secondary harmonies latent in a larger harmonic entity, which is typically of the size of a standard segment: he, for example, uses this technique to identify the tetrachords present in various pentachords.

Randall Dipert critiques Forte's definition of imbrication for its vague method (1977, 10) and instead proposes, with R. M. Whelden, that 'the underlying structure of music is … a sequence of chords' (Dipert and Whelden 1976, 17). A terminological improvement here would be to replace 'chord' with 'verticality', but though this is clearly a crude method of segmentation it is appealing for its rigour. A recent example of this practice in action comes from Dmitri Tymoczko, who appears to apply this to measure 'the relative preponderance of different chord types' (Tymoczko 2011, 166). Tymoczko is frustratingly opaque about the particular nature of some of his analytical procedures, but all indications are that he models music in this way. In fact, Dipert's verticality-sequence model of music has been theorised separately by modern computational analysts and deployed frequently (Conklin 2002; Bigo and Andreatta 2019; Rohrmeier and Cross 2008; White and Quinn 2018; White 2022), and indeed it will appear later in my research.

Segmentation is clearly a much-discussed issue. On one hand, there is segmentation according to surface features. The inherent subjectivity of this approach accounts for many differences in analysis; however, it is also intuitively attractive and defensible. On the other hand, there is systematic segmentation, generating a reproducible and testable method. This too has its weaknesses: Forte himself queries 'whether this technique provides useful information about the music' and suggests that this 'is a question that cannot be answered without reference to other components' (Forte 1973, 84). My research will employ a variety of segmentation methods, depending on the analytical object under scrutiny, but at all times the segmentations will be carried out in an algorithmic manner, and so an awareness of the possible flaws will be an important caveat in the analysis.

2.2.2 Similarity

Analytical work surrounding similarity and equivalence is, thankfully, less fraught than that regarding segmentation. For most authors, if the basic premises of pitch-class set theory are accepted, notions of similarity follow fairly easily. The following discussion will first outline Forte's contribution and then some comments from Boss, before proposing the Discrete Fourier Transform (DFT) as a superior approach to the same basic idea.

For Forte, there is a clear delineation between equivalence and similarity. Two sets are equivalent 'if they are reducible to the same prime form by transposition or by inversion followed by transposition' (Forte 1973, see 5–11 for a fuller discussion). Regarding similarity, Forte defines various relations that allow the analyst to categorise the degree of similarity (Forte 1973, 46, see 26–60 for a comprehensive survey). Boss's contribution is slightly different: he argues that 'what Schoenberg identified as motivic equivalence corresponds exactly to what we call set-class equivalence' (Boss 2019, 39). This is part of his broader view that set classes can be understood as motivic. As such, he argues that in discussing variation techniques, it is crucial to consider 'which of them produce motive-forms more remote from the unvaried motive' (Boss 1992, 135), and so has developed a 'scale of remoteness' based on his identification of four principal categories, each 'a family of functions' (Boss 1992, 133), that Schoenberg deployed. This approach differs from Forte's insofar as it espouses Schoenberg's analytical and compositional tendencies rather than a comprehensive mathematical discussion of possibilities (for an exposition of these ideas see Boss 1992, 132–37). For both authors, the crucial element in measuring degrees of similarity is intervallic. Forte identifies four relations, R_p, R_0, R_1, and R_2, of which the latter three are intervallic and the first is deemed significant only in conjunction with one of the intervallic conditions. Likewise, Boss's variation techniques are organised by intervallic effect: greater intervallic change produces greater remoteness. This is largely unsurprising: in highly chromatic and non-tonal music, pitch relations (i.e. intervals) are often of much greater significance than pitches themselves, and particularly in music which actively sets out to negate the prioritisation of individual pitches.

As such, my own work takes great interest in intervallic effects. This is most obvious in Chapter 5, but with regard to pc sets it reaches its apotheosis with the application of the Discrete Fourier Transform (DFT) to pc sets in Chapters 3 and 6. I will leave a comprehensive introduction to the DFT until closer to the time, but in short it is a method for quantifying the quality of harmonies in six variables, each of which demonstrates a particular harmonic colour (e.g. octatonicity, diatonicity). Emmanuel Amiot (2017) has argued stridently for the advantages of the DFT over the 'poverty' of traditional set-class theory, demonstrating the greater sophistication and sensitivity of the DFT to conventional interval-vector comparison. The DFT still works with pc sets, its magnitudes, which provide the intervallic information, are still invariant to transposition or inversion (Yust 2015a, 3), but having carried out this basic reduction it provides a much more detailed picture of the set's harmonic colour. This colour, of course, is provided by interval content of the pc set, but the DFT provides an interpretative lens. The analyst can thus compare similarity relations not merely in terms of the likeness of overall interval vectors, but with regard to specific harmonic phenomena that arise from the interaction of intervals within a pc set. Amiot provides the example of two sets, (0, 2, 4, 5, 7, 11) and (0, 1, 5, 6, 7, 8). These have the same interval vector value for ic5 (5), which might suggest a strong diatonic pull, but common sense dictates that the first is far more diatonic than the second. This is then revealed by the DFT, whose diatonicity coefficient values are, respectively, 8 and 2. This is a simple example, but with six variables to apply its potential is huge: similarity can be considered in a much more sophisticated and textured way.

2.2.3 Prolongation

The subject of prolongation has inspired what Pople characterises as a 'lake of ink' (Pople 2004, 162). In its traditional neo-Schenkerian guise, prolongation is not pertinent to this project because my research uses replicable empirical approaches which, in my view at least, cannot be meaningfully extended into the neo-Schenkerian realm, which is quite rightly dependent upon the subjectivities of hearing. However, Tymoczko's (2011) concept of macroharmony is highly pertinent to my work. Although, at least in Tymoczko's presentation, macroharmonic analysis does not differentiate between structural and ornamental pitches as in traditional prolongational analysis, it does similarly aim to model the perception of large spans of music. What is more, it similarly assumes that pitches' significance can last beyond their immediate temporal boundaries, and as such can be seen as a related theory.

A wide variety of authors have attempted to apply neo-Schenkerian techniques to the post-tonal repertoire (for a helpful summary, see Roig-Francolí 2001, 58), but success has been limited. Joseph Straus (1987) has outlined four conditions for successful prolongation: 'a pitch-defined basis for determining relative structural weight'; 'a consistent hierarchy of consonant harmonies'; 'a consistent set of relationships between tones of lesser and greater structural weight'; and 'a clear distinction between the vertical and horizontal dimensions' (Straus 1987, 2–5). He argues that in atonal and indeed most post-tonal music, these criteria, particularly the first and second, are typically not met, but proposes instead a weaker theory of 'association', in which non-contiguous elements can be connected by parameters other than pitch, and indeed can do so on a middleground level. Crucially, his theory of association does not argue that any intervening elements prolong the first one; merely that the outer elements are somehow connected. In adapting his earlier work to atonal music, Fred Lerdahl (Lerdahl 1989, 1997) has explored similar ground. He views the heart of prolongation as about hierarchisation: differentiating musical elements in the hierarchy based on their relative significance. In Lerdahl's theory of tonal music, salience is distinguished from stability. The significance of an event in the hierarchy can then be understood in terms of these two variables. In atonal music, however, Lerdahl (Lerdahl 1989) argues that there are no stability criteria (which would be similar to Straus's first and third conditions) and thus salience conditions come to the fore. With no way to caveat salience through stability, salience comes fully to characterise the significance of an event in a musical texture. He has developed this in his later work by arguing that alongside salience conditions, tones can be understood as either ornamental or structural and distinguished according to four perceptual criteria (Lerdahl 1997, 17–20). In my own work, I stick firmly to salience criteria as these are most effectively treated in an algorithmic manner. Although Lerdahl's criteria could likely be quantified, this is simply beyond the scope of my own research.

Weaknesses of atonal prolongation have not totally deterred analysts from attempting to develop pitch-based theories (although they mainly accept Straus' contention that strict prolongation in the traditional sense cannot operate). Pearsall, for example, proposes that individual compositions create their own underlying intervallically based harmonic structures, giving an example of Webern's Op. 9/ii as organised around a fundamental structure characterised by ic1. Considering Schoenberg's music specifically, Boss (1994) has proposed a hierarchy of ornamental harmony based on motivic similarity. Likewise, Lerdahl (Lerdahl 1997) refers to the associational potential of motivic similarity in creating hierarchy. Finally, Miguel Roig-Francolí (2001) has formulated a theory of 'Pitch-Class-Set Extension' (PCSE) where adjacent units are linked by common-tone or chromatic connection, and so subsidiary elements are understood to 'extend' earlier ones. Fairly obviously, neither Pearsall or Boss's approaches could be explored in an empirical manner and so they have little relevance to my work. In the case of Roig-Francolí, I find his criteria for extension simply to be too weak: mere common-tone or chromatic connection seems to me to be insufficient to argue for connection in music that is often highly chromatic.

PCSE does, however, lead to a helpful end. It is important to note that PCSE accomplishes the inverse of traditional prolongation: whereas conventionally an element *a* is prolonged to a new element *c* with an intervening *b* at a subsidiary level in the hierarchy, Roig-Francolí makes no distinction between the structural importance of *a* and *b*, merely arguing that they are part of the same PCSE region, and in some sense therefore constitute a single larger unit. This sort of larger unit has a name: macroharmony, which Tymoczko has defined as 'a harmonic penumbra … extending beyond the boundaries of the temporal instant' (Tymoczko 2011, 154). Across my research I will consider macroharmonies on a variety of temporal levels, from whole movements to small, five-second units. In each case, this concept contends that useful information is revealed by these larger units, which are in some sense 'associated' by contiguity, but not a strict prolongation in the sense of Straus. No pitch-based hierarchy is offered, nor does a macroharmonic analysis differentiate between the vertical and horizontal: a pc is a pc. There are two principal reasons that I view the analysis of macroharmony as an important consideration in Webern's music. Firstly, the famously compressed duration of the individual movements makes a stronger intuitive case for the plausible perception of macroharmony than might be true in a longer work (though long duration is by no means a universal case: for an extreme example, consider the climactic entry of the piano/cello D♮ midway through Section VI from Steve Reich's *Music for 18 Musicians* (Reich 2000, 127), but the same point might be made in a more chromatic context about the gradually undulating macroharmonies of Morton Feldman's late music). More specifically to Webern, however, his sensitivity to the structural properties of the total chromatic as articulated across time is well-noted. With regard to Op. 9, he wrote that 'each "run" of twelve notes marked a division within the piece, idea or theme' (Webern 1963, 51) (for more extensive discussion of this comment, as well as analytical attempts specifically informed by it, see Hallis Jr. 2004). In effect, he is describing an awareness of Tymoczko's macroharmony, and its structural importance. Again, my research is post-hoc, and so while this does not legitimate my interest in Webern's music, it does hint that an analyst may find a more consciously developed macroharmonic language in Webern's music than elsewhere.

2.2.4 Conclusion

Analysts of Webern's music have, perhaps unsurprisingly, tended to congregate around particular much-discussed topics, trading ever-more-subtle critiques. This therefore leaves several areas comparatively unexplored: in particular, the vertical, which, although considered incidentally, has not received such systematic enquiry. Tymoczko views as a significant omission analysis of different chord types in Schoenberg's music (Tymoczko 2011, 183); I propose the same is true with regard to Webern. This focus on the vertical is crucial to my research, but another guiding principle for me is a rigorous, replicable approach. In order to carry out analysis on the scale of this project—considering tens of thousands of verticalities—decisions must be made algorithmically. Occam's Razor is instructive here: the more complicated the segmentation procedure, the more likely it is to produce unhelpful or incomparable segmentations.

As a final note, some thoughts on the epistemological underpinning of this work. Haimo differentiates two types of analytical statement: those that 'presume to reconstruct the composer's thought, methods, or actions' and that focus 'exclusively on the music, making claims about relationships that we perceive' (Haimo 1996, 178). Bailey offers a similar distinction: 'Analysis can be approached either with the intention of cataloguing and defining all the events and relationships that one can perceive (aurally and/or intellectually), or … of examining and elaborating upon the composer's concept' (Bailey 1991, 331). Webern's output is often partitioned according to constructivist phenomena; my own research hopes to go beyond this, sidestepping the fascination with compositional process that has dominated the discussion. As such, the guiding principle of my research is a post-hoc one, even if informed by the biographical. For Haimo, this methodology is characterised as follows: a proposed hypothesis about the musical structure is 'tested by a critique of the analytical premises against the compositional data' (Haimo 1996, 180). Crucially, 'different observers should be able to examine the same work and produce similar analytical results' (Haimo 1996, 180). Bailey goes so far as to argue that 'any attempt to rationalize formal structure without a knowledge of the row and its properties and of the way in which it is used is specious, since the two aspects of any twelve-note work are interdependent' (Bailey 1991, 5). This may be a legitimate criticism, but it seems fair to suggest that with the help of her knowledge of the row, other matters may be considered.

Bailey's division between aural and intellectual perception is also important. She never precisely defines the implications of this binary, and stays clear of any empirical research, so one presumes a fairly common-sense understanding. Throughout her book Bailey is careful to acknowledge features which are perceivable only intellectually. Indeed, when discussing the canons in Op. 31 she goes so far as to question their relevance, positing that as 'music is an essentially aural art form, the validity of music in which the method of organization cannot be heard may be seen as questionable' (Bailey 1991, 119). Given the potential ramifications of applying this logic to many of her findings, it is unsurprising that she neglects to consider the implications of this statement. Meaningful empirical research about perception is clearly beyond the scope of this project; nonetheless, her acknowledgement is an instructive qualification of any such analysis.

2.3 Digital Musicology

Having situated this project within post-tonal analysis, the other dimension is Digital Musicology (DM). The focus of this research is musicological, rather than computational; the aim is to advance understanding of Webern, not digital methods. Nonetheless, some of the digital techniques applied constitute original contributions, and it is important to recognise the relevant scholarly context. The following review will therefore locate this project within the field of DM, then discuss some important methodological concerns, before concluding with some of the relevant critiques levelled at the digital humanities.

2.3.1 Situating this Project

DM is an enormous area, spanning an overwhelming array of subjects and methods that have developed principally since the 1950s. The word ‘digital’ in the title indicates its relationship to the Digital Humanities (DH) more widely, and so it is there that I will begin. Again, an enormous quantity of ink (or perhaps pixels) has been spilled over defining, theorising, celebrating, and critiquing DH, and what follows will not be at all comprehensive, but rather attempt to take account of the pertinent backdrop to set up my own research. As early as 2009, William Pannapacker heralded DH as ‘the first “next big thing” in a long time’ (Pannapacker 2009, para. 1). Somewhat more underwhelmingly, taking stock of the field a decade later Underwood described DH as now ‘a semi-normal thing’ (Underwood 2019), and he and others (Callaway et al. 2020) have suggested that some of the most extreme claims about DH have been superseded by a softer acceptance of the role that the field can play. I will therefore avoid rehearsing some of the more overblown debates about DH’s transformative potential, either through celebration or disavowal. I will begin with a discussion of defining the field, a subject I will approach from three angles: the theoretical, pedagogical, and historical, and then go on to discuss some of the epistemological critiques of DH in general, before turning to Digital Musicology more specifically.

Defining DH, and by extension DM, is a large genre of scholarly writing in itself. Though at a first glance a definition might seem simple, perhaps along the lines of 'work in the humanities that involves a digital element, whether it be in subject or technique', alas the reality is anything but easy. In fact, in a wonderfully 'meta' move, Elizabeth Callaway et al. (2020) have used topic modelling on a corpus of over 300 articles seeking to define DH. Their result is hardly conclusive, though they do uncover four principal areas of discussion in definitions of DH: coding (is it necessary?), community (is it inclusive? Does it have shared values?), distant reading (is this synonymous with DH?), and diversity & inclusion (how successful is DH at this?). The lack of clear definition is actually celebrated by Raphael C. Alvarado (2012), who proposes that DH cannot be defined as a traditional discipline, characterised by 'a set of theoretical concerns and research methods' (Alvarado 2012, para. 1). Simply put, for Alvarado the field is just too big to be seen in terms of a set of practices. Rather, he views it as centred not on particular technologies or methods but rather 'the ongoing, playful encounter with digital representation itself' (Alvarado 2012, para. 13). By this he proposes that integral to the DH approach is a mindset not merely of deploying a digital tool as a means to an end, but full engagement with the medium. This is similar to Underwood's view that 'Math is a way of thinking' (Underwood 2018, para.13): he too argues that rather than merely co-opting tools from other disciplines, to engage in DH is to adopt a fundamentally new way of thinking. This, perhaps obviously, has implications for the pedagogy of DH. One viewpoint following from the conceptions of Alvarado and Underwood is to suggest that to teach DH requires intensive engagement with the technological, be that through coding or statistics (Arnold and Tilton 2019; Underwood 2018). If a discipline is defined in terms of Alvarado's 'theoretical concerns and research methods', this positions DH, therefore, as a distinct entity, with a fundamentally different outlook characterised by these research methods, even if the subject matter (culture, broadly defined) remains the same as the traditional humanities. The change of mindset that a statistical epistemology offers is certainly valuable, and indeed relevant to my own research as questions are framed in terms of the empirically verifiable. Nonetheless, these authors risk falling into the conflation of DH with the use of statistical analysis on humanities subject-matter. Underwood has used the term 'cultural analytics' (Underwood 2018, para. 1) to describe the use of numbers to understand cultural history, which is a helpful label. He suggests, rather vaguely, that it is not helpful to think of this as a subfield of DH, but rather a way of integrating different elements of an education; I propose that in fact it is very helpful to think of it as a subfield of DH, defined specifically by the union of computational and humanities epistemologies. This is a view espoused also by Andrew Piper, who warns that cultural analytics is not merely 'computer science applied to culture' (Piper 2016, para. 3), with the implication that the latter has nothing to contribute to the former, but rather a synthesis of the two that takes on board elements of both fields. From computer science, it accounts for the role and nature of evidence, the clear differentiation of theory from practice, while the humanities encourage the rejection of universalism and an awareness of the constant situatedness of knowledge.

Joining these two fields is integral to cultural analytics, and by extension DH, and leads to the common description of DH as a mixed-methods discipline. J. Berenike Herrmann (2017), for example, has explored the implications of treating DH as fully committed to mixed methods. She identifies the same epistemological gap between DH practitioners trained primarily in computation, and those trained in humanities, and argues instead that a formal paradigm of 'mixed methods' is required, similar to that in the social sciences. She suggests that practitioners should choose the best method(s) from all those available for any given research question: rather than constantly establishing binaries, the field should embrace all possibilities. The result of this would be an approach that maintains 'the basic distinction between quantitative and qualitative methods, while simultaneously transcending it' (Herrmann 2017, para. 14) such that the sum is greater than its parts. This is a view endorsed by Moacir P. De Sà Pereira (2019), who also points out that quantitative methods need not be restricted to 'big data'-style corpus studies, but can be deployed on smaller entities too (again this is a reminder that DH and distant reading are not synonymous). This characterisation as a mixed-methods discipline carries the implication that scholars must be trained in a pantheon of methods, or at least have a broad enough knowledge to be able to evaluate their qualities or work constantly in collaboration with a partner from 'the other side'. This is a laudable ambition, but it is likely to encounter friction from an academic environment that glorifies single authors and is potentially utopian in its outlook. Stephen Robertson (2016) has reimagined the classic image of the 'big tent' as a big house, with different rooms representing not siloed disciplines, but rather areas where scholars working with particular tools or media could gather. This appears to me a more realistic image of the field than the optimistic hypothesis outlined above and makes for a more accurate portrayal of how interdisciplinary research tends to operate.

This remains, however, a model of the field that is organised around tools and subject. An alternative view of the discipline is a social one. This perspective is quite common, and unsurprisingly, perhaps, given the situation of the authors involved, often discusses the discipline in terms of the logistics of the academy, viewing DH as more of a signifier of grouping more than practice. Indeed, Underwood has gone so far as to suggest that the field 'only exists as a social network' (Underwood 2018, para. 9), a view similarly espoused by Ryan Cordell (2016) who advocates for an appreciation of the 'local' and 'peculiar' qualities of a field that, despite a sizeable reputation, remains small in university contexts (Underwood 2019, 97). Whilst this is perhaps a helpful outlook in the institutional discussion of DH, from a more applied perspective it risks navel-gazing.

In opposition to Alvarado and Underwood's maths-based way of thinking, Cordell (2016) has proposed an alternative view, based on the iterative improvement of his teaching. He proposes that instead of focussing on an empirical way of thinking, that effective DH teaching should start with small, practical tasks, and indeed should explicitly avoid attempting a comprehensive rendering of the field and the available techniques. A similar pedagogical mindset has been promoted by the 'minimal computing' movement, where prioritising 'small wins' (Risam and Gil 2022, para. 9) is celebrated as a deliberate strategic move to build confidence amongst students. In advocating for this, these authors are responding to an undergraduate student body that is often cautious in a science-adjacent field, frequently characterised by a general insecurity around maths and technology. To some degree there is a conceptual difference between these outlooks: Alvarado and Underwood are talking in terms of the requirements for a 'fully-formed' academic, while Cordell, Roopika Risam, and Alex Gil are focussed on the pragmatic realities of teaching undergraduates who are not, and likely will not become, DH specialists. Nonetheless, the minimal computing movement (with which, to my knowledge, Cordell does not associate himself) draws attention to the effects of scarcity and seeks to provide a more pragmatic approach to DH, taking account of various limitations on scholars, much like Robertson does. David Golumbia has suggested that there is 'absolutely no undergraduate demand' (Golumbia and Kim 2021, 66) for DH, and uses this to argue that there is no reason to offer DH courses as enrolment will be non-existent (and, by extension, little reason to hire DH scholars). Ironically, this falls into exactly the sort of market-based picture of the university that he often critiques, defining the value of scholars in terms of their ability to teach courses that students have a pre-conceived wish for. This utterly fails to account for the role of leadership in teaching, deferring not only to student wishes, but in the process directly rejecting the possible value of any 'unknown unknowns'. I find this a particularly frustrating line of reasoning because it is so relevant to my own educational path. My engagement with DH came not from some long-held fascination with technology or mathematics, but from an encounter during a summer internship with an analyst who happened to use a digital approach in his own research. Despite having been taught ('traditional') analysis by this professor for a year, I had no idea about his (or my own!) technological predilections, and indeed without that fortuitous extracurricular introduction I would have had no engagement with DH. Students necessarily do not know not only what they do not know, but also what they do not like until they have had some introduction to it, and so Golumbia's reasoning is almost entirely circular.

There are dangers to a utilitarian, tool-based approach to DH, however, which prompt a common criticism of DH as in bed with neoliberal corporate interests. At worst, this comes from a lazy conflation of anything quantifiable with the managerial tendencies of the neoliberal university; a more sophisticated view is offered by Daniel Allington, Sarah Brouillette, and Golumbia (2016) who draw attention to the focus on 'building' in DH as symptomatic of a neoliberal tendency to view the function of the university as providing outputs for business, a view also expounded upon by Richard Grusin (2016). Indeed, these authors explicitly argue that DH is '*not* about … the use of digital or quantitative methodologies to answer research questions in the humanities' (Allington, Brouillette, and Golumbia 2016, para. 3), but rather about coding, engineering, and in a pedagogical role training the next generation of corporate workers. The authors are careful to acknowledge that there is some DH work that rejects this, but propose that what they identify is the '*dominant* current' (Allington, Brouillette, and Golumbia 2016, para. 27). Opposing this, Brian Greenspan (2019) has argued forcefully, correctly identifying that much of the 'output' of DH is of minimal or no use in industry, despite a patina of technological 'relevance'. Wendy Hui Kyong Chun (2016) has also pointed out that, from a training perspective, DH surely fails as its graduates will not out-compete Computer Science specialists. Whether this is cast as a failure despite neoliberal aims, or an indication that DH is less corporatist than suggested remains unclear, but it certainly seems true that developing analytical tools for analysing interval distributions in the music of a long-dead Austrian is hardly likely to set pulses fluttering in Spotify HQ.

The more epistemological side of Allington, Brouilette, and Golumbia's condemnation is in proposing that DH deliberately pushes out critique in favour of the building identified above. In a separate piece, Golumbia (2021) has described his experience of this phenomenon at the University of Virginia, arguing that there was a right-wing institutional desire to deploy DH specifically as a replacement for critical theory, and as part of it any discussion of racial politics. Golumbia acknowledges that merely because certain pioneers in the field sought to use DH in this way does not taint all practitioners with the same set of beliefs, but he is nonetheless critical of what he sees as a repeated rejection of texts and ideas foundational to critical theory. Whether or not this assessment has become universally embraced, the importance of theory and critique in DH is certainly widely accepted in more recent practice. John Hunter (2019), for example, is similarly strident, warning that unless method and critique are constantly synthesised then the field risks losing practitioners to a corporatist climate that seeks only the method. Indeed, he views the critical theoretical contribution of DH as almost its primary offering. If it is corporate interests who set the agenda for conditions of knowledge, it seems unlikely that they will willingly wish to engage critically with the implications of their work (as the recent case of Timnit Gebru's firing from Google illustrates only too well), and so DH must provide that critical perspective. As such, the value of DH must not be defined solely in terms of making, hacking, and engineering, but must include a profound critical engagement with all things digital. Ravynn Stringfield (2021) warns that 'too often projects exist simply because *they can*, with no regard for the potential harm it may do' (Stringfield 2021, 477), and recent examples of the misuse of un(der-)investigated technologies (self-driving cars and facial recognition come to mind most obviously) make this case convincingly. The importance of self-reflexive critical thought is also supported by Clement (2016), who argues for the importance of a distinction between technique, method, and methodology, with a constant integration of methodology. In his telling, technique is the application of a particular method, while methodology is a contextualisation of method in relation to the relevant theory. In particular, he draws attention to the danger of co-opting methods from other fields without the associated methodological discussion. Meanwhile, other scholars draw attention to the importance of the work that typically comes 'after the method': interpretation. David M. Berry et al. (2019) go so far as to encourage the field not only to avoid fetishising the numbers, but to focus on the interpretation that follows, not just encouraging a two-step process, but actually suggesting a re-weighting towards the humanistic act of interpretation. In fact, Lincoln Mullen (2019) even argues that method must always be integrated with interpretation—'braided' in his terminology. Mullen proposes a materialist view that the very act of writing aids the integration of the two. Similarly interested in interpretation, Rachel Sagner Buurma and Anna Tione Levine (2016) have demonstrated how the main focus of archival DH work can be to generate exactly the sort of critical interpretation that Allington, Brouillette, and Golumbia argue that DH seeks to push out by prioritising the possibility of multiple interpretations and providing an interactive flexibility that encourages exactly this. Finally, going one step further to consider the possibility of larger debate, James Dobson (2021) proposes that presenting data and method is not by itself a guarantee of greater interpretability for other scholars, and for humanists it is this interpretability, not merely transparency for its own sake, that is important. Indeed, for some humanists, interrogating the methods and deployment of tools by a scholar in a critical and theoretical context can be more important than the results or accuracy of the outcome. Transparency of method is no silver bullet. These calls for methodological awareness and transparency, particularly as part of a general commitment to the importance of theory, are highly pertinent to my own research. As I discuss shortly, Digital Musicology is comparatively under-theorised and under-developed compared to its peers in literary studies. Several of the tools I deploy are adapted from other fields, particularly from linguistics and literary studies. Indeed, perhaps the best precedent for my work is the literary studies subfield of stylometry, 'the statistical analysis of literary style'

(Holmes 1998, 111). As such, when these adapted methods appear they will be framed by context from their home discipline. Meanwhile, interpretation is crucial to my own work: as I will discuss further below, at its heart this is research about Webern, not about statistics. Even if it uses a statistical mindset, the goal is still to further understanding of this repertoire.

Despite the wide frame of the field described above, DH is traditionally seen as having begun with text-based computing. The first DH project is typically named as Father Roberto Busa's concordance of Thomas Aquinas, produced over 34 years in conjunction with IBM (Rockwell and Passarotti 2019), while Frederick Mosteller and David L. Wallace's (1963) study of the anonymous Federalist Papers was perhaps the most famous early DH project (Hockey 2004), and similar authorship attribution studies remain a major focus of stylometry (Benotto 2021; Halteren et al. 2005). Indeed, as Callaway et al. (2020) demonstrate, DH is at times lazily conflated with text mining, or even more narrowly with the technique of distant reading. This simplistic origin story has been complicated. Arun Jacob (2021) has desanitised the conventional Busa story, tracing both the priest's roots in fascist Italy, and the place of IBM punch-card technology in Nazi bureaucracy. Meanwhile, Kinder and McPherson (2014) have offered a more diverse narrative that embraces early developments in non-textual media like film and audio. Indeed, they even speculate that perhaps the field's godfather should be considered to be John Cage as much as Roberto Busa! That said, my own research falls into a quite traditional picture of DH: using statistical analysis to search for patterns in a corpus of texts. My research employs computational analysis systems, as part of a Music Information Retrieval (MIR) approach (for an exposition of this subfield see Downie 2005). This is typical of MIR: as Anja Volk, Frans Wiering, and Peter Van Kranenburg have described, the field arose in the 1990s/2000s due to the increasing digitisation of music (Volk, Wiering, and Kranenburg 2011, 138).

Within MIR, a long-standing concern has been how to encode music: which Music Representation Language (MRL) to use. A brief sample of the last half-century indicates a persistent pattern, authors repeatedly develop apparently comprehensive systems, which never catch on (Debiasi and Poli 1982; Fuller 1970; Janssens and Landrieu 1976; Mesnage 1993; Pople 2004). In many cases, the interest appears to have been more with developing the system than doing the analysis. In 2004, Cook aptly summarised the situation: 'a sustained burst of initial enthusiasm is followed by running out of money, resulting in software that is sometimes less than fully functional, often less than fully documented, rarely properly supported, and usually soon obsolete' (Cook 2004, 107).

Despite this dismal picture, there has been recent improvement. Increased access to computing and the internet has encouraged greater centralisation, particularly inspiring projects that improve the universal database of encoded scores. Some appear to aspire to near-universal coverage (copyright restrictions aside) (Bonte, Froment, and Schweer; Guo; *ELVIS Project*); others are more specialised (Rodin, Sapp, and Bokulich; Rootham, Jonas, and Gotham). As in earlier periods, there is still some division over MRL. Although XML/MXL has become somewhat standard, other common formats include Humdrum, Musedata, Lilypond, and MEI. In 2015, Laurent Pugin drew attention to this as a serious problem, arguing that 'The development of music computer codes has shown us how different centers of interest and different focuses can lead to countless barely compatible initiatives' (Pugin 2015, para. 8). Nonetheless, conversion between filetypes is increasingly easy, and music encoding software (e.g. Sibelius, Finale, MuseScore) are often able to export in multiple formats, breaking down what were once insurmountable barriers. Indeed, Tim Crawford and Richard Lewis write that in the case of XML, the format 'takes interchange of notation between score-notation programs as its primary remit' (Crawford and Lewis 2016, 282).

With regard to the analysis system, my research deploys music21, an 'open-source object-oriented toolkit built in Python for digital and computational musicology'

(Cuthbert et al. 2012, 1). In their introductory article, Michael Cuthbert & Christopher Ariza wrote 'The music21 project expands the audience for computational musicology by creating a new toolkit … with intuitive simplicity and object-oriented design throughout' (Cuthbert and Ariza 2010, 637). They display a clear awareness of many of the issues with previous systems, as well as musicologists' reluctance, and though their claim that music21 is 'close to a new de facto standard for computer-aided work' (Cuthbert et al. 2012, 1) might be something of an overreach, it has certainly had widespread success.[1] Two of music21's most helpful features are its ability to read a wide variety of filetypes and its integration into a conventional coding environment. The first neutralises debates over format, and as Tymoczko writes in his laudatory review, exemplifies 'the open, idealistic pluralism of the internet's higher self' (Tymoczko 2013, para. 5). The latter feature is beneficial because as operating within the broader Python language allows seamless data processing achieved through conventional coding.

My own research synthesises many of these trends. I am encoding and analysing works as XML files, and, where and when copyright allows, share them freely. Indeed, one of the motivations for exploring Webern's music is the lapsed copyright on many of his works. The data collection in this project is carried out through music21: in each case I am writing new code. These too are available through GitHub (see Appendix G). Meanwhile, the statistical analysis and visualisation of the data is achieved through Python. My hope is that by sharing these algorithms I might avoid Cook's obsolescence, providing useful tools for other scholars.

2.3.2 Corpus Study

The above review has located this project within the boundaries of digital musicology; corpus study, by contrast, is an interdisciplinary approach initially derived from linguistics, which forms the basic process at the heart of this

1. At the time of writing there are over 1000 members in the music21 Google Group, the majority of whom likely use music21.

research. It has been used in a plethora of different fields, to consider a multitude of topics (indeed, White (2022) locates an early example in the code-breaking of Alan Turing) but relies on four main tenets: 1) it is an empirical approach; 2) it employs a large collection of texts ('the corpus') from which patterns are derived; 3) it makes use of computational analysis; 4) it utilises both quantitative and qualitative techniques (Biber, Reppen, and Friginal 2012, para. 2).

In musicology, it has been deployed in a wide variety of contexts, from harmonic tendencies in Bach chorales (Rohrmeier and Cross 2008; White and Quinn 2018), to analysing jazz solos (Weiß et al. 2018), to 'developing an evolutionary model of Western classical music' (Georges 2017, 21). As Cook has pointed out, one of the key strengths of digital musicology is its comparative possibilities, and corpus studies exploit that to its greatest potential (Cook 2004, 107–9). Needless to say, the quality of these studies varies hugely: an empirical foundation is no guarantee of success. The third example cited here (Georges 2017) exemplifies many typical pitfalls: the ambition of the project is positively Icarian; the variables chosen are at best superficially representative of the sought-after features; the corpus is not only disproportionate (20 composers cover 1100-1400; 33 cover 1890-1930), but ludicrously small (500 composers for 1000 years of music); and the evolutionary metaphor that underpins the entire project belies an ignorance of any of the conceptual developments of the last century. Martin Rohrmeier and Ian Cross, by contrast, are worthy of approbation for the limited scope of their work: they consistently acknowledge the boundaries of their research and make no claims beyond this perimeter. It is worth noting that corpus studies have a history in pre-computational analysis, as Robert O. Gjerdingen (2022) has charted, but their potential has been altered radically by recent technological developments.

A major pioneer in corpus studies is David Huron. His work tends to be closely tied to perceptual research, such as a series of corpus studies he carried out on musical voices (Huron 1990a, 1990b, 1991), all inspired by an earlier empirical study of voice perception (Huron 1989). Like most corpus studies, his work tends to focus on music from 1500–1900. As such, although it is not directly relevant to my own research, it stands as an exemplar of successful integration between perceptual and analytical research. There are a few plausible reasons for the interest in earlier music that characterises musical corpus studies. As a comparatively young and specialist field, there is a need to develop academic standing, most easily gained by assessing the music of the canon; logistically, copyright restrictions make pre-twentieth century music more easily accessible. Corpus studies are reliant upon large bodies of encoded files, either encoded by scholars themselves, or from the online databases mentioned above which unsurprisingly tend to skew towards older, more canonical music. Music from pre-1900 tends also to be seen as more stylistically homogeneous than that of the twentieth century. This not only allows for larger corpora, but also means that the literature has developed more coherent theories which respond effectively to empirical analysis. In part, these trends explain my decision to concentrate on the music of Webern: there is a clear unity despite aesthetic differences, and the corpus is in fact comparatively stylistically homogeneous (compared to Stravinsky's or Stockhausen's, for example). As I would be encoding the music, the comparative brevity of Webern's output was appealing, as was the quantity of literature. These trends result, however, in corpus studies focussing on, and so perpetuating, the problematic primacy of the canon. Indeed, this study is as guilty as any in doing so: encoding Webern's works ensures that his music is easier to study than that of other, less represented composers. There is little to offer in defence, beyond the hope that having established some academic 'cachet', scholars will be able to turn these digital tools to more underrepresented realms, as has often been the aspiration, if not the achievement.

In the discussion of style analysis in the previous chapter I critiqued Rosen for a failure to engage with an accurate representation of the repertoire constituting particular style, but this lack of accurate representation has hardly restrained contemporary theorists either. Hepokoski and Darcy's (2006) monumental *Elements of Sonata Theory* claims to represent eighteenth century practice in a radically different way. Their contention is two-part: that analysts should understand there to be a normative sonata practice developed in late eighteenth-century music, and that this was understood as such nineteenth-century music and consciously deformed. Julian Horton (2005)[2] takes issue with the idea of a nineteenth-century understanding of a normative sonata practice. With regard to theory, he argues that disparities between different scholars are simply too great; in the repertoire, he suggests that 'deformations predominate to the virtual exclusion of the normative model' (Horton 2005, 11). Interestingly, this is a similar but subtly different contention from Rosen's understanding of style. Whilst Rosen argues that there is a background body of mediocre compositions, but that these are not of interest in studying or defining style, Horton allows that it is possible that such a 'normative repertoire exists in a hinterland of neglected works by neglected composers' (Horton 2005, 10), but posits that no evidence has been offered to bolster such a claim. It is not my intention to litigate between the two sides here, but it is perhaps worth noting that Hepokoski and Darcy specifically argue that 'The genre thus reconstructed is to be regarded as an implicit and necessary backdrop that functions heuristically. In other words, it exists not literally but rather as something like a (Kantian) regulative principle' (Hepokoski and Darcy 2006, 206). As Seth Monahan (2015) has pointed out, Horton's search for specific incantations of these sonata forms in theory or practice is thus something of a misunderstanding: what Hepokoski and Darcy are illustrating is something far more flexible than the rigid demonstration Horton seeks. In short, Horton misrepresents Hepokoski and Darcy as establishing a single metaphorical (or not!) textbook from which later composers deviated whereas in reality they constantly allow for inevitable variation of different composers, particularly with regard to differing awareness of these formal models.

2. The following discusses Horton's 2005 article in relation to Hepokoski and Darcy's 2006 book; whilst this might seem chronologically obtuse, Horton cites Hepokoski and Darcy (2006) in his first footnote (Horton 2005, 5) as forthcoming, and so it seems fair to assume that at the time of writing he had read the text.

In any case, Horton's major issue is that Hepokoski et al. seek to find homogeneity in nineteenth-century practice, when Horton argues that scholars should look instead for a diversity of individualistic approaches. This may even allow for the use of deformational analysis, if such an analysis takes specific account of a particular composer's formal training. Bailey's analysis of Webern's music suggests such an analysis. She writes that Schoenberg and Webern's musical education stressed three disciplines: *Harmonielehre*, *Kontrapunkt* and *Formenlehre*. Whilst harmony had clearly changed radically over the last century, in Bailey's view 'the forms of Mozart, Haydn and Beethoven had simply been adjusted to serve succeeding generations of composers' (Bailey 1991, 147); indeed, she sees the possible development of one of these strands as necessitating little change in the others. Whilst Hepokoski and Darcy might disagree that nineteenth-century formal practices merely constitute adjustments, both sides are in agreement regarding the basic premise of a normative practice understood by later composers and made manifest in their music.

All this is a long-winded way of contextualising the idea of coherent practice on which corpus studies, and style analysis more broadly, often rely. Indeed, despite the lack of supporting evidence, Hepokoski and Darcy's work implies a quasi-statistical basis: their very ordering of different strategies (first-, second-, third-level defaults) are defined as the options that the composers selected '*more frequently*' (Hepokoski and Darcy 2006, 10, italics mine). As for their definition of style, they argue that sonata theory applies to 'any sonata-form movement from the period of Haydn, Mozart, and Beethoven' and that these procedures 'existed conceptually within the knowledgeable musical community' (Hepokoski and Darcy 2006, 9). As such, they appear to be describing the total practice of the era, mediocrities and all. Nonetheless, with regard to their methodology, they argue that reconstructing the norms that define sonata theory 'must be done inductively, leading to constantly tested-and-retested conclusions … that emerge from the study of the works of the more influential composers of the period' (Hepokoski and Darcy 2006, 605). In effect, then, their strategies are no different from the corpus studies identified above: a cursory glance over their Index of Works reveals a devotion to the canon, with a smattering of examples from less well-known composers. (The point it makes about dialectical forms in later nineteenth-century music is less dependent upon a broader corpus, as it seems fair to assume that these composers were engaging more with Beethoven than Stamitz.) Whilst it might allow for the relevance of statistics, however, on the whole *Sonata Theory* does an impressive job steering clear of any actual statistics in favour of the authors' omniscience. Much like Rosen, Hepokoski and Darcy are clear that they can eschew real empirical enquiry in favour of their own opinions.

By contrast, an exemplary model for the value of empirical research comes from none other than Horton's (2021) recent study of eighteenth- and nineteenth-century piano concerti. His subject is the Type 5 Sonata Form, as defined by Hepokoski and Darcy (2006, 430–602), but also discussed, in different terminology, by a host of others including Donald Francis Tovey (1936) and indeed Rosen (Rosen 1971, 172). Horton outlines the traditional perspective on the form which prioritises the monotonal R1 of Mozart's concerti over a modulating R1, rarely found in the canonical concerti. His contribution, however, is to open up the works under discussion from the concerti of Mozart and Beethoven to include a corpus of 87 composers, including many that I am sure Rosen would write off as 'minor'. By taking this wider perspective, Horton finds that the traditional approach to the Classical concerto confuses matters in order to prioritise Mozart's and some of Beethoven's works in the genre. In fact, rather than holding up the monotonal R1 as the normative strategy, from an empirical perspective it is the modulating R1 that constituted the 'generic orthodoxy' (Horton 2021, 398), Mozart's solution is merely a 'minority practice' (Horton 2021, 406). The implications of this are significant: as Horton points out, this exposes a major flaw in the common practice of analysis, the development of stylistic models based on selected works preferred by the analyst. This supposedly neutral model is then used to critique other contributions to the repertoire which, unsurprisingly, are often found wanting. Indeed, it is a fabulous example of motivated reasoning that does not so much put the cart before the horse as do so and then complain about the insufficiency of the cart. It also obscures the place of the canonical works in relation to their context: in Horton's case, Beethoven's concerti make bafflingly little sense when viewed from a Mozartean perspective but are easily comprehensible in their actual context. This viewpoint also encourages the analyst to consider the implications of reception: again drawing from Horton's example, Mozart's concerti had a relatively slow dissemination in the early nineteenth century, so to claim, as many have, that early nineteenth-century works are a response to Mozart's is ridiculous. Horton's revisionism, then, is a good example of several of the strengths White (2022, 20–23) identifies in corpus analysis: in particular, an individual observation is successfully contextualised against a broader backdrop, this backdrop is then better understood through the use of a large dataset, and the result challenges biases latent in music analysis.

If Horton's work suffers, from an empirical perspective its major failing is in the construction of the corpus. Whilst certainly better grounded than previous work on the area, 87 concerti is still a small figure, and Horton provides no explanation as to how they were chosen, which is standard practice in corpus analysis, and with good reason. In a true statistical analysis, a sample is carefully chosen and then weighted according to relevant variables such that the analyst can be confident it represents the period under consideration. As far as is made clear, no such attention has been paid to the selection of these works, and this throws up problems. From the perspective of chronological representation only four works represent the final decade of this 56-year span, when an even distribution would have 16 works in this period. Likewise, there are issues with uneven representation of composers: five composers contribute only one work each, while Dussek is represented by 12, and Cramer, Ries, and Moscheles eight each, such that the works of those four composers take up slightly over 40% of the pieces under consideration. It seems plausible that the composer would be an important variable to weight for, and so these nine composers are, respectively, significant underweighted and overweighted in the sample. Gender and racial representation are clearly appalling in a corpus comprised wholly of white men. As such, while this obviously has a better claim to be a representative corpus of the Classical era than those implicitly considered by Rosen or Tovey, from a statistical perspective, significant questions remain as to the extent to which this sample truly represents the total population.

2.3.3 What can this tell us?

To draw this together it is important to consider, if briefly, the epistemological claims that corpus studies and the digital humanities make. White points out that in such an intrinsically interdisciplinary field, the 'evidentiary ground rules shift underneath an analyst's feet as they move between different kinds of logics' (White 2022, 30), and so analysts must always be aware of these changes in framing. Caroline Bassett has been an important critic of the digital humanities, warning that 'Big Data … promises to render various forms of human expertise increasingly unnecessary' (Bassett 2015, 549) and makes claims to provide 'solutions beyond dispute' (Bassett 2015, 550). It hardly needs asserting that these are dangerous claims to make, and yet it is not atypical for those working in computational fields to make statements that transcend the ambitious to hubristic universalising (this is certainly not restricted to musicology: twenty years of Silicon Valley demonstrates as much). Catherine D'Ignazio and Lauren Klein, meanwhile, have proposed principles that embody a feminist approach to data research. Drawing on feminist theory they have problematised various aspects of data research as it is often carried out. Particularly relevant to my research are their exhortations to embrace pluralism and 'self-disclosure' (D'Ignazio and Klein 2016, paras. 12–14) and to consider the situated context of all knowledge (D'Ignazio and Klein 2016, paras. 19–21). Their particular contribution is locating these in a data-specific manner, with regard to data collection and processing, and data visualisation and presentation. These authors provide good examples, then, of the focus I explored above on integrating a humanistic perspective with an empirical mindset.

In this spirit, there are two further areas that require critical discussion. The first important recognition is the reliance in my research on texts as source material. Post-structuralist thought has successfully destabilised analysts' default tendencies to use a score as a proxy for, representation of, or even incarnation of the hallowed 'music itself'. It is certainly the case that the basic approach here could analyse recordings, as, for example, Cook (2017) does in his study of Op. 27, or even performances. In the case of Webern, however, much of the music remains chronically under-recorded and so, with a few exceptions, an analysis using these sources would be less a study of harmony in the music of Webern than, for the large-scale works, a study of harmony in the music of Webern as (usually) conducted by Pierre Boulez and Robert Craft. In any case, Webern is specific enough with regard to the parameters under consideration (primarily pitch and duration) that the use of scores is justifiable. Likewise, the condition of his published works is good enough that few significant editorial decisions have had to be made regarding the scores; those that have will be acknowledged and explained where relevant, as will the constitution of the corpus itself, particularly with regard to revisions and unpublished works. Nonetheless, this recognition has an important impact on qualifying the knowledge that this project produces. Both textual analysis and Big Data encourage certainty beyond their means, and so it remains paramount to recognise their limitations, particularly in assessing the outcomes of the project.

Secondly, it is worth enquiring briefly into the theoretical implications of this sort of large-scale pattern-recognition. This approach has its genesis in Franco Moretti's 'distant reading' (Moretti 2000a, 2000b). Moretti coined this term to describe his analytical approach, based on statistics and computational analysis, and crucially posed as an alternative to the conventional practice in literary studies of 'close reading', characterised by deep engagement with a text of limited size. I largely subscribe to his contention that the sort of close reading carried out in traditional analysis limits the scope of the research: 'close reading … necessarily depends on an extremely small canon' (Moretti 2000a, 57). By contrast, this corpus study achieves a musical equivalent of distant reading, which might be thought of as 'distant listening', although as the analysis in this thesis is primarily score-based rather than focused on recordings, reading remains an apposite term that I will continue to use. This has significant weaknesses: the analytical algorithm only encompasses those parameters that can be predicted and measured; edge cases remain unseen by a human, in fact, all cases are unseen by a human once the algorithm is applied; the resulting algorithm is inevitably cruder than the complex assessment a human carries out, both consciously and unconsciously. However, this algorithmic approach has significant strengths too, foremost amongst them transparency and replicability. In designing an analytical process like this, the analyst can be totally explicit about their decision-making, both in their writing, and indeed to themselves. Every step can be outlined and justified, and in the longer-term, this massively increases the likelihood of gradual improvement, as other analysts build on and develop the sophistication of these algorithms. What is more, other analysts can repeat the same analysis and find the same result. The ability to replicate a study like this is a cornerstone of the scientific method, and finally provides music analysis with the scientific rigour to which areas of the discipline have long aspired. Above, I mentioned Dobson's concern for interpretability and providing this sort of transparency of method is a crucial part of this. Indeed, Alexandra Juhasz (2021) makes the case that DH requires exactly this 'self-reflexive praxis' (a more holistic version of D'Ignazio & Klein's 'self-disclosure'). Her work admirably demonstrates what this sort of transparent interpretability might look like, covering everything from the traditional acknowledgements of funding and collaborators through features of identity into a discussion of her audience, and the mediated manner in which they might access her work. Juhasz's case is perhaps extreme: her primary audience is incarcerated, which brings with it complex instability, but her transparency remains an important reminder of this self-reflective approach.

N. Katherine Hayles (2012) has complicated Moretti's close/distant binary by identifying instead two types of distant reading: one 'involving a human assisted by machines, the other involving computer algorithms assisted by humans' (Hayles 2012, 72). So-called 'hyper' reading provides a good example of the former. Hyper reading is a digitally native form of reading in which the human reader is assisted by a computer to filter, skim, juxtapose, and fragment in the reading process, thus managing multiple information streams and typically engaging with a large amount of source material at speed. The classic corpus study approach exemplified by my own research is, by contrast, an example of the latter phenomenon. Detailed interpretation is comparatively absent from the algorithms themselves, although Hayles makes the valid point that humanistic interpretation and algorithmic data collection often operate in more of a symbiotic relationship than the literature typically depicts. Exploratory data analysis (EDA) (Müllensiefen and Frieler 2022, 4–16) is a good example of this: the analyst will often use basic EDA techniques to 'get a sense' of the dataset, this will inform the hypotheses they develop, and then they will develop more targeted algorithms to consider these topics. Huron (2013) has condemned this type of exploratory analysis, proposing that it degrades the dataset under consideration, because running successive statistical tests (whether exploratory or not) on the same corpus increases the probability that a test will incorrectly identify significance. Alan Marsden (2022), however, has argued against Huron, correctly pointing out that if successive tests are truly independent then they are unlinked and so have no related implications. However, if later tests are based on earlier ones (as in the workflow I outlined above) then the proposed theory 'becomes a theory of *that corpus* rather than a theory with wider applicability' (Marsden 2022, 38, italics his). In many studies this is problematic because they are treated as a representative sample of a wider population; in my own research this is not at issue as I am interested strictly in Webern *qua* Webern, I do not use Webern's corpus as a sample of some larger body of work.

More fundamentally, Moretti suggests that the distance involved '*is a condition of knowledge*' (Moretti 2000a, 57, italics his) and in so being governs what the knower is able to know. In Moretti's telling, distant reading allows the analyst to focus on the very small and the very large, whilst 'the text itself disappears' (57 Moretti 2000a). In order to trace these large-scale stylistic phenomena, the rich detail of the individual text has to be abandoned. Following Weber, he argues that in order to develop any abstract theoretical knowledge, this complexity must be left behind. With regard to the act of distant reading itself, this seems largely correct. This obscures a huge amount of detail that is left to the computer to assess, and means that the interaction with the music itself, the movements, is always at least one level removed. This allows scholars to consider, however, the broader style, and to quantify stylistic features and trace their change across time, in a manner that would be impossible manually. Developing this subject, Michael Gavin (2020) has delved deeply into the implications of reading text from such a distance in his provocatively titled 'Is there a text in my data?'. Gavin's subject is the theoretical implications of counting words. This is highly pertinent for my own research as it counts similar small items, be they pitches, intervals, or harmonies, and considers them along the lines of the 'bag-of-words' model (Manning, Raghavan, and Schütze 2008, 107), familiar from literary studies, whereby the collection of items that comprise a piece are treated as an unordered collection. This is an easy model to attack. Intuitively, it seems that the gap between this model and the subjective experience of reading, an intrinsically linear experience, is so vast as to be disqualifying in itself, or that even if it is accepted, the results of such an analysis must be fundamentally impoverished by this reduction of complexity. Along similar lines to Moretti, however, Gavin makes the case that though this difference is certainly vast, that is not a problem. In fact, Gavin persuasively argues that, using the tools that literary studies have developed to process word counts, the analyst can increase complexity. Rather than the analyst's subject being a single text, it is instead a large-scale dataset. In this dataset (in Gavin's case, a standard vector model) each text, indeed each word, is represented by its relationship to the corpus as a whole and this therefore provides wide-reaching information about the individual texts and their broader context. In Gavin's words, 'every instance is stamped with an image of the whole against which its unique properties become more clearly visible' (Gavin 2020, para. 35). It is exactly this lack of complexity that weakens McKenzie's aforementioned study of Webern's works on a foundational level. Although he sets out with an ambition to achieve meaningful comparison across the corpus, the reality of this is rather underwhelming. Comparisons are left to off-hand comments offered, with little evidence, about often highly superficial features of Webern's style. There is no developed comparative work precisely because of the method of working: the thrust of the thesis is in the comments about individual works because McKenzie needs to outline his subjective experience of the music, which leaves precious little space for a comparative discussion. More fundamentally, it would likely be impossible for McKenzie actually to consider each of these 31 works in relation to each other. This wide frame of reference is exactly what a computer can achieve that a single human cannot.

There is another contribution of distant reading, however. I propose that one of the very strengths of distant reading is its use *in conjunction with* close reading. For me, one of the most exciting contributions of distant reading is its ability to contextualise individual works in relation to the broader corpus. Moretti argues that there is an ethical issue presented by close reading, that valorises a limited canon, with the attendant problems of selection that accompany this. My suggestion is that close reading informed by a distant analysis allows the analyst to choose those works on which to focus with an understanding of their place in the wider context that is based not on the vagaries of reception, but rather on some empirical factor. Thus, the analyst can choose to read closely those works which are perhaps most typical, or most unusual, secure in the knowledge of how they fit into the broader body of work under consideration. Of course, the same selection problems bedevil the choice of corpus: works encoded to a sufficient level of accuracy, in the right file-format, and easily available tend to be those of the pre-existing canon (though there are some efforts to rectify this, of which the OpenScore Lieder Corpus is a notable example (Rootham, Jonas, and Gotham)). In this thesis, the corpus is restricted to the works of a single composer, Anton Webern. Though he is indisputably perceived as a high-status member of the traditional Austro-Germanic canon, as I discussed in the Introduction within his own body of work certain pieces have been formed into something of a 'Webern canon'. In that context, this thesis seeks to rectify that by treating each movement equally and assessing almost all of them. When I home in on particular movements to read closely for a case study, I make my selection based on the wider context, and the place of the movements within that. Thus, I can discuss movements as typical or unusual, and in the process of doing so I will often expose prior scholarly work that makes unfounded or even incorrect claims.

Chapter 3

Chromaticism: Counting Pitch Classes

3.1 Introduction

Attempting to quantify degrees and types of chromaticism has been a long-standing project for music analysts. In particular, there has been an endless fascination with that body of music written around 1910 that seems to straddle various 'worlds' of harmony: not tonal in a common practice sense; not atonal in a classical sense; not even unambiguously modal. Various authors have attempted to apply computational methods to this repertoire in a bid to deploy technological power to cut through the knot of possibilities. Pople's (2004) prototypical *Tonalities* project, for example, sought to automate the identification of chords and, as he put it, gamuts. The hope was that a computational approach would allow 'an explicit competition between a number of more-or-less viable hypotheses about successive fragments of music' (Cross and Russ 2004, 148). More recently, Tymoczko has proposed a number of computational tools to assess aspects of what he terms 'macroharmony' (Tymoczko 2011, 158–191), though further development of these ideas has been limited. There is a more extensive conceptual treatment of macroharmony in Chapter 2 Section 2.2.3, but in short it is a term pioneered by Tymoczko (2011) to characterise harmonic entities that are larger than the chords or motives on which analysts tend to concentrate. These entities, or segments, are defined by the analyst. On the wide end of the spectrum, macroharmonies can encompass as large a passage of music as the analyst views to be helpful, up to all the notes in a piece; on the local level Tymoczko offers no strict definition as to what makes a macroharmony distinct from a 'normal' harmonic segment. I propose that for a macroharmony to be conceptually valid, if it is short in duration, it must be a segment large enough and/or varied enough that it can clearly be subdivided into multiple distinct smaller harmonies. Thus, the total content of a sequence of a few chords could be considered a small macroharmony while each individual chord likely could not; likewise, a short and stable motive from a longer melody would not be considered to articulate a macroharmony, while the entire melody itself might.

In the following chapter, I will first consider macroharmonies using the same underlying technique on two levels. The technique is the pitch-class distribution, the weighted distribution of all pcs used in a particular segment. The two applications of this technique are first to assess whole-movement macroharmonies as individual entities, and secondly to look at pc circulation, which describes how these macroharmonies are constructed and prolonged. The pc distribution provides a sense of the overall harmonic palette of a segment: it reveals how many pcs are used, and how (un)evenly they are deployed, in the process indicating how chromatic the music is. Pc circulation is a comparatively blunt tool that gives an indication of the rate of harmonic change across a work by revealing how many pcs tend to be in circulation for a given window of time, and thus how chromatic the music is. As is a common preoccupation throughout this thesis, across all of these areas I am particularly interested in the way the macroharmony of these works changes across the corpus, and particularly what impact is made by the advent of the dodecaphonic technique, first introduced in Op. 17.

Before going any further, however, I will briefly discuss the term chromaticism. Dictionary definitions, and indeed colloquial usage, tend to categorise chromaticism as one face of a coin, the obverse being diatonicism (Dunsby and Whittall 1988; Dyson and Drabkin 2001; Kennedy and Bourne Kennedy 2013).

As such, chromaticism is seen to be a technique for colouring, extending, and ornamenting the standard tonal (or at least modal) pitch language. If a non-chromatic scale definitionally uses some subset (A) of the 12 available pcs, chromaticism describes the use of that complementary subset (A′). From a quantitative standpoint, then, the degree of chromaticism might be calculated by the significance of these chromatic pitches in comparison to diatonic ones. If fully diatonic describes music that never deviates from the ordained subset of pcs, fully chromatic thus describes music that refuses to distinguish A from A′, and in so doing eradicates A as a meaningful entity. So far, nothing too unusual. This is roughly the same conception of chromaticism that is presented by Richard Cohn, though he allies it to an historical tale in which 'chords and scales are transformed into sets and set classes' (Cohn 2012, 207). Again, with a principal focus on nineteenth-century music, the 'age of Webern' (Cohn 2012, 205), as he puts it, represents a terminus. This produces a situation, however, in which any music that rejects establishing a basic hierarchy of pitches is described simply as 'chromatic', with no further differentiation. This is problematic because, as I will go on to show, music that truly refuses to differentiate between pcs is actually extremely rare, even if music that refuses to establish, or even propose, a tonic is much more common. Indeed, even much of the dodecaphonic music typically held up as the archetype of this attitude does not really satisfy the condition of true pc equality. For analysts of early twentieth-century music it is imperative to distinguish between these various musical styles, and not treat them as one undifferentiated, chromatic morass. In the context of Webern, there is clearly an enormous difference between the early Lieder, for example Opp. 3 & 4, and the *Gesänge*, Op. 23. In my own research, then, I propose, initially at least, to adapt the spectrum outlined above. Rather than a line running from diatonicism to chromaticism, I think in terms of varying degrees of chromaticism. Instead of conceptualising chromaticism in terms of inclusion relations, whereby a pc is *either* diatonic or chromatic, chromaticism can be thought of as taking account of a situation in which all pcs are available, but some degree of hierarchisation takes place. Thus, music in which all pcs are treated wholly equally from every perspective (the platonic ideal of early dodecaphonicism, perhaps), would be understood to display maximum chromaticism, whereas music in which there is more differentiation in the significance ascribed to different pcs is understood to be less chromatic.

3.2 The Corpus: Works and Encoding

3.2.1 What to include?

A crucial step in any corpus study is defining the corpus. As I have outlined above, the underlying premise of my research is to consider everything that Webern wrote to provide a holistic perspective on his practice. Whilst this might seem simple, alas determining what counts is not quite as simple as that ambition suggests. An expansive perspective might consider literally every note: not only works that conventionally fall on the margins like unpublished works or juvenilia, but unfinished works, sketches, indeed any time Webern placed a notehead on a stave. This could certainly be an exciting, if logistically and perhaps ethically dubious project. Although sketches are typically considered 'fair game' in the analysis of individual works, incorporating them into this sort of project would provoke difficult questions about weighting: should an early draft of a passage be accorded the same significance in characterising 'Webern' as a finished version of a piece, worked over repeatedly and released to the composer's satisfaction? Indeed, while sketch study can clearly be immensely valuable in describing a composer's process, and thus the history of an individual work, does a draft actually represent their aesthetic position, if they then actively *reject* given features of that draft? If the answer is no, circumspection should surely be extended to works that a composer ultimately chose either not to publish, or indeed to publish and later retract. Clearly a composer's aesthetic sense and preference tends to change over their career, and tracing those changes is part of the aim of this project, but I propose that there is a category difference between a composer experimenting through sketching before coming down with a finished work (even if that takes years or decades), and a composer concluding a work and then reconsidering it. In one the composer's perspective on the piece remains unstable enough that they themself view it as still in the process of being created; in the other they have decided (even if due to outside pressure from performers, commissioners, or others) that it is ready for a public airing. This might seem to present some simple criteria then: sketches are out, finished pieces are in, irrespective of whether they were later withdrawn. Again, however, the picture is not quite so simple. In the Webern catalogue, there are both works with and without Opus numbers, and as the composer had a hand in assigning the Opus numbers, it seems reasonable to suggest that these works were deemed more significant by the composer. Indeed, this speculation is lent further credence as the process of numbering was one that Webern carried on during his lifetime, and only began to take on its present form starting in 1920 with his commitment to Universal Edition (Meyer and Shreffler 1993a, 3755), rather than being a fully retrospective act (the title page of a 1913 copy of Op. 6 (reproduced in Beale 1966, 36), for example, lists it as Op. 4). Unnumbered pieces include both complete works (*Im Sommerwind*; the 1905 String Quartet) and unfinished ones (the 1914 Cello Sonata, the 1924 *Kinderstück*), but even the complete works were not always played, as is the case for the aforementioned Quartet, not premiered until 1962 (Moldenhauer and Moldenhauer 1978, 723). This raises potentially unanswerable questions: in particular, had Webern always had a publisher, would any of the complete unnumbered early works have made it in, rather than being rejected by the Webern of the 1920s? Did Webern reject them because he found them to be weak, and so unrepresentative, on their own terms, or merely from his new aesthetic position? Recent work has highlighted their importance (Wedler 2017) in Webern's compositional development, and pieces like the *Langsamer Satz* are as frequently performed as much of the numbered music, suggesting that from audience perspectives, at least, some of them belong in Webern's oeuvre.

Nonetheless, making a fixed determination seems impossible, and so in my research the corpus is restricted cleanly to the 31 works with opus numbers that constitute Webern's published oeuvre, and thus the largest body of work that can uncontroversially be seen as representing Webern's output. None of this is to say that this other material is not worthy of interrogation; I outlined the value of the sketches above, but an issue with the corpus I use is its paucity of tonal music, which could be greatly bolstered by the inclusion of some of Webern's earlier music. This corpus also privileges the revised, published versions of the early works. Meyer and Shreffler (Meyer and Shreffler 1993a, 1993b) have convincingly shown that Webern's revisions, carried out between 1915 and the mid-1920s, sought to 'update' his works informed by serial experiments and his new, classicist aesthetic. As with the unpublished works, future research comparing different versions of these works could lend an insightful perspective and follow Meyer & Shreffler's exhortation to '[broaden] the object of analysis' (Meyer and Shreffler 1993b, 376). In the following research, the corpus is therefore privileged towards Webern's later aesthetic, even though his revisions had limited impact on the pitch and durational matters under observation. Finally, movements of multi-movement works are considered individually: each movement is sufficiently harmonically self-dependent that it is more meaningful to analyse it on its own terms, much as Bailey (1991, 12) understands serial movements having their own prime row-forms.

3.2.2 How to encode it?

In encoding the scores used for this project, my principle was to recreate the relevant Universal Edition score as closely as possible. On the whole, this was a fairly simple process that required little editorial intervention. This was particularly true of pitch and duration notation; the greatest ambiguity surrounded the exact placement of dynamic and tempo markings, and whilst the first has little bearing on this project, in practice the second could usually be inferred from proximate musical gestures. Nonetheless, in order to use these scores in music21, various alterations had to be made to the scores. At the heart of this project lies the methodological assumption that scores can be viably used as proxies for sounds: these interventions seek to minimise the difference between these two. I outline these here both to ensure full transparency of method, and also as a guide for similar work.

The first group of alterations are small-scale local interventions that interpret particular notational phenomena. Trills have been notated as dyads, assigning both pitches the duration of the full note. Fast alternations between notes (Figure 3.1) have similarly been modelled as a sustaining a dyad (Figure 3.2). In both cases it is understood that the alternations should not be read as multiple new attacks. Percussion trills have been assumed to be a prolongation of one note unless a second pitch is explicitly indicated. This is rarely relevant, but applies, for example, to the glockenspiel trills in Op. 10/ii. Only one has a secondary pitch indicated. Cross-referencing this with the autograph manuscript indicates that while Webern was punctilious about clarifying secondary pitches in other instruments (e.g. the clarinet in b. 2) none of the glockenspiel trills have secondary pitches indicated, unlike in the published UE score. One can only presume, therefore, that Webern inserted these prior to publication. (Frankly, I would not be surprised if it were actually an editorial error, the E♭ inserted to ensure that the glockenspiel matched the piccolo, but in the absence of any further evidence, this remains pure speculation.)

Turning to the piano, arpeggiations have been removed from chords. In part this is to standardise the inevitable variety of performance, in part because the aural effect is still of one harmony. The sustaining pedal presents further difficulty. My approach has been to prolong notes for the indicated period in any case where Webern indicates use of pedal, and to assume that performers otherwise would not use the pedal. Though this latter assumption is certainly speculative, it seems legitimate lest they cloud the often ascetic textures. Indeed, the only works for piano with no pedal markings are Opp. 11 and 27, and surely neither would benefit from unmarked prolongation.

Figure 3.1: Op. 10/iii, bb. 1–3, original version. © Reproduced by kind permission of Universal Edition A.G., Wien.

Figure 3.2: Op. 10/iii, bb. 1–3 processed version.

String instruments present some further interpretative matters. Harmonics have been spelled out at sounding pitch, typically notated by Webern, but otherwise easy to infer. Timbral distortions, for example *Am Steg*, have been modelled as having no pitch effect. The reality of this is hardly so clear cut: playing on the bridge often emphasises the second partial at the expense of the fundamental, resulting in an effective transposition up an octave. However, this is highly irregular, often changing within the course of a note, and so is impossible to model consistently.

Octave transpositions do, however, inspire a tangle of interpretative judgements with regard to Webern's notation. It is not until Op. 14 that his scores begin to be notated at sounding pitch, and even then all is not clear. The following is a survey of those instruments where transpositional matters are not always clear, with some discussion of my reasoning in each case. In the case of ambiguities, the general principle I have adopted is to minimise octave doubling in favour of unison doubling where possible, given Webern's general apathy towards doubling at the octave, particularly in the more contrapuntal, later music.

To begin with the easiest cases, the guitar is transposed down an octave in Op. 10; in Opp. 18 & 19 it is specifically marked as sounding at the written pitch, which also confirms Webern's knowledge of the traditional transposition. Similarly, the piccolo is transposed up an octave, contrabassoon and contrabass down, until the final three orchestral pieces (Opp. 29, 30, and 31), where everything sounds precisely at pitch. French horns present a problem with regard to the bass clef. I have opted to read it in the traditional (now outdated) manner that transposes up a fourth rather than down a fifth. This only occurs in Op. 13, and is principally justified by the first note, which would otherwise give a sounding A1.

The knottiest problems come, however, with the celesta, glockenspiel, and xylophone. I will discuss each of the relevant pieces in turn. As before, Opp. 30 and 31 are simple, as they are notated at sounding pitch, other pieces, less so. With regard to Op. 6, my policy has been to read the celesta as transposing up one octave, as would be expected. The evidence for this is limited, but rests on places where reading it at concert pitch would create octave doublings in the harmony (e.g. movt. i b. 15; movt. v bb. 11–12). The glockenspiel is harder to judge, and will be discussed below, in light of the other works. Op. 10 presents some clearer evidence. I have read the celesta as transposing up one octave (e.g. movt. i bb. 1 & 10, movt. ii b. 5 & 7, movt. v b. 10), and the xylophone and glockenspiel both up one octave (the traditional transposition in the case of the xylophone). Movt. v provides the justification here: in bb. 8–9 they are doubled by the harp, bb. 16–17 the violins. In Op. 13 I have maintained the same transpositions as Op. 10. There are at least three clues in the celesta part that point towards this: movt. i b. 17, movt. ii bb. 21–22 and 24–25. In the first case this is a matter of doubling, in the second and third, to match the surrounding texture. For the glockenspiel, movt. i b. 17 suggests a single octave transposition, though movt. ii presents a contradiction, as bb. 24–25 and 28 double the violin at different octaves, so a transposition of either one or two octaves would result in one of these passages doubling at the octave rather than the unison. Finally, despite the lack of clarifying markings, in Opp. 26 & 29 I have read the celesta and glockenspiel at pitch, matching the other instruments discussed above, and supported by some examples (e.g. Op. 26 bb. 12–13). Returning to the glockenspiel in Op. 6, the situation is far from clear. Traditionally the glockenspiel transposes up two octaves, but to maintain consistency with Opp. 10 and 13, in this piece I read it as transposing by one octave. Counter to this, two examples suggest that an argument could be made to read it at concert pitch: movt. iii bb. 5–6, and movt. v bb. 19–21. In the former case, transposing would create contrapuntal parallel octaves; in the latter, consulting the earlier 1909 score shows the glockenspiel (written) doubling the trumpet exactly. In the latter case the removal of the trumpet removes this melodic octave doubling; in the former, this is perhaps the exception that proves the rule. Regina Busch has written of octaves in the *Bagatellen* that they 'can no longer be taken for granted is an inadequate observation; that they can no longer occur or should occur is going too far' (Busch 1991, 14); unusual, then, but not fully absent. Many of these decisions are speculative. They rely on an assumption about Webern's practice (that he disliked octave doublings) that is largely inferred from an intuitive knowledge of his music and style. Nonetheless, there is enough evidence to suggest that he consistently notated the celesta, xylophone, and glockenspiel each sounding an octave higher than written, until the final four orchestral works, all written at sounding pitch.

The final matter to mention here is a peculiar quirk of the Sibelius-XML-music21 chain which, although easy to miss, wreaks havoc if unattended. The matter in question is full-bar rests. In Sibelius, as is notational convention, irrespective of

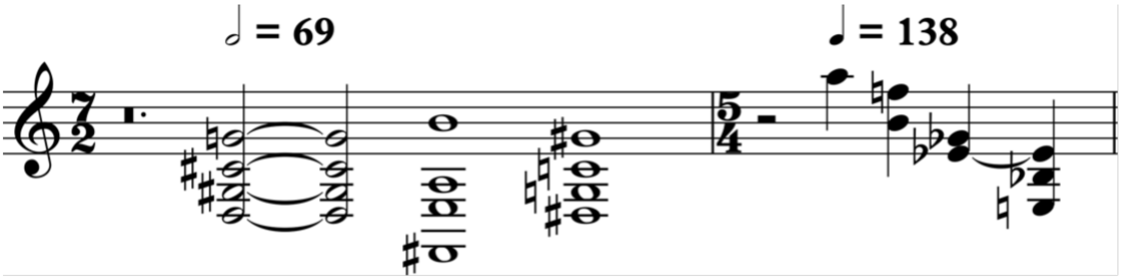

Figure 3.3: Op. 29/i, bb. 1-2, chordified, with problems.

time signature these are denoted with a semibreve-shaped rest, located in the middle of the bar. In exporting a file to XML, these are therefore defined as 'whole' rests. When importing these into music21, however, these 'whole' rests are understood to be 'whole-note' rests, with a duration of four crotchets. As such, any part with such a bar in a time signature with fewer than four crotchet beats suddenly gains extra silent beats at the end of the bar. Not only does this extend the duration of the given work and introduce all sorts of unintended silences, but it often misaligns the parts (e.g. Figure 3.3). My solution to this was to use a plug-in to replace full-bar rests with rests of the correct duration spelled in quavers, a crude response, but an effective one.

Many of these editorial decisions rest on my personal reading of Webern's music. Whilst I have tried to hew as closely as possible to the realities of the score, in certain cases interpretation and inference play an important part, particularly in realising these scores as proxies for sounding realities. It is fully possible that other scholars would interpret some of these features in other ways; in many cases there are no definitive interpretations, and so I have opted for those interventions that I think make most sense in the context of the broader corpus.

3.3 Pitch-Class Distributions: Whole Movements

3.3.1 Introduction

This chapter deploys pc distributions, a technique with a significant pedigree in the analysis of tonal (Albrecht and Huron 2014; Albrecht and Shanahan 2013;

Perttu 2007) and neo-tonal (Tymoczko 2011) music, to consider macroharmonic effects in Webern's music. Pc distributions might seem to be relevant only to tonal and neo-tonal repertoire, particularly as they are typically deployed to support key-finding. Instead, the contention of this chapter is that pc distributions can provide new information about atonal music. They are an effective way of considering the degree of chromaticism, and might particularly provide insights into the so-called 'free atonal' music,[1] especially in comparison to the dodecaphonic music. To achieve this, having gathered the data, descriptive statistical analysis is used to consider Webern's practice, with particular concern for when and how the style changes, and the impact of pre-conceived boundaries such as the onset of first atonality and then dodecaphonicism.

3.3.2 Method

A pc distribution is, as the name suggests, the distribution of pcs in a given piece (in tonal music this is often expressed in terms of scale degrees). These proportions may be calculated in terms of instances of each pc, or, as in this research, total durations. In summary, this algorithm counts all of the individual notes in a given movement, records their pc value and duration, and summarises the cumulative proportion of each pc as a proportion of the duration of all the notes. It is important to note that the duration of all the notes is not the duration of the piece: rests are not counted, and each of the notes in a simultaneity is counted individually, therefore providing a greater duration that in the original. As an analytical method, this is therefore conceptually simple and easy to deploy. Whereas for Tymoczko, with his focus on neo-tonal composers, overall pc content (i.e. which pcs are included and which are omitted) has some relevance, in an atonal context this is largely meaningless (they are always all included). This research, therefore, is more concerned with the overall features of pc distributions. There are important questions, however, about the practical application of pc distributions: how do you count a pc? The most basic and commonly used

1. *Pace* Forte (1973, x) I will use this term to refer to the music from Op. 3 to Op. 15, those works that are accepted as atonal, but were not composed with the dodecaphonic technique.

approach is merely to count the frequency of each pc. This is simple and easy to do computationally, but it has the enormous drawback of excluding any durational difference: a semiquaver is viewed, from the perspective of the pc distribution, as having identical significance to a semibreve. Common sense indicates that there is an issue here. Somehow pitch classes need to be weighted in order to take account of other features.

The underlying aim is to accord pcs a value that represents their significance to the analyst. As Matt Chiu points out, there are two principal ways to do this: musical weights, or mathematical ones (Chiu 2021, para. 2.3). The former deploy scholarly knowledge about the perception of music to inform weighting. As discussed in Chapter 2, as early as 1989, Lerdahl introduced ten such 'salience conditions' (Lerdahl 1989, 73–74) which can lend significance to a harmony. Examples include relative volume, timbral prominence, metrical position, duration, or motivic importance. The ability to deploy these computationally varies: relative duration is easy to apply; relative motivic importance is very difficult. Metrical position is comparatively easy in certain repertoires, but in Webern less so. As Bailey puts it with regard to some of the later music, perhaps risking understatement in her phrasing, 'Clearly the relationship between rhythm and metre in this music is complex and invites consideration' (Bailey 1995, 252). It definitely seems the case that the sort of basic metrical hierarchy hypothesised by Chiu (2021, para. 2.3.3), though he eventually does not adopt it in his own analysis, cannot be applied to this repertoire.

The alternative approach to deploying these musical features is to use a mathematical function, in particular a monotonic one (a function that preserves the order of values). Chiu suggests frequency-rank as one possibility, citing Tymoczko's use of it in pc profiles[2] (Tymoczko 2011, 170–172). Here, the twelve pcs are ordered by relative frequency and assigned a weight according to their rank (so, the most frequent pc would be weighted at 12, the second-most frequent

2. Note that pc profiles are distinct from pc distributions in being a theoretical rather than analytical construct.

at 11, etc.). The obvious issue with frequency-rank weighting in this context is that it eradicates differences between degrees of frequency. A pc distribution with frequencies of (0 1 2 0 0 0 0 0 0 0 0 0) would be appear to be identical to one with frequencies of (0 1 100 0 0 0 0 0 0 0 0 0). In an analytical context, this throws out so much information as to risk being useless. The alternative option that Chiu suggests, and adopts, is to use the $log_2(x + 1)$ function (Figure 3.4). As Chiu argues, the value of using a logarithmic function is that as the frequency of a pitch class increases, its weighting gradually decreases, which mimics our experience of music: 'after establishing a macroharmony, repeating a pitch does little to perceptually disturb our sense of the macroharmony' (Chiu 2021, para. 2.3.5). There are diminishing returns to the contribution offered by repeating a pitch class. To my ears, this sort of weighting is very valuable in considering macroharmony on the local level that Chiu does (he assesses passages of 12 beats, or approximately 10 seconds). I am less convinced, however, that it applies to my experience of macroharmony across an entire movement, which is more influenced by the gradual accumulation of various smaller-scale macroharmonies than as if experienced as one entity. I therefore apply multiple approaches in my research. As this chapter considers pc distributions of full movements they are weighted according to duration. These are calculated in seconds according to the metronome marks in the scores. Though analysts often use the relative proportions of written note durations (a quaver being half the duration of a crotchet, for example), this fails to allow for tempo changes which are frequent in this corpus.

Regarding preconceptions about pitch-class distributions, the literature is clear that tonal works are characterised by variety. Figure 3.5 is a distribution using data from Joshua Albrecht & Daniel Shanahan's (2013) corpus study of 625 major mode and 357 minor mode works. Although the corpus is weighted towards earlier music, it does include more chromatic music by Brahms, Chopin, and Kabalevsky. The distribution clearly outlines the diatonic scale, particularly emphasising the tonic and dominant. A historical analysis by Albrecht & David

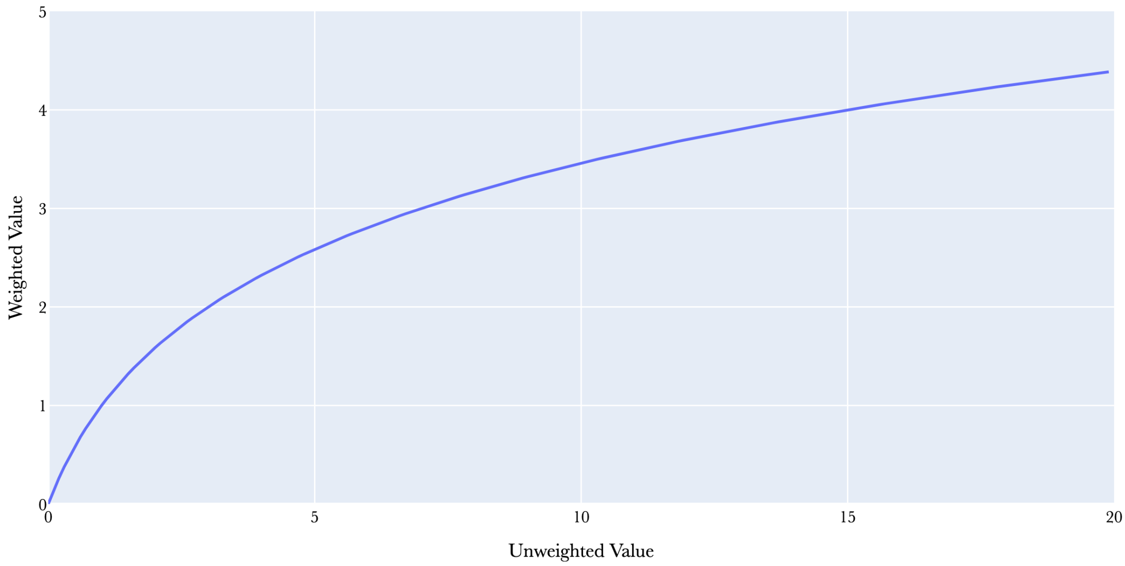

Figure 3.4: $log_2(x+1)$.

Huron (2014) has shown that, despite detailed changes in distributions, these overarching features persist across the period 1400–1750. White (2013) has also explored the different archetypal distributions across whole movements, phrase beginnings, and phrase endings, finding that although the relative preponderance of secondary pitches in the scale varies, the distribution always outlines the diatonic scale and emphasises the tonic and dominant, with the third-rank position always going to the mediant degree. Whether these archetypal distributions will define music as chromatic as Webern's Opp. 1 and 2 is less clear, but a fairly strong correlation might reasonably be expected. Conversely, I might anticipate that a dodecaphonic work would have a significantly less varied distribution. Whilst a composer could write a classically dodecaphonic piece that replicated the distribution of Figure 3.5, as Tymoczko points out, dodecaphonicism was in fact 'explicitly designed to promote flat pitch-class profiles' (Tymoczko 2011, 183). As for the freely atonal music, expectations are much less clear: it is possible that the advent of atonality encouraged a swift move to pc equality, but it is equally feasible that there was a gradual change over time; clustering the corpus will help with this enquiry. Much of this analysis assumes a relationship between duration and significance. This is not wholly determinative;

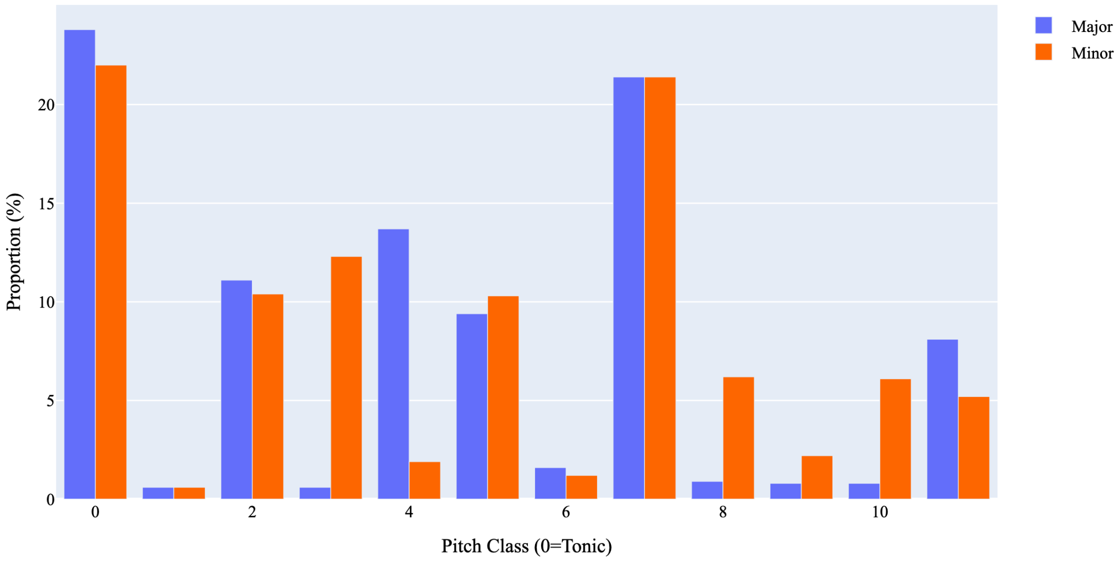

Figure 3.5: The relative frequency of scale degrees (data from Albrecht and Shanahan 2013, 67).

nonetheless, patterns in the overall distribution certainly play an important part in shaping the musical surface.

There is some debate in the literature about what to sample in order to consider pc distributions. In most cases, scholars have sought to find typical distributions of keys. Modulatory passages can cause issues with the count, and so scholars often sample only beginnings or endings of works, arguing that these will present more apt distributions (Albrecht and Huron 2014; Perttu 2007). Jason Yust's work has interrogated this further, suggesting that, in tonal music, entire pieces tend to have a flatter distribution than a combination of beginnings and endings, but that whilst endings behave independently, beginnings and whole pieces are similar. In the present paper, the focus is not on creating archetypal distributions, but instead on comparing the specific distributions of these works and so these are measured across movements. Yust's finding that whole piece distributions approximate those of beginnings is interesting in this regard: his speculation is that 'the typical choices of contrasting key areas tend to reinforce the distributional properties of the home key in some respects and cancel one another out in others' (Yust 2019, 13). Whether the same is true of atonal music is beyond the scope of my research, but this legitimates comparisons between these distributions and archetypal tonal ones.

3.3.3 Results

The following summary outlines the key results; the full data is given in Appendix B. In Figures 3.6, 3.7, and 3.9 the corpus is ordered chronologically, largely by date of composition as recorded in Hans and Rosaleen Moldenhauer's biography (Moldenhauer and Moldenhauer 1978, 700–705).[3] This is largely equivalent to the ordering of Opus numbers, though not universally so. The only major uncertainty is for Opp. 3 & 4 for which the dates of composition are unclear. These have therefore been sorted by their eventual position in the published works. There is one error of which I am aware in the Moldenhauers' list: Op. 17/i. They list Op. 17/i as preceding Op. 16/v (October 1924) and Op. 16/i (November 1924), recording that there are no extant sketches for Op. 17/i but that in a handwritten catalogue list by Webern, this movement is listed as 'Autumn 1924' (Moldenhauer and Moldenhauer 1978, 315). Shreffler, however, notes that sketches discovered later in fact reveal the date of completion to be 10th December 1924, thus positioning Op. 17/i after the two canons (Shreffler 1994a, 285).[4]

3.3.3.1 Spread

The data gathered for pc distributions can be shown by a bar chart for each piece. In considering the corpus as a whole, however, it is helpful to reduce each movement's pc distribution to a single variable. As the subject of interest here is

3. For this list see Appendix A.

4. Throughout this thesis, chronological values have been recorded with regard to rank order in the corpus, rather than date of composition. The differences in chronological gaps between movements is thus not accounted for in either the charts or correlation calculations (i.e. two adjacent movements will be ranked chronologically as x and $x \pm 1$ irrespective of whether they were composed one month or one decade apart). This is in part due to incomplete data for the corpus as a whole, but also because it raises significant questions about which date is relevant: should it be the date of compositional completion, initiation, publication, premiere? Some average of these? For different movements, different dates might be more or less relevant, and to ascribe a single time to these works gives what I view as a misleading level of accuracy.

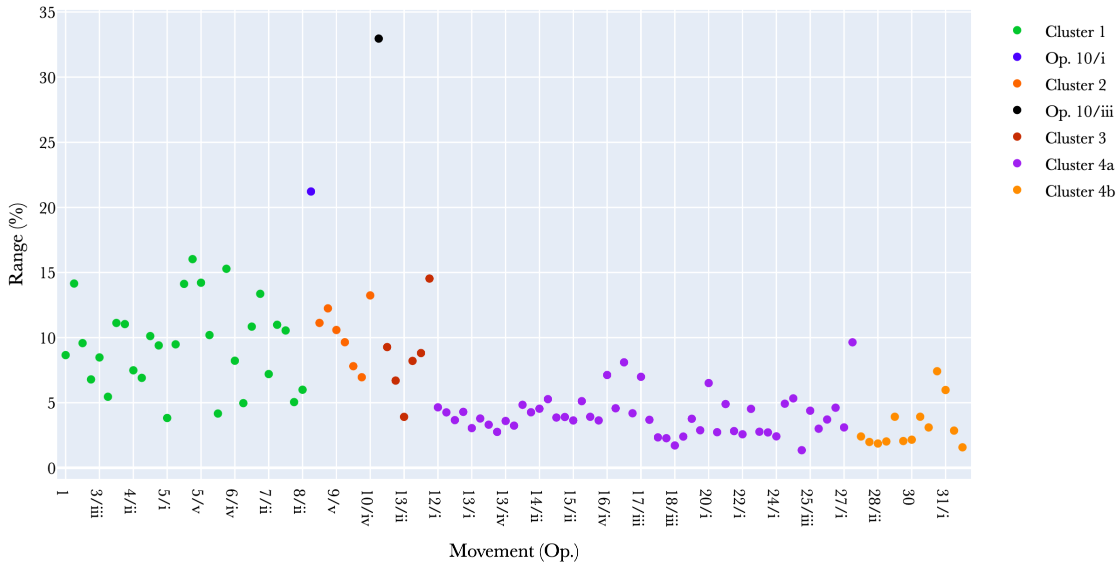

Figure 3.6: Pc distribution ranges.

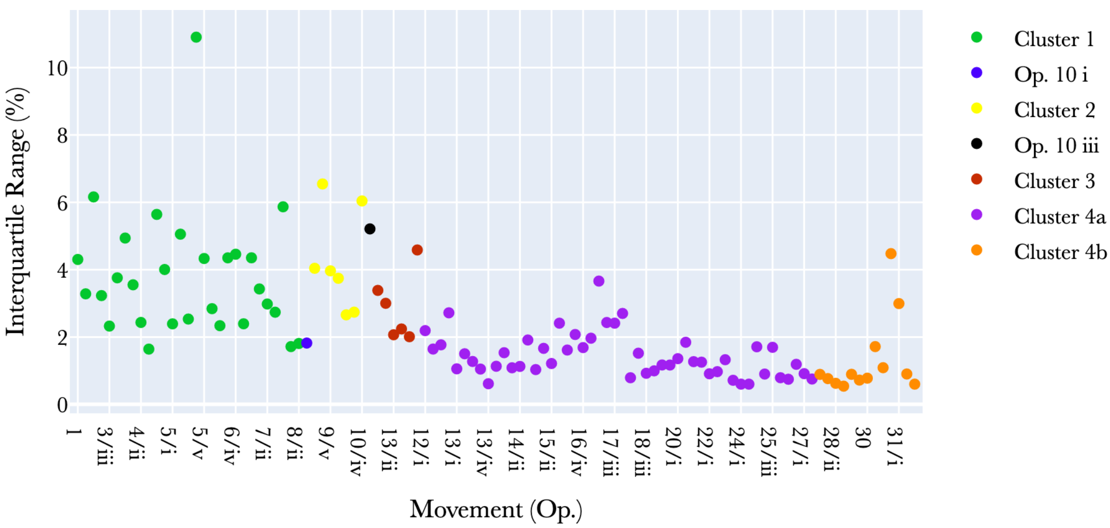

Figure 3.7: Pc distribution IQRs.

the variety within the distribution, the relevant metric is the distribution spread. As the results are nonparametric, this is calculated as the range of values: the difference between the proportions of the most and least common pitch classes. The greater the range, the more variation there is. Using the range to measure variety is a crude tool as it only takes account of the two extreme values and so does not express the degree of internal coherence, and so it can be complemented by the use of the inter-quartile range (IQR) which measures the difference between the first and third quartile, and so gives a sense of the internal variety of the distribution. As mentioned, tonal music is expected to have comparatively high spread values, corresponding to pc proportion discrepancies; dodecaphonic music, by contrast, would likely have a small range. Figure 3.6 is a scatter plot of the ranges, and Figure 3.7 of the IQR values. Clusters will be discussed below and derive from Figure 3.9. The general pattern is clear: despite some noise, distribution spreads decline across the corpus; indeed, the correlation between corpus position and range is a strong inverse correlation of -0.70.[5] The correlation between corpus position and IQR is -0.73, similarly strong. In summary, therefore, the results are largely as expected.

A further point of interest is which movements are unusually varied. These movements are those with an anomalous range, calculated according to Tukey's Rule.[6] Interestingly, both of these come from Op. 10: movements i and iii, with spreads of 21.2% and 33.0%, respectively, although they do not have anomalous IQR values. Figure 3.8 shows their distributions which reveals this disparity: for the first movement, the distribution is characterised by one anomalously high value above an otherwise typical distribution; for Op. 10/iii, both pc 8 and 9 are unusually high. One final movement with an unusual value is Op. 5/iv, which has a particularly high IQR (its range is also very high, though not anomalous).

3.3.3.2 Clustering

Variability-Based Neighbour Clustering (VNC) can be used to cluster the corpus empirically. Clustering reveals the level of similarity between the distributions of the works in the corpus. VNC is a hierarchical agglomerative approach that was specifically developed by Stefan Gries and Martin Hilpert (2008) to preserve corpus position and thus facilitate diachronic analysis (for an example of its application see van Hulle and Kestemont 2016).[7] Figure 3.9 displays this

5. All correlations used are Spearman correlations given to two decimal places, unless otherwise noted.

6. An anomaly is thus defined as a value lying outside $1.5 * IQR$ below the first or above the third quartile.

7. This code is by Folgert Karsdorp (https://github.com/fbkarsdorp/diachronic-text-analysis).

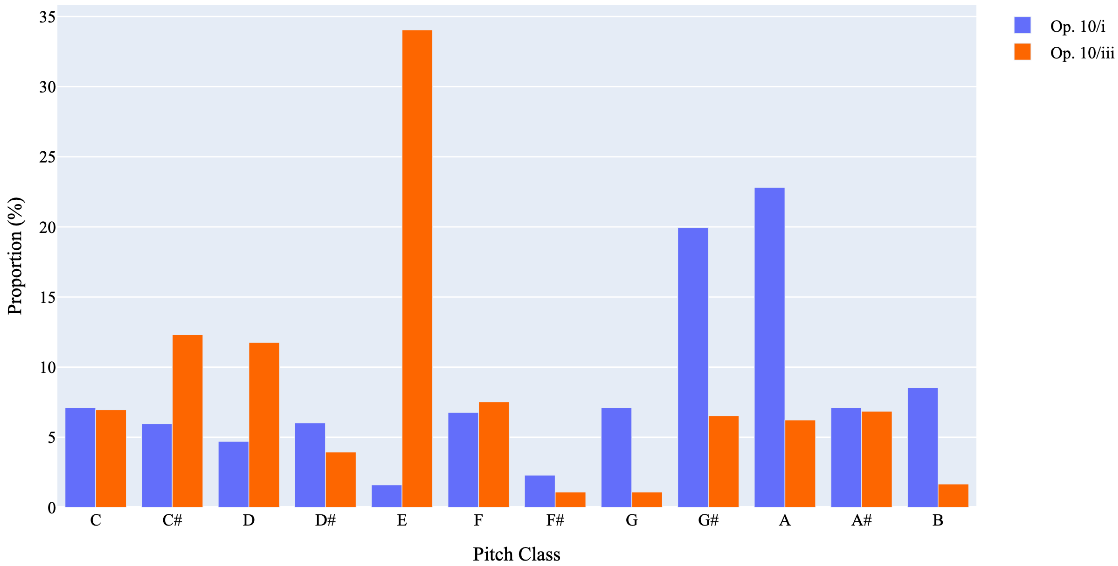

Figure 3.8: Op. 10/i and iii pc distributions.

clustering applied to Webern's corpus using complete pc distributions for each movement.[8]

The implications of this clustering will be discussed below; at this stage, it is worth drawing out the structure of the various clusters. As in the example in the introductory chapter of this thesis, in an agglomerative approach the number of clusters does not have to be pre-determined; rather, this is left to the analyst to discern based on the results of the algorithm, as presented in Figure 3.9. In this case, the picture is somewhat messy, with several movements clearly dissimilar to their neighbours (e.g. Op. 10/iii, Op. 27/ii). Nonetheless, there are several principal clusters that can be discerned. On the highest level, the corpus is partitioned between Opp. 10/iii and 9/ii. Given the anomalous value of Op. 10/iii, this is not hugely surprising. However, even if the three anomalous movements identified above are removed, the highest-level partition remains between Op. 10/iv & Op. 9/ii. Subdividing these two principal regions further reveals other groupings: in the latter half, Op. 9/ii–Op. 11/iii and Op. 12/i–Op. 31/iii, which can be further subdivided between Op. 27/ii and Op. 28/iii; the

8. Following Dirk van Hulle and Mike Kestemont (2016, n. 7) I use Ward linkage; the distance measure, however, is Manhattan distance to account for working with twelve dimensions.

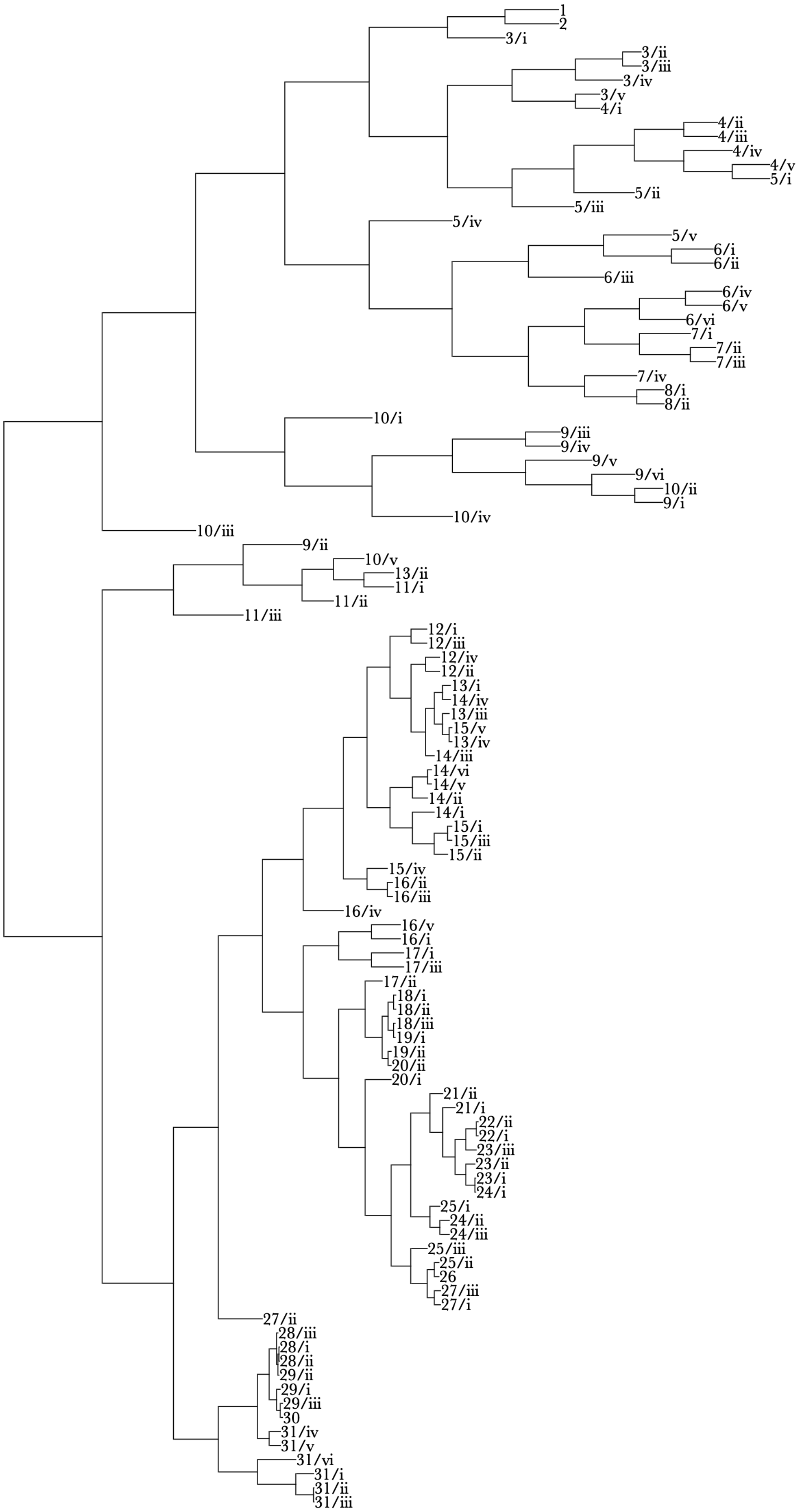

Figure 3.9: Variability-based neighbour clustering dendrogram.

first half is less uniform, but suggests two main clusters, Op. 1–Op. 8/ii and Op. 9/iii–Op. 10/iv, alongside the three anomalous movements.

3.3.4 Discussion

Having outlined the results, I will now consider the implications of this data, seeking to identify trends and consider its more overt musical relevance. This research is fundamentally post-hoc analysis, but it is buttressed by Webern's well-noted sensitivity to the structural functions of the total chromatic (Paccione 1988; Whittall 1983). I will start by looking at the overall picture: the major break points in the corpus, and their comparison with more traditional places of partition: the introduction of atonality; the onset of dodecaphony; and the question of a late style. I will then go on to consider some works in detail: Op. 21/i as an exemplar of Webern's dodecaphonic technique, and the two anomalous movements from Op. 10.

Turning first to clustering, Figure 3.9 proposes that the highest-level clustering partition lies around Opp. 10 & 11, rather than the principal partition in the corpus lying between tonal and atonal music, or indeed the freely atonal and dodecaphonic music. This is an observation supported by Figure 3.7, though not totally by Figure 3.6. Although the scatter plots are somewhat harder to parse, there is clearly much more variety in Clusters 1 and 2 than the later music, which is characterised by much smaller spread values (both range and IQR) and therefore much flatter distributions. The differentiation in groupings between Figure 3.6, which has a precipitous drop between Op. 11/iii and Op. 12/i, Figure 3.7, which has a more general decline, and Figure 3.9, which focuses attention slightly earlier, demonstrates the importance of a pluralistic approach. These three tools point to differing phenomena in the corpus, and though the overall trend is the same, with less hierarchical pc distributions later on, these three measurements distinguish between the different ways in which this happens. Nonetheless, inevitably there is a limit to what can be learnt here: analysis of pc distributions is only one approach, which focusses only on a very partial aspect of the music. The following discussion will thus be limited to such higher-level clusters, which therefore subsume any statistical noise generated by individual oddities into the greater whole; making extensive commentary on the lower-level divisions, the break between Op. 3/iv and v, say, risks elevating minor fluctuations out of their context. Shreffler has described the instrumental miniatures Opp. 9–11 as a 'crisis', inspiring a period of significant experimentation in Op. 12 and the ensuing Lieder in which Webern sought to establish a new style (Shreffler 1994b, 11). Adorno similarly describes Op. 12 as introducing a 'new expansiveness' (Adorno 1999, 99) in Webern's composition. This research generally supports that picture, with a separate cluster comprised of a later subsection of the instrumental miniatures Opp. 9/ii–11/iii. It is worth noting, however, that whether or not the two anomalies from Op. 10 (movts. i and iii) are included, this does not come with a clean break between Op. 11 and Op. 12, as the range values suggest. Instead, this later subsection is included in the second main division of the corpus. An explanation for this is provided by comparing Figure 3.6 and Figure 3.7. In Figure 3.6 there is a noticeable gap between Op. 11/iii and Op. 12/i, which suggests that the range values differ significantly in the Lieder from the instrumental miniatures; in Figure 3.7, however, the disparity is far less extreme, indicating that the IQR values are much more similar. This demonstrates, therefore, that while Webern clearly developed a new approach around this time, the difference between Op. 11/iii and Op. 12/i was less extreme than other comparable pairs in the corpus as a whole, and thus hints that the shift was more of a gradual process that came about through the instrumental miniatures and into the Lieder.

Helpful context for this period of time is also provided by the unpublished music listed by the Moldenhauers. Even between 1913 and 1914, i.e. between Op. 9/i and Op. 12/i, there is a large quantity of work that Webern did not include in his official oeuvre. This includes several pieces that were published posthumously but unnumbered: a set of five unnumbered orchestral pieces, a set of three orchestral songs, and the cello sonata (Moldenhauer and Moldenhauer 1978, 724–25); meanwhile, of extant sketches there are a work for women's chorus and instruments, eight orchestral fragments, a string quartet movement, and five works for voice and instrument(s) (Moldenhauer and Moldenhauer 1978, 736–39). Whether or not this is an unusual quantity of rejected material is difficult to ascertain given the limited survival of Webern's sketches, but it certainly supports an image of a composer experimenting, not always successfully, in a bid for a new style. Shreffler describes the sketches as presenting a wide variety of musical ideas, even across multiple sketches for what would become the same piece (Shreffler 1994b, 41), and so in all this suggests that Webern was in a particularly fecund period. In sum, then, this research certainly confirms the view that the miniatures were an extreme and required a change going forwards, but rather than this being a radical shift between two particular movements, it suggests that Webern was in a particularly exploratory part of his career, developing a new style which crystallised as he continued composing Lieder. Overall, then, Webern's earlier and later practice can be grouped apart from methods of construction: the first group encompassing tonal and some freely atonal music, leading up to the 'crisis'; the second covering the new freely atonal style and dodecaphonicism. On the highest level, it seems quite remarkable that the greatest change in the chromaticism of Webern's music came not with the onset of systematic dodecaphony, but rather through his own aesthetic intuition.

As for the start of the corpus, a similar picture appears. The partition between Op. 2 and Op. 3/i is at a comparatively low hierarchical level in the clustering diagram, indicating it is of little structural importance, meanwhile Figure 3.6 and Figure 3.7 show no particular difference between the tonal music and the early atonal music. Indeed, the distributions of these two tonal works (Figure 3.10) are quite dissimilar from typical tonal distributions. Op. 1 has a correlation of 0.61 with Albrecht & Shanahan's (2013) minor distribution, ordered with a D tonic. At first glance this seems quite high and is bolstered by some tonic/dominant emphasis and diatonic patterning in the distribution. As for Op. 2, its correlation with the major distribution, ordered with a G tonic, is 0.47, and though there is

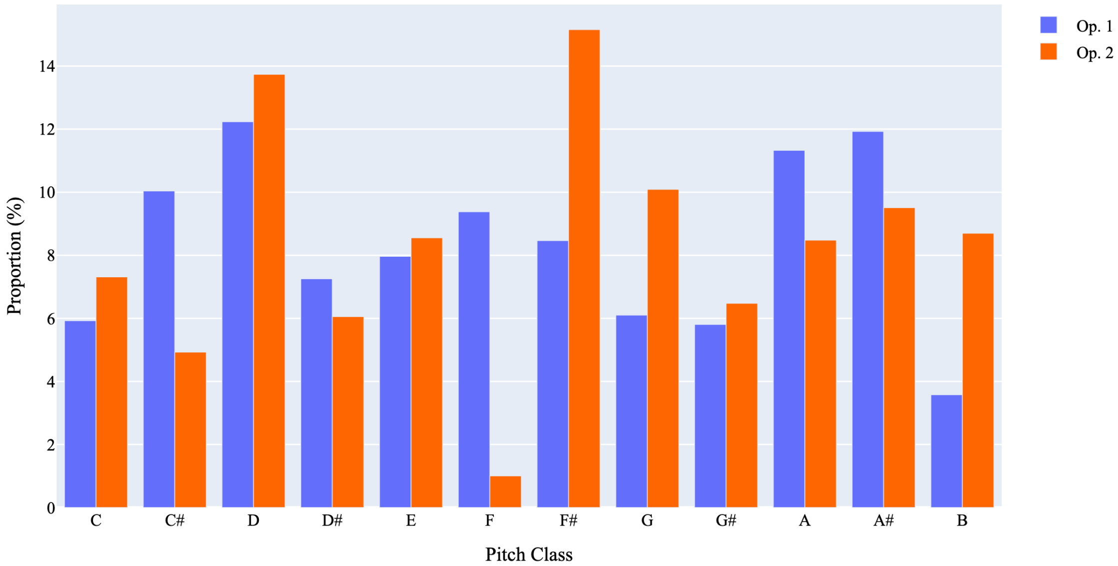

Figure 3.10: Opp. 1 & 2 pc distributions.

some dominant emphasis, it is otherwise distinct from the archetypal distribution, with an exceedingly high leading-note value. Even a peremptory glance at the piece hints at the sort of relatively flat distribution that might be created by such chromaticism: 11/12 pcs have been introduced by the end of the second bar, saturating the chromatic field. McKenzie accurately describes the tonality of Op. 2 as 'ambiguous most of the way through' (McKenzie 1960, 82), and these correlation values certainly indicate the increased chromaticism of the latter work. A tonal-atonal partition is the typical chronology between these two works and Op. 3, but this research shows this partition to be insufficient from the perspective of chromaticism as the two sides cannot be so easily distinguished. That said, a sample size of two tonal works is hardly representative and would be helped by significant augmentation with some of the *Werke ohne Opus*.

The final chronological period that merits discussion is the introduction of the dodecaphonic technique in Op. 17/i and the surrounding works. The first observation to make here is the comparative uniformity of the period that stretches all the way through Clusters 4a and 4b in Figure 3.6 and Figure 3.7, which are characterised by low ranges and low medians. For context, the range of values in Figure 3.6's Clusters 1–3 is 12.2% and in Cluster 4 is 8.3%; the median range values, meanwhile, are 9.4% and 3.7%, respectively. As such, whilst there is a somewhat less variety in later movement range values, the later ranges tend to be drastically much smaller in value, indicating significantly less variation in the pc distributions of those movements. The distributions of the dodecaphonic works themselves therefore confirm expectations: though there is inevitably some variation, this is minimal, and particularly in light of Figure 3.6, Webern's claim that 'All twelve notes have equal rights' (Webern 1963, 52) certainly seems apt. Shreffler (1994a) has drawn a picture of Webern experimenting with, and gradually adopting dodecaphonicism through Op. 15/iv and the canons of Op. 16, and this research strongly supports this. Along similar lines, writing about the Op. 14 songs, Adorno proposes that they 'sound like twelve-tone music, freed from every trace of the traditional language of music, every predominance of one tone or another' (Adorno 1999, 100), indeed that 'the distance from here to twelve-tone music was minimal, that it was no alien principle' (Adorno 1999, 100). Across this period, Webern's exposure to the dodecaphonic method was intense. As well as experimenting with the technique himself, the Moldenhauers record that during early 1924, so between the composition of Op. 15 and Op. 16, Webern was involved in coaching Schoenberg's serial Serenade Op. 24 (310–312). Even though he followed such close exposure to the dodecaphonic technique with the freely atonal, but highly structured, canons, Webern was clearly alive to the intensified focus on negating tonicisation through even pc emphasis. Shreffler is keen to note that Webern's adoption of dodecaphonicism should not be seen as a linear path, but more an experimental period in which various approaches were explored. Indeed Webern himself described the difficulty of choosing whether to adopt dodecaphonicism as comparable to 'taking the decision to marry' (Webern 1963, 54). Either way, the outcome from the perspective of this research is quite clear: Webern achieved the distributional effect that dodecaphonicism produced independently of it: there is no radical shift with the onset of dodecaphonicism in Op. 17, nor the partially dodecaphonic Op. 15/iv, nor the highly structured canons of Op. 16.

Having discussed the move into dodecaphonic composition, the dodecaphonic pieces themselves demand some consideration. Cluster 4 is large. Originating with Op. 12/i it encompasses all of the mid-period Lieder and the dodecaphonic music. As discussed above, however, Figure 3.6 and Figure 3.7 show that Cluster 4 is very consistent, especially as compared to the preceding music. Nonetheless, I have divided it into two sub-clusters, 4a and 4b, around the principal internal division, which lies between Op. 27 and Op. 28. A division located this close to the end of the corpus immediately raises the question of whether there is some sort of late style here. The Moldenhauers trace the composition of Opp. 24 and 25 as often overlapping (Moldenhauer and Moldenhauer 1978, 438), and a four month break (filled by Bach arrangement) separated the completion of Op. 25 (November 1934) from the commencement of work on Op. 26 (February 1935). This, along with the natural grouping of the three choral-orchestral settings of Jone's poetry (Opp. 26, 29, and 31) that Boulez draws attention to (Boulez 1968, 386–87) and Hoskisson describes as unified by a 'late choral style' (Hoskisson 2017, 11), has often encouraged scholars to chart Webern's final period as starting with Op. 26, a view with which Bailey (1995) accords. Other divisions have been proposed, however. Though Adorno does not make any claims for a 'late' period, he views Webern's works as changing for the worse after Op. 20 (Adorno 1999, 102).[9] Taking a more analytical and less historical perspective, Brian Moseley (2018) has proposed that the final four works of this corpus should be grouped on the grounds that Webern used row cycles to structure the music far more extensively than in earlier works. This is an interesting decision because it aligns exactly with Figure 3.9. Cross-referencing this with Figure 3.6 and Figure 3.7 does not immediately clarify why the clustering algorithm has identified this grouping: in both cases the values are marginally lower, with range medians of 3.78% for 4a and 2.41% for 4b, and IQR medians of 1.26% and 0.89%, respectively. However, running a Mann-Whitney U test on the range data

9. This article began life as one of Adorno's *Hessischer Rundfunk* radio broadcasts, which took place in the late 1950s and 1960s. As such, it post-dates his seminal essay on late style in Beethoven by at least twenty years (Adorno 2002).

for the two clusters reveals a p-value below the standard significance threshold of 5% ($U = 456$, $p = 0.03$), and thus that there is sufficient evidence to reject the null hypothesis, that there is no difference between the two groups. This suggests, therefore, that the difference between these two value sets is statistically significant. Fundamentally, these differences are minor. The variation in the post-Op. 12 works compared to the earlier three clusters is much smaller, but nonetheless it is an interesting finding that this approach finds the greatest turning point in the latter half of the corpus to be with Op. 28, rather than with the advent of the dodecaphonic technique (on which more to come shortly). From a cursory glance, Webern does not appear to have a radically different 'late style' in the manner of Beethoven or Schumann, and when tripartite partitions are applied to his work, they typically segment off the dodecaphonic music in its entirety as the third group. Here, instead, there is a proposal for an understanding of his music that emphasises continuity earlier on and draws attention to the innovation in his practice right up to the end. As mentioned above, Moseley (2018) argues that these final four works display a compositional shift in Webern's practice, towards ever-greater structuring. Moseley identifies Webern's use of chained row cycles as a method by which Webern could structure spans of time larger than the typical 12-note series with the features of the row. There is no reason *per se* to assume that this would create more even pc distributions, and assessing whether there is a predictive relationship here is beyond the scope of my research (and may well run into issues of statistical power, given the small sample sizes). Indeed, although Moseley correctly points out that row chain cycles 'produce serial structures larger than the original row that contain no adjacent pitch repetitions' (Moseley 2018, 171) they, almost definitionally, will limit the (non-durational) frequency of some pcs at the expense of others.[10] Again, this has only a limited relationship to the durationally weighted pc distributions of my research, as other compositional decisions, primarily about note durations, have a crucial role to play that Moseley does not consider. Nonetheless, the lower medians for range and IQR values

10. As one brief example, the chain in Moseley's Fig. 1a (Moseley 2018, 168) features pcs 0, 1, 2, and 11 three times each, whilst the remainder appear only twice.

suggest that Webern's control of the macroharmony of his works had reached new heights, an interest surely supported by his sophisticated exploration of large-scale structuring, demonstrated through the row chain cycles.

Looking at the dodecaphonic music more broadly, it is clear that this section of the corpus is characterised by flat pc distributions. This rather complicates Bailey's contention that through intersection and *Ausfälle* Webern limited certain pcs. Without a way into his head it is impossible to discern whether her supposition that 'it was not the equality of the twelve notes that was of primary concern to Webern' (Bailey 1991, 146) is accurate. She suggests that this feature emerged as part of Webern's dodecaphonic style; in fact, within the dodecaphonic works the correlation between corpus position and range is -0.06 and IQR is -0.34. These are weak correlations, indicating that though there was minor change, it was hardly a significant feature of Webern's developing style. Meanwhile, the median values are 2.9% for range and 0.9% for IQR. This therefore demonstrates that not only was there no meaningful change in pc distributions across Webern's dodecaphonic works, but also that the dodecaphonic works are characterised by highly even distributions. This indicates, therefore, that Webern's style was, from the perspective of chromaticism, consistent across the dodecaphonic period. Adorno suggests that there is a 'clear-cut caesura' (Adorno 1999, 101) that comes with Op. 21, but at least from this viewpoint, there is far more continuity than change.

The overall picture clearly negates Bailey's proposition that Webern regularly prioritised certain pcs at the expense of others. It remains plausible that this phenomenon might take place in occasional works, however, and as a relatively typical movement, Op. 21/i is a reasonable candidate. This movement has a range of 4.90% and an IQR of 1.27%, slightly higher than the mean values for a dodecaphonic work; the most common pcs in the distribution are E and C, while A♯ and B are used comparatively sparingly. Across the movement there are 34 pitches that are used once rather than twice to elide two row statements, these intersections are given in Table 3.1. There is no consistent relationship between pc frequency and the count or duration of intersections: as two examples, E, the most frequent pc in the distribution, is subjected to intersection (therefore potentially *lessening* its frequency in the movement) four times with a total duration of 6.5 crotchet beats; G♯ is only used for intersection twice with a total duration of 1.5 beats and is nonetheless the third most frequent pc. Bailey writes that Webern's dodecaphonicism is characterised by 'the deliberate manipulation of the relative importance of the twelve pitches through … a reduction in the aural presence of certain of the twelve notes through intersection and *Ausfälle*' (Bailey 1991, 145). This research suggests that there is simply no evidence for such a claim. Not only does Webern use 11/12 pcs in intersections, but there is little durational difference between those that he does use (from 1.5 beats for G♯ to 11 for D♯). This is not necessarily to be expected: there are 32 row statements in Op. 21/i, and thus the potential for 384 pitches. Only four are not elided with another row through intersection, and it would therefore be quite feasible to shape the pc distribution through those elisions, comprising as they do almost 10% of pitches in the movement.

Pitch Class	Intersection Count	Duration (♩)	Bar(s)	Instrument
C	2	3 3	11 50–51	vcl. vln. I
C♯	2	1 3	18 62	vln. II vln. II
D	4	1 1 4 0.5	11 20 52–53 62	hp. hp. hn. I vln. I
D♯	4	2 2 3.5 3.5	12 14 52 54	vcl. vla. vln. I vcl.
E	4	1 1 4 0.5	13 22 54–55 64	hp. hp. cl. vla.
F	2	1 1.5	16 60	vla. b. cl.

F♯	2	3 3	13 52–53	vla. cl.
G	4	1 1 1 0.5	14 23a 55 64	hp. vcl. cl. vla.
G♯	2	1 0.5	17 60	vla. b. cl.
A♯	4	1 1 acciaccatura 1 acciaccatura 0.5	19 36 38 62	vln. II vln. II vln. I vln. II
B	4	1 1 1 0.5	12 21 53 62	hp. vla. hn. I vln. I

Table 3.1: Row intersections in Op. 21/i.

To conclude this discussion of whole-movement pc distributions, I will consider the two anomalous movements identified above. As mentioned previously, these movements (Op. 10/i and iii) have distributions that are characterised by a single particularly prominent note. In each case this is musically apparent as a pedal: the Celesta G♯–A trill in the first movement, and the E shared variously between guitar, bells, celesta and harp in the third movement. The effect of this is to provide a central timbral and harmonic focal point, a note with a high degree of salience, to use Lerdahl's (1997) term, that anchors the otherwise aphoristic gestures that make up these movements. There are no similar pedals in either of the other movements of Op. 10, hence their smaller ranges. (Although a superficial look at Op. 10/iv might suggest that the repeated As in the clarinet might be a suitable candidate, although A is the most common pc, it is only marginally more common than B♭, and there are a cluster of other pcs (C, C♯, F♯, G♯, B) which are prolonged for relatively long durations.) Meyer & Shreffler describe how Op. 10 consists of movements from two previously separate orchestral pieces; these subcollections, however, are i and iv, and ii, iii, and v. They warn 'against finding specific motivic connections between movements'

(Meyer and Shreffler 1993b, 357), yet it is notable that this phenomenon of a focal pedal-point cuts across the compositional boundaries in this collection. In fact, a similar, though far less extreme phenomenon, appears in surrounding works. McKenzie argues that Op. 9 is unified by a 'rotating ostinato figure' (McKenzie 1960, 189) which he identifies as a pair of notes a major second apart in movts. i, ii, and iii, and a minor ninth apart in movt. iv. The exact gestures to which he refers in movts. i, iii, and iv are never clarified, but it seems reasonable to surmise he means, respectively, the A-B viola figure in b. 2, the B♭-C violin I figure in bb. 2–4, and the C♯–D violin II figure in bb. 5–6. In ii he states that the ostinato is the E♭-F figure in b. 2 in vln. I, though it is crucial to note that the apparent clarity of this motif is muddied by the adjacent D and E provided by viola and cello. In each case these are minimal gestures, which befit the scale of the movements, but question the terminology of 'ostinato'. More significantly for my own research, none of these figures has the effect on the pc distribution of the pedal figures in Op. 10: in each case, the relevant pitches are no more important in the pc distribution than others, and indeed are often relatively insignificant. In several cases, however, there are other static or repetitive figures which are more influential on the distribution. The best example is the collection of three pedal pitches in Op. 9/iv, the violin I E in bb. 3–5, the viola G in bb. 5–6, and the cello F♯ in bb. 5–7, which are three of the four most prominent pcs in that movement's pc distribution (respectively, 12%, 13%, and 17%). Turning to Op. 11, in the first movement the pc D appears in 7/9 bars and comprises 14% of the distribution; in the second B♭ appears in 6/13 bars and again makes up 14% of the distribution. It is crucial to note that none of these figures is as high as the pedal pitches in Op. 10, which make up 23% in mvt. i and a full 34% in mvt. iii. As such, although the prolonged pitches in Opp. 9 and 11 seem to be less extreme versions of the same idea, it is certainly questionable whether they have the same dominating effect. Nonetheless, a hypothetical equal distribution of pcs would assign 8% to each pc, so values almost double that are still of note. Indeed, it seems to be the case that across many movements from this period Webern was experimenting with using the same technique: a single focal pitch around which other motifs could be organised. The manner in which this is deployed varies, however. Of Op. 10, Forte writes that 'long sustained single notes like this are not uncommon in Webern's atonal works' (Forte 1998, 384). Whether or not this is comprehensively true, a variety of different practices can be observed across Opp. 9–11. In Op. 9/iv, the relevant pitches are largely temporally constrained: outside of their pedal appearances, E is used only twice, and F♯ and G only once. Meanwhile, the pedal appearances themselves are continuous and confined to a single instrument and octave. The same pattern is true in Op. 10/i: G♯ is used almost exclusively by the celesta, excepting two colour doublings in flute (b. 8) and cello (b. 10) which occur at the same octave level; A has one seemingly unrelated appearance, in bb. 4–5 an octave higher than the celesta in the clarinet, but its other occurrences (b. 8, cello; b. 9, trumpet; bb. 9 and 11, harp) are all at the same pitch level as the celesta and either colour it, or function as a prolongation of that pitch. In Op. 10/iii the situation is slightly different: the pedal E, almost omnipresent in the movement, is shared across several instruments (guitar, bells, harp, celesta) which comprise two timbral groups (strings and bells) at the same octave. Pc E does occur briefly in other parts and octaves: the harmonium in b. 4; the cello in b. 5; and the viola at the pedal octave in b. 6. Meanwhile, in Op. 11/i and Op. 11/ii the relevant pitches both occur in 3 different octaves and are dispersed widely across the movements, featuring both in chords (in all manner of positions) and melodic fragments. As such, across this succession of movements Webern can be seen exploring this technique in a variety of guises: in some movements he makes a significant aural focus of this pitch, in others it functions as much more of a background entity, simply more relevant to much of the writing than other pcs. There is an obvious objection to this: in Op. 10/iii and perhaps Op. 10/i this elevation of a single pc is a meaningful characteristic, but it is inevitable that one pc will take precedence over the others. Indeed, particularly in movements like those from Op. 11 where there is little emphasis from other parameters, these pitches cannot be of much structural significance. The riposte to this is simply that, as Figure 3.6 shows, Webern was frequently writing music with less emphasis on any individual pc and had been doing so for many years by this point. What is more, whilst one pc will, almost always, be used more frequently than any other, the disparity in these movements is, as mentioned above, significant. In fact, in Opp. 10 and 11 these are statistically significant: in all of these cases the pc highlighted above is a statistical anomaly, indicating that this is not some random coincidence, but has unusual prominence. In Op. 9/iv none of the three pcs discussed above are anomalous, but attention is clearly drawn to them by the features identified above. Finally, it is worth noting Webern's repeated restriction of individual pcs to single pitch levels. This tendency to 'freeze' pitches is often remarked upon in the later music (Op. 21 perhaps most famously), so it is fascinating to observe it in prototypical form this early in the corpus.

3.4 Pitch-Class Circulation

3.4.1 Introduction

Pitch-Class Circulation is an analytical technique developed and deployed solely by Tymoczko. In his words, it assesses 'how fast a piece of music moves through the available notes' (Tymoczko 2011, 159). In rough terms, it segments a piece into successive windows of a given length and records the average number of pcs used in each window. This process is then repeated for successively longer windows, which provides another perspective on how chromatic the music is: in the context of Webern, how frequently the total chromatic circulates. Tymoczko's deployment of this technique is relatively superficial: it receives only a couple of pages in his book, and as far as I can find has not been taken further by its originator or other scholars. While it is certainly a limited technique, I suggest that it can provide a novel and unique perspective on this repertoire, and so I have formalised and developed Tymoczko's method more transparently, as will be discussed below. This chapter will also explore subsidiary information that can be derived from the basic implementation: how frequently the total chromatic rotates across a movement, and what these statements comprise as a proportion of a total movement. My discussion of the total chromatic here will complement Robert Harry Hallis Jr.'s (2004) historical and analytical study of Webern's self-proclaimed 'run' technique, which supposedly sought to use the completion of the total chromatic as a structural device.

3.4.2 Method

As outlined above, pc circulation is a useful but underdeveloped tool, and there are some areas of Tymoczko's implementation that deserve discussion. There is a general tendency in his monograph to eschew details of the method in favour of greater readability. Given the pedagogical nature of the text that is understandable, but leaves some lacunae which I will fill in when implementing these ideas myself. The most significant is his measurement of musical windows. Tymoczko describes pc circulation as measuring pc content 'over various spans of musical time' (Tymoczko 2011, 158), but goes on to explain that the unit used for establishing windows is note onset, which is not a reliable proxy for time but rather follows event density. As an example, 3-unit windows would include both a succession of three consecutive semiquavers, and a quaver followed by a minim followed by a breve. These are clearly not the same span of musical time and, I argue, are divorced from a reasonable perception of music. Tymoczko acknowledges in a footnote that pc circulation graphs are 'susceptible to tempo' (Tymoczko 2011, 159), but this is only because of his implementation. This subject is missing from discussion in the body of the text as the chosen example is the first 6 notes of the monophonic theme from BWV 779, all of which are quavers, and therefore does not indicate that the following run of semiquavers would change the temporal spans. Secondly, the use of this theme avoids any questions of rests: in his method, presumably notes either side of a long silence are deemed to be just as connected as two adjacent notes. Thirdly, his example, as a monophonic theme, does not illuminate how he considers simultaneities. A footnote explains that 'it is necessary to "linearize" simultaneous attacks, so that one comes before the other' (Tymoczko 2011, 159); again, this is an artifact of the note-onset windows. Finally, his text does not explain what happens in cases where the number of onsets in a work does not equally divide into the window-size he is calculating (for example, what happens to the final window in a window-size 3 analysis of a piece with 200 note onsets? Are the final two notes ignored from the analysis? Is the final window comprised of 2 notes? Is the final window comprised of those 2 notes plus, perhaps, the antepenultimate note of the piece, which is therefore double-counted?).[11] Tymoczko takes account of these various issues by suggesting that 'these graphs should be taken with a grain of salt', and that, with regard to flattening out simultaneities into a linear arpeggiation, he is 'hoping that the size of the data set will wash away any inaccuracies introduced by the process' (Tymoczko 2011, 159). Clearly it is wise to be circumspect when using novel tools like these, but, as I explore below, with a more transparent implementation there is no reason to be so underconfident.

To apply this method in my research, I have had to develop a more sophisticated approach, which involves two principal modifications to Tymoczko's method. Firstly, I use temporally defined windows, rather than calculating them according either to Tymoczko's note-onsets, or the standard, though flawed, practice in computational musicology of notated durations. Secondly, I use median averages rather than means. As a measure of central tendency, means have two issues: they assume a normal distribution, and are susceptible to the influence of outlier values. In the initial graphs I use this latter phenomenon is less problematic, but later on it becomes crucial. The algorithm used here is therefore as follows: the movement is modelled as a series of rests and verticalities, and then for each in a series of integer window sizes from 1 to the number of seconds in the movement, the movement is divided into consecutive windows of that length. In most cases, the duration of the movement does not divide equally by the window size, and so where this happens the windows are slightly overlapped, to ensure each window is

11. In private correspondence Tymoczko has clarified to me that his implementation uses overlapping windows incremented by one note, so this is not an issue.

Figure 3.11: Pc circulation example bars.

of the full length. Although overlapping is not problematic, it does not add to the analysis, and this is preferable to other solutions (e.g. truncating the final window). The windows are positioned to ensure that the overlap is the same between each pair of consecutive windows, and the minimum number of total windows is used. From each of these windows the number of pitch classes used is then counted and the mean average recorded. To demonstrate, such an analysis of Figure 3.11 is shown in Table 3.2. It is important to note that, unlike in the pc distributions discussed above, pcs are not weighted by frequency or duration. Although weighting is my usual practice, pc circulation is focussed specifically on identifying the typical size of the subset of pcs used in a given temporal span, not how this subset is used. This provides another perspective on the chromaticism of a movement by indicating how chromatically saturated the harmonic space is: as a model, it understands even the brief use of one pc to be a meaningful event that expands the harmonic palette irrevocably. To model a piece as a series of verticalities represented on one stave, the algorithm uses music21's 'chordify' tool. Although the result is visually abrasive (Figure 3.12 and Figure 3.13) this is irrelevant in a computational analysis. These data can then be plotted as line graphs: Figure 3.14 is Tymoczko's such graph. Tonal and dodecaphonic music are clearly radically different; how this might replicate across Webern's corpus is less clear, however. As is common, Tymoczko's work suffers from a tiny sample size: Webern's Op. 27 might represent all serial music; equally, it might be an outlier. (Putting aside the issue of conflating all three movements.) Intuitively, dodecaphonic construction would be expected to lead to this sort of fast convergence, but Tymoczko's work is silent on free atonal music, or indeed how much dodecaphonic music might vary.

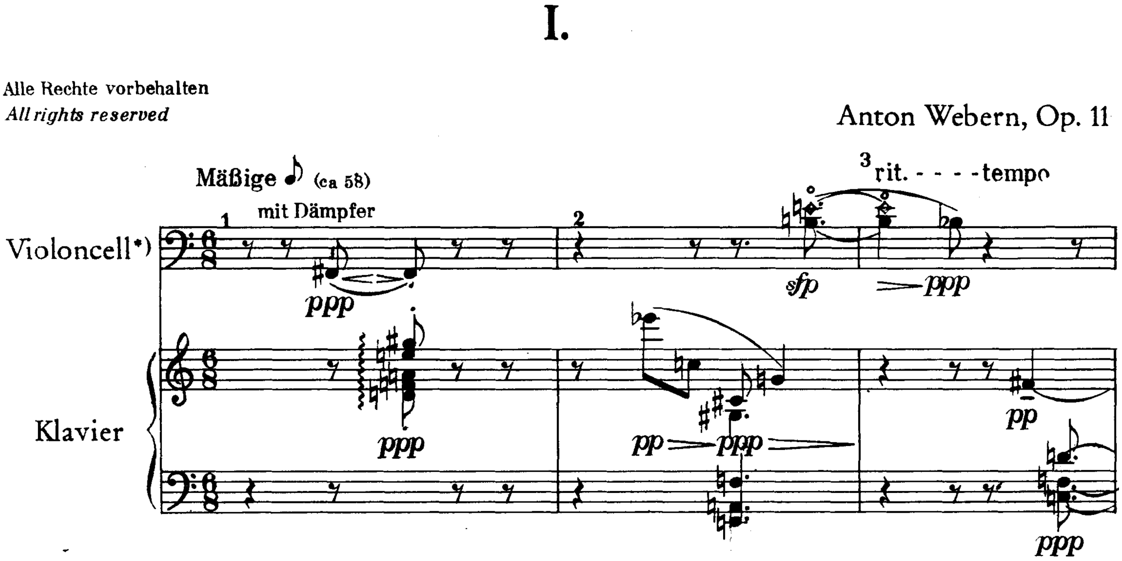

Figure 3.12: Op. 11, bb. 1-3, original. © Reproduced by kind permission of Universal Edition A.G., Wien.

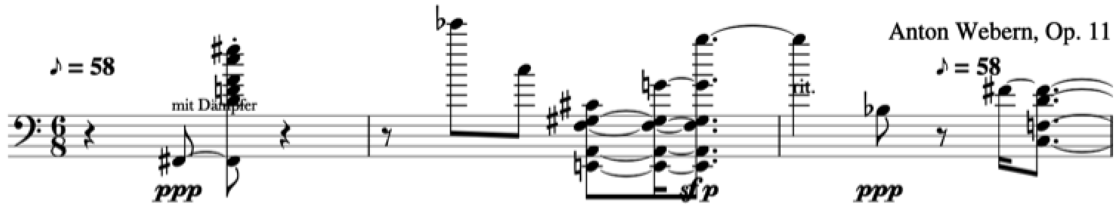

Figure 3.13: Op. 11, bb. 1-3, chordified.

Window Size (s)	Window 1	Window 2	Window 3	Window 4	Window 5	Median PC Count
1	A	B	C♯, D♯	E	E, A	1
2	A, B	B, C♯, D♯	E, A			2
3	A, B, C♯, D♯	C♯, D♯, E, A				4
4	A, B, C♯, D♯	B, C♯, D♯, E, A				4.5
5	A, B, C♯, D♯, E,					5

Table 3.2: Pc circulation example calculations.

3.4.3 Results and Discussion

Figure 3.15 is a Pitch-Class Circulation graph for the corpus in this project, cropped to the first 100 windows. In full, the graph could extend to the

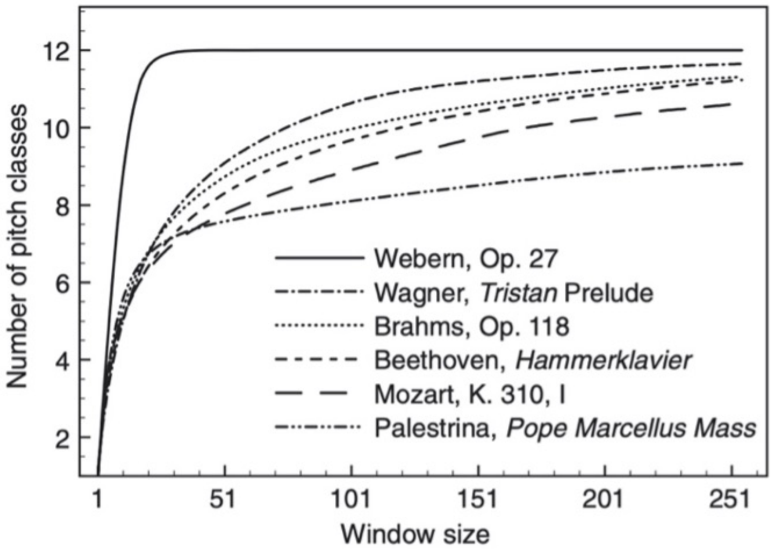

Figure 3.14: Pc circulation graph, from Tymoczko (2011, 160). © 2011 by Oxford University Press, Inc. Reproduced with permission of the Licensor through PLSclear.

maximum window size of 537, but that region is simply a flat line, with works gradually dropping out as their duration is reached. The most obvious implication is the remarkable consistency, with most of the movements quickly tending to a mean average of 12. Recalling Figure 3.14, it is evident that Op. 27 is fairly typical: not only do all but one of Webern's works reach a mean of 12, but they all do so very quickly. Deriving more information graphically is difficult; the only observation that remains is to compare Figure 3.15 with Figure 3.16, a graph of the dodecaphonic subset of the corpus. On the whole the two graphs are very similar, but the comparison shows how quickly the vast majority of the dodecaphonic works converge compared to the rest of the corpus. This indicates, therefore, both that dodecaphonic works are highly consistent, and that there was clearly some more variety in the tonal and free atonal works.

An obvious disparity between Tymoczko's graph and my own is that his lines never descend, whereas mine, on occasion do. This makes sense, although it requires a little bit of thinking about as it may initially seem counterintuitive. In general, as is usually the case, average pc content would be expected to increase as window sizes increase. This is obviously true at the extreme ends of the x-axis,

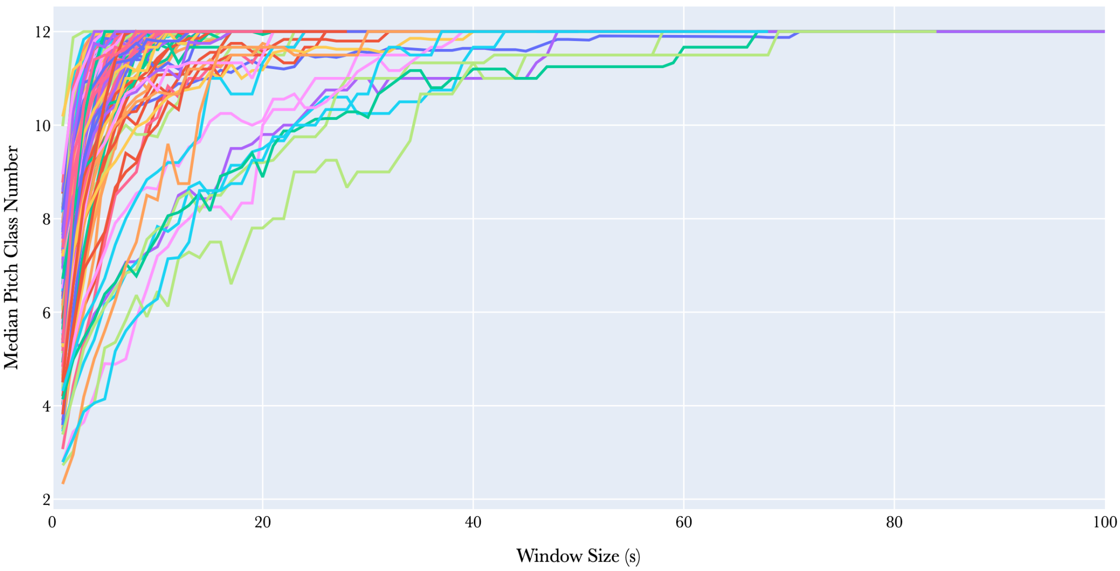

Figure 3.15: Median pitch class count for different window sizes for each movement.

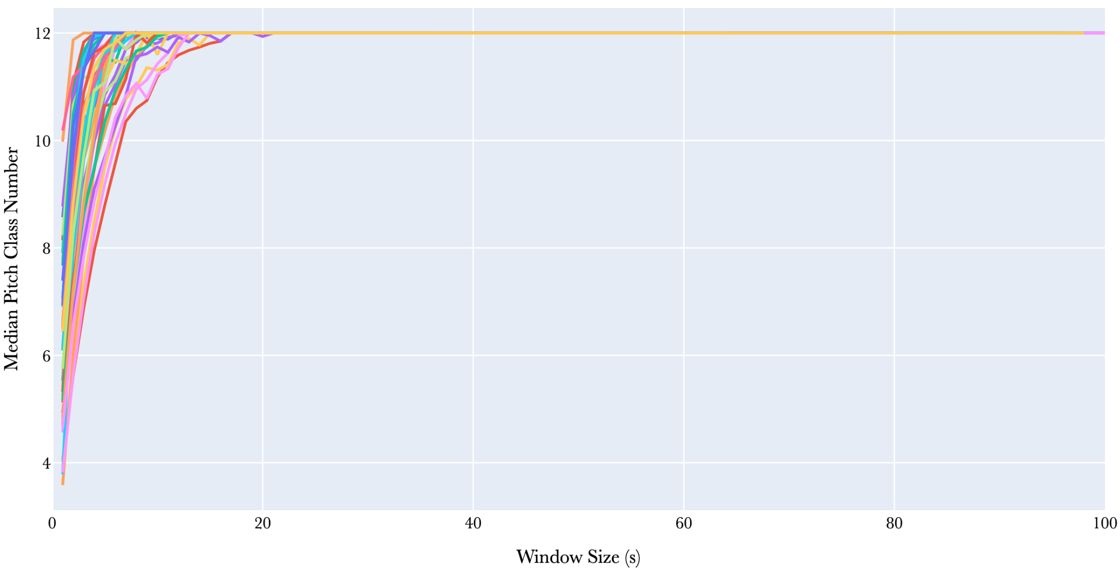

Figure 3.16: Median pitch class count for different window sizes for dodecaphonic works.

but in general as window size gets bigger, each window would be expected to encompass all the pcs in the previous window size, with the possible addition of some more. There are some edge cases, however, where an increase in pc window size moves a pc from a window where it was the only representative of that pc to a window where it duplicates a pc, thus lowering the number of pcs in the first window while the second window stays the same, and so the average falls. Why this phenomenon does not occur in Tymoczko's analysis, I do not know (it is not due to his use of note onsets compared to my own use of temporally defined windows, both could provide this outcome). It may be a vagary of the repertoire he considers, or he may have applied some sort of smoothing to his curves, but without any published code or further transparency about his method, it is impossible to evaluate. That it is a common occurrence in my own analysis is likely to be due to the short duration of many of the works, which thus makes average results more susceptible to individual peculiarities.

Deriving more information graphically is difficult, but turning to numerical enquiry, a helpful value is the window size at which a given movement reaches its peak average pc value (12 in all cases). This can be interpreted as indicating how quickly a movement cycles through the total chromatic, irrespective of tempo or duration. It is here that the value of medians really overtakes means: a mean average will only reach its maximum value (12) once all windows include that value. In practice, this results in a situation where the minimum window size with a value of 12 is merely the window size of the longest statement of the total chromatic, failing to take account of other statements. Using a median avoids this issue. Figure 3.17 plots these values, which indicates a moderate negative correlation between this value and corpus position, -0.45. Whilst this is hardly determinative, it indicates that later works typically converge faster, perhaps due to the impact of the dodecaphonic technique. Even a cursory look at Figure 3.17 indicates that there is remarkable consistency amongst the dodecaphonic works, and frankly all the works from Op. 12 onwards. To test this empirically, I can compare these three segments (Opp. 1–11, 12–16, and 17–31) using two pairwise t-tests, which compare segments 1 and 2, and 2 and 3. In both cases the null hypothesis is that the means are the same, and thus the variation is simply explained by random variation; the alternative hypothesis is that the difference in means is not 0, and thus there is some other explanation (in this case, an aesthetic change in Webern's practice). I adopt a significance level of 0.001. Comparing the first two segments of the corpus returns a statistically significant result with this level, a test statistic of 3.81. This is sufficient evidence to reject the null

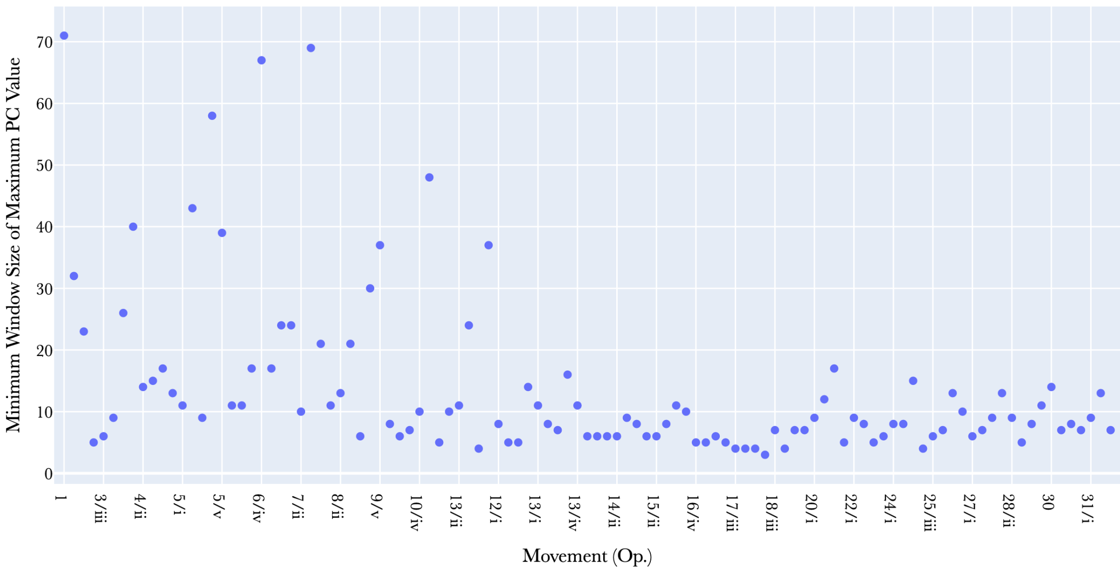

Figure 3.17: Smallest window size with greatest median value.

hypothesis in this case. For the latter two segments, however, the result is not statistically significant, providing no evidence to reject the null hypothesis. This finding is particularly notable given this graph does not control for tempo or movement-duration, but suggests that whatever the context, in general Webern moves through statements of the total chromatic at roughly the same speed. This is not, I would suggest, an intuitive result: movements either with slower tempi, or with longer durations, might reasonably be expected to move through the total chromatic at slower speeds.

Turning to the other end of the scale, anomalously high values are also useful, of which there are 9. The movement with the highest value, Op. 7/iii, points to an interesting feature of the music: the extremely slow rate at which Webern unfurls the total chromatic. The entire movement is only 84 seconds long, so a window size of 69s is almost the entirety of the movement. Although Webern introduces 11 pcs in the first 6 bars, it is not until the ante-penultimate bar (b. 12) that he finally completes the total chromatic, and as he does this, the piece ends. One of Webern's most famous proclamations refers to his compositional process in Op. 9, where he stated that 'I had the feeling, "When all twelve notes have gone by, the piece is over"' (Webern 1963, 51), and indeed wrote out and crossed off each note in the course of composition. This has undergone much dissection: Hallis Jr. (2004, 87) has pointed out that the only sketch evidence apparent for such a method comes much later, in 1914, and has speculated that Webern ascribed this approach to himself retrospectively as part of a bid to craft a coherent autobiography that led linearly to dodecaphony. Nonetheless, whether or not Webern was crossing off a list of notes, in Op. 7/iii does constitute a piece ending as soon as the total chromatic is completed. This on its own is hardly a novel insight, Arnold Whittall reached the same conclusion decades ago (Whittall 1983, 736), pointing out that although Webern had been discussing Op. 9, this was relevant most clearly to Op. 7/iii. What Whittall does not do, however, is provide any comprehensive context for this: is Op. 7/iii typical, or unusual? Are there other movements that do the same? Returning to Webern's own commentary, he proposed that across his freely atonal period he used 'runs' (essentially equivalent to a gradually unrolled statement of the total chromatic)[12] of the total chromatic as a structural device (Webern 1963, 51). In his telling, Op. 7/iii would therefore be simply one example of this, if perhaps an extreme one. Paul Paccione (1988) has charted a similar approach in Op. 9/v (another anomalous movement, with a value of 37s in Figure 3.17), where two statements of the total chromatic are used to structure the two sections of the movement, and Robert Hanson (1983) has likewise explored the use of the total chromatic in Op. 10/iv (not an anomaly, although this may be down to the short duration of the movement itself, on which more later). Though Hanson's interest is more specifically in Webern's use of chromatic voice leading to establish and then expand chromatic clusters, this leads naturally to the role of the total chromatic, where he is keen to locate the seeds of dodecaphonic thinking. Nonetheless, it is Hallis Jr. who has explored the implications of Webern's comments most extensively. I will briefly summarise his approach and findings, before complementing it with my own analysis. Hallis Jr. argues that in Webern's music, runs perform a structural function. In particular, the final pc from a total chromatic statement is often revealed in 'the closing

12. For more extensive definition of what Webern conceived of as a 'run', see Hallis Jr. (2004, 346–47).

cadence of the section' (Hallis Jr. 2004, 342). Runs are thus often allied with formal boundaries articulated by other means. Hallis Jr. does suggest that there is an evolution in the run technique, but that they are relevant only to a subset of the corpus, Op. 7–Op. 16. Across this subset of the corpus, he identifies a change in Op. 12. In the earlier works of this set, a single statement of the total chromatic, a single run, is typically used to structure the harmonic content of a section. Pc repetition is permitted, although as the equalisation of pcs was a further goal this inevitably contributed to a minimisation of repetition. The effect of this was the temporal contraction that can be observed in these works, as Joseph Earl Rogers (2005) has discussed with regard to Op. 9/i. Op. 11/iii, of course, is the apotheosis of this, after which Webern uses runs in the ensuing Lieder in a more expansive way: multiple runs are used in a single formal section, thus contributing to the equalisation of the pc hierarchy. Clearly this foreshadows the dodecaphonic period, where swift rotation of the total chromatic becomes a guiding principle.

Hallis Jr.'s analysis is a prompt to consider further the structural function of the total chromatic in the corpus as a whole. Inevitably his work lacks comprehensiveness, where my own research can step in. Rather than analysing runs, specifically, I will build on the data presented in Figure 3.17. First, however, a reply to his comment that run technique is only relevant from Op. 7 onwards. Hallis Jr. takes Op. 5/ii as his example for the irrelevance of run technique, arguing that the runs presented here do not coincide with structurally significant sections of the piece. On this I must disagree, for reasons both subjective and objective. The objective reason comes from Hallis Jr.'s unfortunate miscounting of pcs: though he suggests that all twelve pcs are introduced in the first three bars, completed by the entry of the viola F♯, this is incorrect. His Figure 16 (Hallis Jr. 2004, 370) indicates that he misreads the viola C♯ at the end of b. 2 as a B♭. In actual fact, B♭ does not arrive until the final chord of b. 4, the close of the first section, and exactly the sort of structural articulation Hallis Jr. concentrates on elsewhere. From here, the subjective interpretation: I am in agreement that the next run, leading up to the violin I G♯ in b. 8, does not conclude with any significant structural articulation. Following two and a half bars dominated aurally by the D-E♭ motif and its derivatives, and an expressive outburst in the second violin, from the latter half of b. 7 the music moves into a new realm, dominated by the rocking E♭-F in the second violin. The G♯ does not appear until a few notes into this section. Certainly it is the high point of the violin I's phrase, indeed no note will sound as high for the remainder of the movement, and it receives metrical support by placement on a downbeat, but at least to my ears it does not *feel* like the sort of structural conclusion that might be expected. Quite the opposite is presented by the third run, which concludes with the B♭ in b. 12. Not only is it significant that this third run concludes with the addition of the same pitch as the first, but in many ways this B♭, then supported by the C-B♭ appoggiatura in the second violin, might be seen as concluding the movement, with the final material, marked *verklingend* and *kaum hörbar*, as coda material. Emphasised again by metrical placement, register, the following silence, instrumentation (the only point at which all four instruments have landed a chord together since the conclusion of the first run in b. 4), as well as the aforementioned violin II appoggiatura, the importance of the run's conclusion seems unmistakeable to me. Certainly Webern's technique is somewhat less strict here: the notes that follow spell out half of a further run, and the second run in the movement seems to lack particular structural significance, but it seems clear that Webern is alive to the structural functions of the total chromatic as early as Op. 5/ii. Likewise, looking the other way, Wedler (2023, 98–99) has shown how in one of Webern's early dodecaphonic experiments, the row is specifically deployed to coincide with structural moments in the text.

All this lends credence to the utility of pc circulation as an investigative technique outside the narrow band of works considered by Hallis Jr. Above, I introduced the values in Figure 3.17 as the duration of typical statements of the total chromatic. If the duration of each movement is then divided by this value, a proportional perspective is gained on how frequently the total chromatic circulates during the movement, thus controlling for the duration of movements. For works like Op. 7/iii in which the pcs are dispersed gradually across the work, values are low; for those in which the total chromatic is circulating more regularly, values are higher. This improves on Hallis Jr.'s method in a few ways. Firstly, it continues to take account of sections of works that do not deploy the run technique. Hallis Jr.'s analysis of Op. 13/iii (Hallis Jr. 2004, 391), to take merely one example, explores runs in the prelude, opening vocal phrase, and final phrase, but leaves the centre of the work unexplored. Given his specific interest in the run technique this is perfectly forgivable, but fails therefore to provide a comprehensive perspective on the use of the total chromatic (as distinct from runs specifically) in this song. My analysis is also more sensitive to run statements of different lengths: whereas the implied frequency counts of Hallis Jr's approach merely indicate how many runs there are in a movement, the values in Figure 3.18 are able to indicate how much of the piece these statements, on average, take up. More broadly, applying this technique to the entire corpus contextualises those works in which the run technique clearly is relevant as a constructive device. It can demonstrate whether this has a significant effect on the total chromatic in the resulting music, and explore this in the other works of the corpus. On a technical basis, reading Hallis Jr.'s analysis and comparing it to the music gives an informal sense of the proportional role of the runs, but this remains speculative and unquantified. As such, the reader has a rough sense of the relationship between movement duration and run count, but no precision here. My own analysis aims to provide exactly this precision.

The resulting values are presented in Figure 3.18. Unsurprisingly, in the relevant movements there is a strong complementary relationship with Hallis Jr.'s analysis.[13] The first feature the chart indicates is that this is more a phenomenon

13. Indeed, to formalise this I have calculated the correlation between the number of runs identified by Hallis Jr. and the value given for that movement in Figure 3.18, which is a strong positive correlation of 0.81. I have only included those movements for which he carries out a comprehensive run analysis. For transparency's sake, the movements considered and their run count are as follows: 7/i, 2; /ii, 9; /iii, 1; /iv, 4; 8/i, 7; /ii, 7; 9/i, 4; /ii, 4; /iii, 3; /iv, 2; /v, 2; /vi, 3; 10/i, 3; /ii, 2; /iii, 3; /iv, 2; /v, 7; 11/i, 4; /ii, 3; /iii, 1; 12/i, 6; /ii, 13; /iii, 8; /iv, 6; 13/ii, 8;

of the earlier music than the later music. This is not hugely surprising: a classically dodecaphonic work with only a single row statement would likely be brief in the extreme, and clearly in the dodecaphonic music Webern was fascinated by the use of multiple row forms and dodecaphony as a larger structuring device. Without exception, then, the dodecaphonic works all have noticeably higher values (median 22.81) than the earlier music (6.51). On a slightly more detailed level, there is a clear difference between the mid-period Lieder and the music that precedes it, which displays significant continuity. This runs counter to Hallis Jr.'s proposition that there is a change in Op. 7 and the following works. Instead, the picture is quite clear: in this earlier music statements of the total chromatic tend to take up around 16% of a movement. Lacking this context, Hallis Jr. misses the point that even if earlier works deploy the total chromatic in different ways (and, as above, I am not convinced this is necessarily always true), Webern continues to deploy the total chromatic at the same rate in music where, apparently, he does not use the run technique. Without a comprehensive run-based analysis of Opp. 1–6 it is, as usual, impossible to assess the situation. Nonetheless, Figure 3.18 is clear that in this first period of Webern's output there is far more continuity than Hallis Jr. suggests. He is not alone in differentiating works in this period: Whittall contrasts Op. 7/iii to Op. 11/iii, arguing that even this latter aphoristic movement is not controlled in such an extreme manner by a single statement of the total chromatic. On this he is correct, but Figure 3.18 indicates that this is not by much (Op. 7/iii has a value of 1.22, Op. 11/iii of 1.30). In fact, the twelfth pc (A, in this case) is again only introduced in the ante-penultimate bar, after which only two more notes are sounded. These two works, constituting the minima in Figure 3.18, do seem to be extreme cases: no other movement has a value under 2 (or in other words, for every other movement, statements of the total chromatic typically take up less than half of the movement). There are two movements, however, with values not much above this: Op. 9/v, and Op. 10/iv, both with values of 2.11.

14/i, 5, /iii, 7; 15/iii, 8; /iv, 4; 16/i, 4; /ii, 4; /iii, 5; /v, 4.

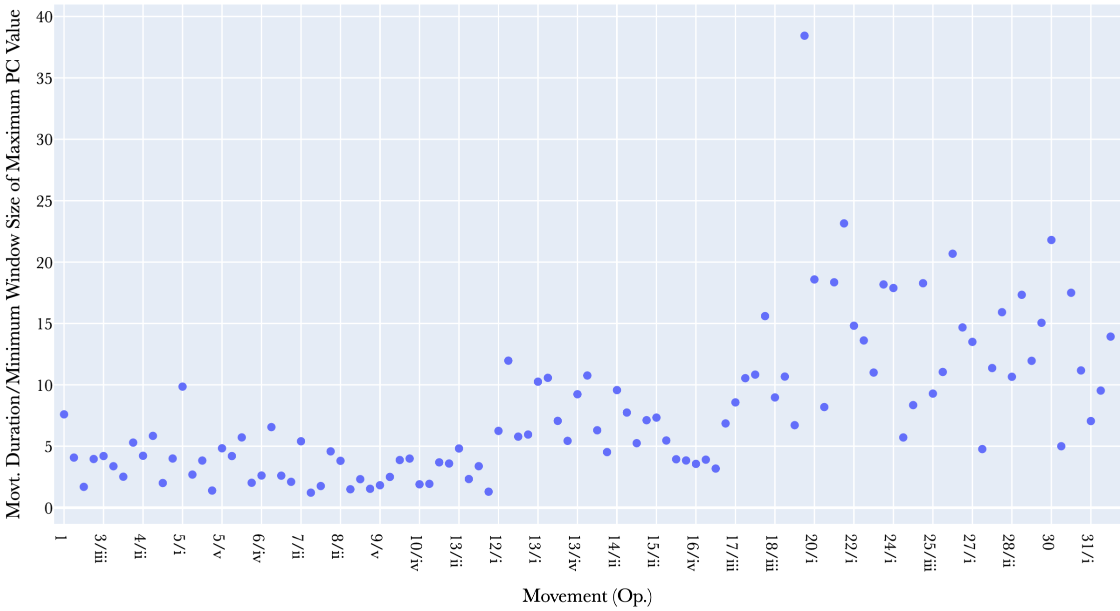

Figure 3.18: The average proportion of a movement in which the total chromatic is stated.

In the case of Op. 10/iv, there are two statements of the total chromatic, which do not seem to align with major structural moments. The first unfolding of the total chromatic is completed with the B♮ in the trumpet entry in b. 2. This fast revelation of the aggregate does not conclude with any significance, beyond the change of timbre in the melodic material from one Mahlerian instrument, the mandolin, to another, the trumpet. To my ears, this serves to emphasise continuity over structural distinction. Hallis Jr. locates the conclusion of this run at the end of the trumpet's melody, allowing for various pc repetitions and so counting the start of the second run from the trombone G♯ in b. 3 and the congruent change of tempo (and not counting the repeated As in the clarinet that, at least temporally, fall in this latter section). Hallis Jr. argues that the second run in the movement is incomplete as it lacks both a C♯ and an A♯. In the case of the former, this relies on ignoring the clarinet trill, a not entirely indefensible position, but one that I, and for that matter Hanson (1983), disagree with. As for the A♯, this is another misreading, in this case of his own Figure 8 (Hallis Jr. 2004, 359) which correctly identifies the pre-antepenultimate note as a B♭. In both cases, the conclusion of the run does not coincide with a moment of much structural significance, which appears to be an unusual situation in Webern's work. In Op.

9/v, the use of the total chromatic is rather more structurally significant. The first statement concludes with the viola B♭ in b. 7, the second concludes, incomplete, at the end of the work. However, the pc missing from that second statement is B♭. Using the windowed method of Figure 3.18, rather than a run-based method where the slate is wiped clean after the conclusion of a run, indicates the continuity that this creates. Because that missing B♭ is positioned at the conclusion of the first statement, and coincidentally at the centre of the movement, its influence bleeds on into the second half of the movement, hence the value of 2.11 rather than a value closer to 1.

At the opposite extreme is Op. 20/ii, with a value in Figure 3.18 of 89.69. Bailey describes this movement as 'one of the most difficult to follow aurally' (Bailey 1991, 155), and Roger Smalley (1975, 18) proposes that in both movements of the Trio this relative incomprehensibility is down to the obscuring of significant structural moments (reprises of the rondo-theme in the first movement; the arrival of the recapitulation in the second). However, it seems reasonable to suggest that at least a supporting factor is the extremely high rate of chromatic turnover, extended for a long time. Clearly my work is not an investigation of dodecaphonic perception, and so in the absence of any evidence I would not wish to argue that there is necessarily a relationship between chromatic turnover and aural difficulty (indeed, one might argue that rapid chromatic saturation merely reorientates the listener towards other features, for example intervals, as Charles Bodman Rae (1999, 49–57) has discussed in the music of Lutoławski). Nonetheless, what Figure 3.18 draws attention to is that the rate at which the total chromatic is sounded is remarkably fast (its value in Figure 3.17 is 3, suggesting that the total chromatic rotates every 3s), and this is prolonged for a long duration (performances last approximately 6 minutes). Only a few movements have a faster value in Figure 3.17, with six movements having a value of 2s, and most extremely, Op. 19/i, which has a value of 1s, but none of these movements extend for anything near the duration of Op. 20/ii, all lasting under 90 seconds. Op. 20/ii (1927) is the first purely instrumental movement in the corpus since Op. 11/iii (1914), and was Webern's first use of a 'traditional' instrumental form in a dodecaphonic context (Bailey 1991, 155–63). This clearly gave him the confidence, and structural underpinning, to write an extended movement without the motivation of a text, as was so often the case for him, and exclusively so for the previous decade. Indeed, no prior instrumental movement in the corpus is as long as Op. 20/ii, except for the *Passacaglia*, Op. 1, which is obviously a tonal work, though interestingly again organised by a traditional structure. Curiously, the other dodecaphonic movements with equivalent durations, Opp. 21/i, 26, and 30, all have values in Figure 3.17 higher than the median and double that of Op. 20/ii (respectively, 10s, 6s, and 7s): it was only in the second movement of the Trio that Webern felt able to sustain the frenetic energy provided by such rapid chromatic turnover, for an extended period of time. Indeed, the correlation between movement duration and the values in Figure 3.17 is a moderately strong one of 0.62: clearly this relationship is not restricted to the edge cases of long-duration movements but holds elsewhere too.

3.5 Conclusion

This chapter has assessed macroharmony and chromaticism in Webern's music using two techniques: whole-movement pc distributions, and pc circulation. I opened by conceptualising of chromaticism as part of a spectrum, rather than a binary state, and this chapter has clearly shown that this is a valid and important way to understand this repertoire. Indeed, both of these analytical methods have been developed to explore earlier tonal music, so it is a novel and valuable contribution to apply them to this repertoire. Across Webern's corpus, pc distributions become flatter, thereby moving further along the spectrum away from any vestiges of diatonicism towards the chromatic end. The extreme high-point of this comes with Op. 24/iii, which has a pc distribution range of slightly over 1%, thus using pcs almost totally evenly. Although Figure 3.6 demonstrates this general pattern, it is worth pointing out that it is not a hard-and-fast rule. Op. 27/ii, for example, has a range of values of almost 10%, whilst a movement as early as Op. 5/i has less than 4% difference between its most and least common pcs. These are works notable for bucking the historical trend, but this chapter has also drawn attention to those movements that are unusual in the context of the corpus as a whole, most obviously the first and third movements from Op. 10. I will return to this historical trend in Chapter 7, and so I will not comment on it further here; nonetheless, the overall pattern is obvious: the later music is undoubtedly more chromatic, even if it took a winding path getting there.

Pc distributions have been explored reasonably extensively in the analytical literature, if not with regard to this repertoire. The same cannot be said of pc circulation, which has largely been ignored in the decade since its introduction. Although in its simplest form the information it reveals is limited, which may explain its general lack of take-up, this chapter has shown that by extracting secondary information it reveals novel insights about the use of the total chromatic in a structural manner. This is obviously hugely pertinent to Webern, but could well be explored in other repertoire too. The comparison to Hallis Jr.'s analysis also implicitly makes the case for rigorous empirical analysis. While his more informal approach certainly offers valuable insights, it falls victim to the weaknesses of close reading. Indeed, the difference between these analyses makes clear the advantages of distant reading that I discussed in Chapter 2. There I critiqued McKenzie's work for its reliance on manual comparison, but exactly the same concern is relevant here. Figure 3.18 shows a clear historical development that provides wide context for the more detailed discussion I have offered of select movements across the chapter. Thus, Whittall's comparison of the third movements from Opp. 7 and 11 is placed in a much broader context that more accurately describes their place within the repertoire, as well as drawing attention to other movements that operate in similar ways.

Chapter 4

The Wider View: Macroharmonies

4.1 Introduction

In the previous chapter, I considered chromaticism primarily from the perspective of distribution spreads, and looked at how these are established across the temporal span of a movement. Varied distributions, with high spread values, are associated with the prioritisation of a subset of pcs, and, in this repertoire, tend therefore to lessen the chromaticism of the harmony; more even distributions, with low spread values, by contrast indicate a more chromatic harmony as there is minimal prioritisation of individual pcs. This is a helpful tool: as I have explored, used on a whole-movement basis it describes an important feature of harmony in this repertoire, but it is nonetheless rather crude. A more sophisticated picture is provided by the DFT. Rather than provide a single, spread-based measure of chromaticism, the DFT instead separates out different harmonic possibilities and quantifies each in turn, providing the analyst with a clear sense of the relative strength of each. As such, I can build on the previous discussion of chromaticism by asking not only how Webern's large-scale macroharmony relates to the overall possible space, the total chromatic, but also, in those cases where there is some degree of prioritisation, what the implications of this are in terms of harmonic colour. The DFT is particularly helpful for providing answers that can be understood in more traditional terms such as diatonicity or octatonicity, and therefore allowing the analyst to understand Webern's music through the lens of other types of harmony, rather than fixating on the traditional set-classes and row structures that predominate in analyses of this music.

The DFT is explained in more detail below, but a very brief summary is likely to be helpful before going any further. As used in this paper, the DFT is applied to a point in twelve-dimensional space representing a pitch-class set or distribution, where each dimension represents a pitch-class. When applied to pitch-class sets, these are encoded in a binary manner, with 0 indicating the absence of a pitch-class and 1 the presence of it. As an example, a C major triad would be encoded as follows: (1 0 0 0 1 0 0 1 0 0 0 0). If the input is a pitch-class distribution, these values can instead be scaled in any desired manner. Very simply, the DFT then returns seven non-trivial components, notated as f_0–f_6. Each component has two features: the magnitude, notated as $|f_x|$, and the phase, notated as ϕ_x. The magnitude indicates 'how strongly clustered the set is on the basis of a single interval type' and phases 'indicate where on the pitch-class circle the center of that cluster is' (Yust 2015b, 128). The magnitude of each component is thus associated with a particular interval type: for example, f_5 is associated with the ability of a vector to be mapped on a circle of fifths, and thus the 'diatonicity' of the harmony it represents; f_4 is associated with the vector's proximity to the three diminished seventh chords, and thus can indicate the octatonicism of the harmony. The phase, meanwhile, gives a sense of the most significant pcs in the harmony under consideration.

In his discussion of how to consider macroharmony, Tymoczko suggests that there are at least four important questions to ask, of which the following research will deal with two. Firstly, how structurally similar are the macroharmonies in a work? And secondly, what are the intervallic qualities of these macroharmonies (this is a reframing of Tymoczko's question of how consonant or dissonant are the macroharmonies to take a more expanded view). Both of these questions can be approached by deploying magnitude information from the DFT. This provides a way to quantify different types of harmonic quality in a pc distribution, thus exploding the basic 'chromatic/diatonic' binary, or the chromaticism spectrum introduced in the previous chapter, into six variables, each addressing different types of harmony.

I will begin, therefore, by assessing large-scale macroharmonic features: in particular, applying the DFT to the whole-movement pc distributions introduced above. This allows me to consider any potential large-scale transformation across Webern's output. The 'freely atonal' works (Opp. 3–16) are of most interest here, but the DFT will also be applied to pc distributions in the tonal and dodecaphonic works. After this large-scale approach, the DFT is applied to Op. 5/iv as a case study movement to chart changes in localised macroharmonies across the work. This takes a windowing method with 5-second overlapping windows, following Chiu (2021), to track the relative strengths of the differing components as a movement progresses, and allow for a more detailed perspective. The chapter then puts these methods together, to consider the degree of intra-movement variation across the corpus as a whole. Finally, I offer a brief comparative analysis of Op. 27/ii, demonstrating some phenomena typical of the later music.

4.2 A Brief Introduction to the Discrete Fourier Transform

4.2.1 How has the DFT been used?

The DFT has by now received rather extensive treatment in the analytical literature, so the following survey will seek only to contextualise it and provide a basic introduction to some of its key tenets. An introduction that focuses on the theoretical and mathematical aspects of the DFT can be found in Amiot's (2016) book.

The DFT has its origins in the analysis of pitch signals, where it allows for the derivation of frequency information from a waveform. Its use for score-based analysis was first posed by David Lewin, initially in a very early article (1959), and then again at the end of his life (2001). Despite the wide chronological gulf between the two articles in which Lewin dealt with the DFT, it received little attention (from Lewin or anyone else) until a pair of articles by Ian Quinn (2006, 2007) introduced the technique to music analysts in a more practicable way. Quinn's aim was to find a way to quantify 'chord quality' and to embed these qualities in a broader 'quality space': the two features (magnitude and phase) of a DFT component allowed for exactly that. This interest in spatial metaphors has been another fruitful contribution of the DFT, though it will not be employed in my research beyond the basic use of graphs.[1] Following Quinn's breakthrough, the DFT has been applied to a wide variety of repertoire: Yust has led the charge, using it variously to consider music by Bartók and Webern (2015a), Schubert (2015b), and Debussy (2017a). Following him, Jennifer Harding (2020) has used the DFT to compare music by Mozart and Messiaen, Chiu (2021) to assess macroharmonic features of the Domine Jesu from Duruflé's Requiem, and Oliver Chandler (forthcoming) to look at Malcolm Arnold's symphonies. More expansively, Yust has also used it in a large-scale corpus study manner to track changing elements of pitch class distributions from the sixteenth to the nineteenth centuries (2019), and in conjunction with Mark Gotham to assess the constituent parts (tetrachords and hexachords, mostly) of serial rows (2021). Though Amiot's (2016) monograph is more focussed on the theoretical, it uses a wide variety of examples, and has also suggested how the DFT might be expanded out of pitch analysis to consider rhythmic features, a path that Yust has recently began to tread to consider the music of Steve Reich (2021a, 2021b). Though the DFT clearly has things to say about a wide variety of repertoire, it is notable that the endless fascination with that transitional music from the early twentieth century is

1. Yust (2015b) convincingly lays out the case for the DFT's utility in a spatial approach to modelling harmony. Yust includes a valuable critique of the traditional *Tonnetz* system: crucially, that it fails to differentiate between harmonies of different type as everything is reduced to a triad, and instead shows how the DFT can model pitch space whilst allowing for these functional differences. To borrow Yust's example, in a DFT-based space, the E major triad is thus located in a different point from the E major diatonic collection, and from the E major dominant seventh chord, whilst on a *Tonnetz* they all occupy the same space. Furthermore, as Yust points out in the discussion of weighting, if desired it could even be the case that an E major triad is located in two different places depending on whether the key is A major or E major.

reflected by these repertoire choices. Although there certainly are examples from the common practice period, the examples analysts opt for are predominantly those ambiguous works that provide so much grist for the analytical mill.

4.2.2 What does the DFT do?

With the scholarly context thus surveyed, I will now expand upon the brief précis offered in the introduction as to the workings of the DFT. For a description of the DFT that is 'user-friendly' but no less thorough for it, I recommend the introductory passage from Yust's article on Schubert (127–138 Yust 2015b). The following introduction is largely a summary of Yust's writing, but with the aim merely of providing enough information to facilitate an understanding of the following research.

The basic premise of the DFT is to plot the twelve pitch-class values representing a harmony as points along the x-axis of a bar-chart, with the y-axis representing their absence (a value of 0) or presence (a value of 1). The DFT then plots twelve sinusoids (the twelve components) that cycle through this bar-chart in various ways. The component number indicates the number of cycles the sinusoid has; the magnitude represents the height of the sinusoid (and thus the 'strength' for that component); the phase gives the position of the sinusoid in relation to the x-axis (and thus the pitch-class position for that component). Each component therefore represents the set that is perfectly even with regard to octave division, with a cardinality dictated by the component number that is closest to the target harmony. The likelihood is that in most cases a target harmony will not be perfectly even: the aim of the DFT is to get the peaks of the sinusoid as close to these points as possible. Thus, an instance of f_3 represents a trichord that perfectly divides the octave into three, in other words, an augmented triad. Given a set-class—to borrow Yust's example, let's take a C major triad—the DFT then plots that augmented triad which is closest to it: in this case, one that is approximately C quarter-flat, E quarter-flat, and G quarter-sharp. As mentioned, there are twelve possible components returned by the DFT, but only seven of these are useful. f_0 represents the cardinality of the harmony; f_1–f_6 represent six intervallic features; finally, the DFT is symmetrical, so f_7–f_{12} reflect f_1–f_6 and therefore can be disregarded.

Before going any further, I will run through the features of each of the components from f_1–f_6. They have each become associated in the literature with some harmonic collection that derives from their interval quality (for some illustrative prototypical harmonies, see Yust 2016, 221; for a more detailed discussion of their implications, see Yust 2019, 3).

f_0, as mentioned above, represents the cardinality of the harmony, which allows the analyst to normalise the relative strength of the other components given the size of the harmony.

f_1 has been referred to as 'chromaticity' and refers to the degree to which a harmony is concentrated in a specific area of a pitch-class circle. Thus, while a cluster harmony will have high $|f_1|$, a more even harmony (a diminished 7th chord, for example) will have lower $|f_1|$. As Yust points out, it is important not to conflate 'chromaticity' and an even use of the twelve pitch classes: the effect of that, rather, is to lower the magnitude of all components compared to f_0.

f_2 divides the octave in half, which Yust has described as 'dyadicity' (Yust 2017a, 158). It is thus strongest if there is a prominent tritone or perfect fourth/fifth in the harmony. In 'post-tonal' music, high $|f_2|$ thus reveals a harmony built on 'stacked perfect and augmented fourths' (Yust 2019, 3).

f_3 has been discussed above as strongest when a harmony is close to a stack of thirds. For obvious reasons it is therefore referred to as 'triadicity' and is an important indicator of tonality, in conjunction with f_5. In non-tonal contexts, it can also demonstrate proximity to augmented triads or a hexatonic scale.

f_4 relates to diminished seventh chords, and so, in post-tonal music, is a useful indicator of 'octatonicity', whilst in tonal music it can assess the strength of seventh harmonies.

f_5 has again been mentioned previously and indicates the 'diatonicity' of the harmony by measuring the ability of the harmony to be mapped on the cycle of fifths. In conjunction with f_3 and f_2 it is the prime measure of tonality.

f_6, finally, measures the two whole-tone collections against each other. In lieu of a better phrase it receives the moniker 'whole-tone quality'.[2]

I have hinted that certain features can be used in combination, and indeed Chiu has correctly pointed out that the combinations of components can in fact be just as important as individual ones, and thus that associating each component with a harmonic quality in this manner is rather reductive (Chiu 2021, para. 1.3). I am sure that future research will develop our understanding of these relationships and their impacts, but as they are relatively undeveloped for now we must merely remain alive to these possibilities. Conversely, confusion is sometimes produced by the apparent similarities between these different harmonic qualities. Dyadicity and diatonicity, for example, are often assumed to represent the same information, or f_3 and f_6. Although a whole-tone quality might often arise from strong triadic relationships, harmonies do exist that maximise one while keeping the other comparatively low: (0 2 6 8), for example, as a whole-tone subset has a high value for f_6 despite a low value for f_3; (0 1 4 7) has the reverse characteristics.

Thus far I have talked about the DFT in terms of analysing harmonies, where the presence of a pitch class is a binary proposition: either present or absent. In actual fact, there is no need to limit ourselves in this way: the DFT really analyses pitch-class distributions, of which a fixed harmony is merely a special case. The final technical notes to make about weighting are firstly that I follow Amiot (Amiot 2017, 159), himself following Yust (2016, 223 in particular n. 6), in squaring magnitude values, and secondly that when I consider changing macroharmonies across a movement, adopting the smaller-scale windowing

2. This at least avoids the inelegant neologism 'whole-toneicity'.

procedure, distributions are first weighted by duration and then by the same $log_2(x + 1)$ function that Chiu deploys.

4.3 Whole-Movement Macroharmonies

The first stage of this analysis is to apply the DFT to those same whole-movement pc distributions introduced in the previous chapter. As mentioned, f_0 reflects the cardinality of the distribution, which is therefore 100 for each movement (distributions are measured in percentages to normalise for differing movement durations), and so will be discarded from this analysis. It is worth noting at this point that total power (the sum of all squared magnitudes) still differs between movements, which allows the comparison of the absolute strength of each magnitude between movements. This magnitude data is given in Appendix C.

4.3.1 Change over time?

Figure 1 charts the squared magnitudes of the six remaining DFT components for each movement, organised chronologically as above. The first and most obvious comment is the extreme heterogeneity of the graph. A useful comparison here is to Yust's article on pc distributions in tonal works from Byrd to Rachmaninoff. His Figure 6 (Yust 2019, 7) shows very stable component features, almost always ranked in the same order of strength, and largely with little change over time (the one major exception is f_5, which falls gradually across his corpus). At first glance, Figure 4.1 has no such clarity, though under careful inspection some details do reveal themselves. Firstly, squared magnitudes are clearly higher earlier in the corpus than later. Yust suggests that 'flatter distributions will be observable as decreases in all components' (Yust 2019, 3) and we can clearly see that this is happening here, as a result of the decline in distribution spreads identified earlier in this chapter. Table 4.1 confirms this with more detail: all squared component magnitudes have a negative correlation with corpus position, and they almost all fall within a very close span [-.54,-.59]. The exception to this picture is f_6, which declines at a much slower pace, though as the median value

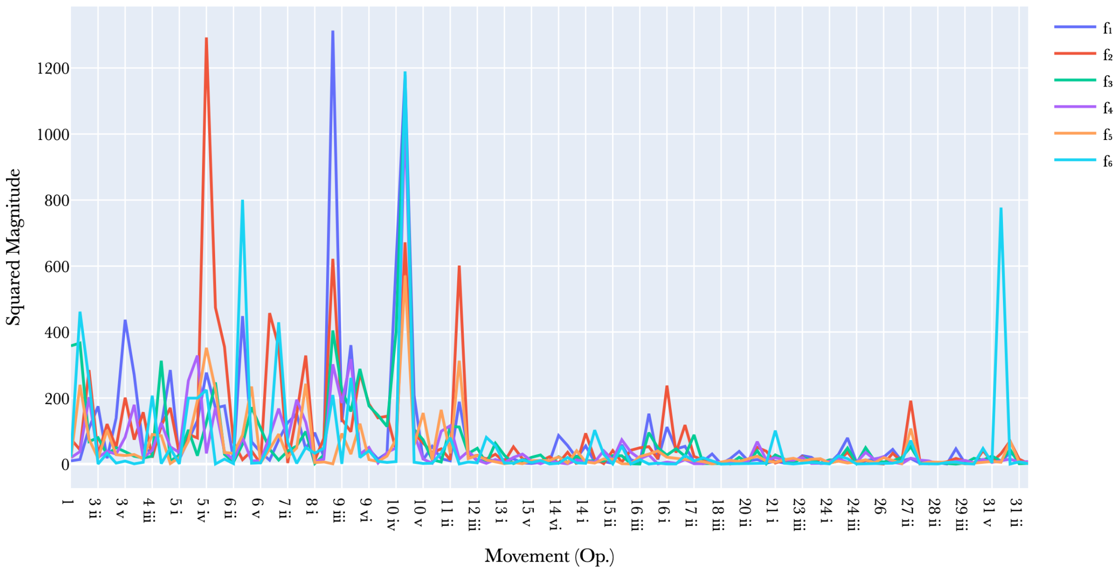

Figure 4.1: DFT squared magnitudes for each movement.

for f_6 is already lower than for the other components, this is perhaps unsurprising. Interestingly, when the squared magnitudes are normalised for total power, thus taking account of progressively flatter distributions and only comparing squared magnitudes to each other in the context of a given movement, the results are rather different. Table 4.2 indicates that there is no significant pattern of change in any component. This suggests that, on the whole, the decline is due to the comparatively flatter distributions later in the corpus, rather than Webern seeking to minimise any particular component and so shift the macroharmonic quality of the music. Figure 4.1 does suggest, however, that the decline does not take place at a gradual rate: instead, there is a significant drop-off around Op. 12. Again, this accords with the earlier description of changes across this corpus.

Corpus	Component	Median	Correlation with Corpus Position
Total	f_1	18.1	-0.56
	f_2	31.7	-0.58
	f_3	24.1	-0.55
	f_4	14.8	-0.59
	f_5	15.4	-0.54
	f_6	4.7	-0.35
Opp. 1–11	f_1	85.4	-0.01
	f_2	95.4	0.06
	f_3	76.5	0.17
	f_4	49.5	-0.03
	f_5	47.0	0.06
	f_6	20.7	-0.02
Opp. 12–31	f_1	9.7	-0.15
	f_2	11.6	-0.20
	f_3	10.3	-0.19
	f_4	9.5	-0.09
	f_5	8.8	-0.19
	f_6	2.7	-0.18

Table 4.1: Summary statistics for squared component magnitudes.

Component	Median	Correlation with Corpus Position
f_1	14.2	-0.02
f_2	17.7	-0.10
f_3	13.0	0.03
f_4	10.6	0.09
f_5	10.2	0.13
f_6	4.0	-0.04

Table 4.2: Summary statistics for total power-normalised squared component magnitudes.

4.3.2 Characteristic components

The second principal observation is that there is no obvious priority amongst the six lines in Figure 4.1: although there are no tall peaks in f_3, f_4 and f_5, the six components frequently overlap. Table 4.1 and Table 4.2 suggest that f_2 is the most powerful component across the corpus, though it is notable that none of these values are unusually high compared to each other (none are statistical anomalies). In broad terms, the prominence of f_2 speaks to an important quartal quality in Webern's harmony. As an initial observation, this isn't hugely shocking, Webern himself acknowledged the importance of quartal foundations to his harmony following from hearing Schoenberg's *Kammersymphonie No. 1* in 1906 (Webern 1963, 48), though it is encouraging that in large terms the DFT has identified a feature of Webern's music often thought to be characteristic. This is also a good point at which to draw attention to the methodological difference dividing this paper from most previous analysis. Whereas 'standard' work on harmony in this repertoire tends to take as its object motives, chords, or otherwise-justified pc sets, the object of this paper is the pc distribution, a summary of the macroharmony that Webern generates. The finding, therefore, that features of localised harmony are reflected on a global scale is an important one and will be considered in more detail below.

Turning to the other components, a feature of Table 4.1 and Table 4.2 that is perhaps more surprising is the relative strength of $|f_3|$. Associated with triadicity, it is often an indicator of strong tonality, especially in conjunction with $|f_2|$ and $|f_5|$ (Yust 2017b, 171–74). Considering first the tonal works, it is unsurprising, then, that Op. 2 has comparatively high $|f_3|$ and $|f_5|$, although it is notable that the former is marginally stronger than the latter. In a large-scale corpus study of common practice period music, Yust's results suggest that in tonal music $|f_5|$ is typically at least four times as powerful as $|f_3|$, even in music as chromatic as Brahms. In Op. 1, the case is even more extreme: whilst it too has high $|f_3|$, its $|f_5|$ is very low compared to what we might expect, a seventeenth the size of $|f_3|$, and lower than $|f_2|$ and $|f_6|$. This suggests that in both cases the pc distribution emphasises triadicity over the diatonicity we might typically expect from 'tonal' music. Yust has found that this tendency is true in the music of Wagner, Scriabin, and particularly Liszt, all especially in minor mode music (Yust 2017b, 172 see in particular note 3), as of course Webern's Op. 1 is, but not Op. 2. To take this question of trying to quantify tonality further, Yust has proposed a 'tonal index' based on his observations in corpus study analysis of tonal music (Yust 2017b, 176–78). The index suggests that in tonal music $Ph_2 + Ph_3 - Ph_5 \approx 0$,[3]. In the corpus of works that Yust considers, index values are largely within [-1, 1] and tend to average ±0.5. If we apply this to the works under consideration here, Op. 1 comes out with an index value of 1.05 and Op. 2 with a value of -1.87, which fits with prior expectations that minor modes are associated with positive values and major modes negative ones. Within the realm of the expected, then, but on its outer edges. Without a broader body of Webern's tonal music, it is difficult to take these results much further. Clearly they both display the same feature of reduced diatonicity as compared to triadicity, but without being able to compare this to the rest of Webern's tonal music, or indeed a broader corpus of tonal Viennese music of the period it is impossible to asses to what extent this is an idiosyncratic feature of Webern's music, or simply of these two works. Nonetheless, given the evidence from Wagner, Scriabin, and Liszt, this does seem to suggest that it might be a tendency of late-Romantic music, with Brahms as the outlier. Rather than solely looking backwards, however, we can look forwards and compare these works to Webern's early 'post-tonal' music. We can formalise the relationship between $|f_3|$ and $|f_5|$ as a ratio: the relative size of $|f_5|$ compared to $|f_3|$. For Op. 1 this is 0.06, for Op. 2 it is 0.65; for Brahms' major music it is 4.50; for Brahms' minor music it is 2.17. For the first stage of Webern's post-tonal music (Op. 3–Op. 11) the median value is 0.79: though there is certainly significant variation within this (the range spans 0.00 for Op. 10/i to 26.96 for Op. 11/i), this does suggest that a high $|f_3|$ compared to $|f_5|$ is a consistent feature of Webern's macroharmonic style. Minimising diatonicity was obviously an important aim for Webern and his peers, and so this is hardly surprising.

The other notable point to make about these median values is the relative unimportance of f_4, an indicator of octatonicism. In Forte's monograph, *The Atonal Music of Anton Webern* (1998), he asks: 'Did [Webern] replace tonality with another construct that enabled him to create new music, while satisfying the innate, perhaps ever genetic, sense of order … that is so apparent in his later

3. $Ph_n = 6(\phi_n)/\pi$ which converts the phase to a 0–12 scale (Yust 2015a, 208).

dodecaphonic music?' (Forte 1998, 3). The answer that Forte offers is the extensive use of the octatonic set (8-28) and its various subsets, which he suggests both implicitly and explicitly. In his discussion of Op. 13/iv, for example, he suggests that 'Webern had been composing with the octatonic as his primary pitch resource since at least 1907' (Forte 1998, 296). Whether Forte means to suggest that the octatonic tendencies he finds in Webern's music were consciously intentional is unclear: though his language suggests this, his broader commitment to post-hoc formalist analysis would suggest this might be a slip of the tongue, although he has a long history of this type of epistemological slippage (for extensive critique along these lines, see Haimo 1996). It is noteworthy, therefore, that the DFT does not reflect this suggested significance of the octatonic. Nor do the correlations imply that $|f_4|$ declines to an unusually extreme extent across the corpus, which might suggest a major disparity between the freely atonal music of Forte's purview, and the dodecaphonic music. Of course, as mentioned above, Forte's analytical objects are the particular pc sets he segments from the movements under his consideration; in this present discussion, the DFT is instead considering the overall macroharmony of each movement. As such, the two findings are not necessarily contradictory; rather, they offer different, though related, perspectives.

4.4 A Case Study: Op. 5/iv

4.4.1 The Academic Background

Op. 5/iv sticks out as a prominent spike in Figure 4.1, and is therefore a useful example to consider in some more detail.[4] This is, as many authors have noted, one of the most analysed movements in Webern's entire corpus, and so this also provides a wealth of interpretative material with which to converse. As early as 1962, George Perle (1968, 16–18) included a brief though rich analysis of the movement as an example of 'free atonality', which was quickly followed by Forte (1964, 173–77) who analysed the work as an example of how his 'theory of

4. I suggest that a score will be helpful to follow this discussion.

set-complexes' could explain elements of the structure. Elmar Budde (1989) followed in 1972 with an analysis that adopts certain tenets from set-theory (although despite some reference to Perle there is no explicit acknowledgement of this), David W. Beach (1979) then provided another set-theoretic analysis, and shortly after Lewin (1982) posed a quasi-serial interpretation of the elusive 'flyaway' motif that bounds each section of the movement's ternary form (the arpeggiated figure first presented in semiquavers in b. 6, then triplet/duplet quavers in b. 10, and finally duplet and quintuplet semiquavers in bb. 12–13). Other authors have used the movement as examples of more general theories: David Clampitt (1999) to introduce his ideas of Q-relations; Richard S. Parks (1998) to elaborate his theory of pc set genera; and Stephen C. Brown (2013) to discuss ic1/ic5 relations. In fact, even Yust (2015a) has covered this movement, as one brief example of the application of the DFT to post-tonal music. The most pertinent articles to this discussion are those of Perle, Forte, Budde, Beach, Lewin, and of course Yust, which I will briefly summarise before deploying the DFT in my own analysis.

The first two analyses are rather similar in their aims and findings. Both seek to establish a harmonic basis for the ternary structure implied by other parameters (principally textural and melodic). There are differences in emphasis between the two approaches: Forte's focus is on sets that are related through K_s subcomplexes (later replaced with the similar though not identical K_h (Forte 1973, 96–100)). He is also particularly interested in finding structural similarities between the local and global features of the harmony. The first example of this is the 'incomparable' relation,[5] demonstrated on the small scale by 3-2 sounding against 4-8 and 4-9, and on the large scale by B section sets, which in Forte's analysis are incomparable with any A section sets of cardinality greater than 3. The second is the alternation between emphasising ic4 and ic6: on the smallest scale this is represented by the relationship between 3-4 and 3-5. These are the

5. A relation between two sets of differing cardinalities where it is impossible to map the smaller set into the larger whilst preserving interval content (Forte 1964, 159–60).

two subsets of 4-8 which is an important set in the work, but these two sets are also prominent on their own terms: as one example, the series of trichords from the final quaver of b. 4 to the first quaver of b. 6 read 3-5, 3-5, 3-4, 3-5, 3-5, 3-8, 3-5, 3-4 (and in Brown's view the 3-8 results from a suspension, and so does not even qualify as a 'full-fledged harmony' (S. C. Brown 2013, 28)). On a broader scale, Forte finds the A section to be characterised by ic6 and the B section by ic4. This is demonstrated, with just one example, by the superset 6-34, which occurs only in the B section and has high ic4. Forte's analysis is insightful and makes a convincing claim for the harmonic articulation of structure, though it is marred by the abrasive writing style that leaves much to the reader's inference, and occasional errors.[6]

The aim of Beach's article is to demonstrate ideas from *The Structure of Atonal Music* which had been published six years previously, whilst Beach was studying with Forte (Berry 2009, 214). Thanks to its pedagogical intent, it has a much clearer and more accessible writing style than Forte's. On the whole, it only diverges infrequently from the earlier analysis, although there are some implied disagreements about segmentation. He too draws attention to the importance of sets 4-8 and 4-9 (and their superset 5-7) as characterising the A section, and indeed the importance of these sets as a 'major unifying factor' (Beach 1979, 19). Meanwhile, he finds the B section to be defined by sets 5-30 and 4-17 (together comprising the superset 8-24). Just like Forte, Beach draws attention to the 'differentiation of intervallic content' between the sets that characterise these two sections: the climax of his analysis is the finding that 5-7 and 5-30, respectively crucial to the A and B sections, are in the relation R_0 (that is, they have no interval vectors in common (Forte 1973, 49)). The result of this is to suggest that

6. As just two small examples: The statement '3-4 … is one of the two 3-note subsets of 4-9' (Forte 1964, 174) is incorrect (presumably a typo). He surely means to write 4-8 (4-9 of course only has one trichordal subset, 3-5). Likewise, the assertion that 'A vertical or diagonal cross-section taken at any point except the middle section (bars 7–9) will yield either 3-4 or 3-5' (Forte 1964, 174–75) is blatantly incorrect. This is even more bizarre given it follows a discussion of 3-2, selected from a diagonal segmentation of b. 2. The work does display the intervallic stability that Forte claims, but it is a shame that unnecessary universalising of this sort tarnishes an otherwise insightful analysis.

'the piece as a whole does not exhibit a connected structure' (Beach 1979, 21); interestingly this somewhat contradicts Perle's analysis, the emphasis of which seems to be on finding connections across the movement as a whole, and in particular in arguing that the material of the B section is to some extent derived from that of the A section through the transitional 'flyaway' motif. Perle therefore offers a more linear analysis than Beach or Forte: thematic elements, presented vertically and horizontally (i.e. as 'chords' and 'motifs') are successively varied in what almost amounts to a quasi-developing variation. Formal articulation, for Perle, is thus achieved through 'timbres, spatial relations, and rhythmic and dynamic details' (Perle 1968, 18) more than through the harmonic and melodic structure of Forte and Beach.

As mentioned, it is the flyaway motif (Figure 4.2) that fascinates Lewin.[7] This is also the principal focus for Budde. Budde argues that the interval structure of the flyaway motif is in part derived from the first violin's opening bars: the C-E major third from b. 1, the E-F♯ major second from the top notes of bb. 1–2 (bolstered by the viola in b. 2), and the F♯-B from the fifth in b. 2. The B-C♯ tone is apparently motivated by the aforementioned E-F♯ tone, while the C♯-G-B♭ all point to the second section of the movement. The first half of this analysis, at least, seems comparatively convincing. The repetition of the E-F♯ at the same pitch level at which it appeared in the viola supports a link back to earlier in the movement, and there would seem something almost cadential about concluding the A section with a return to its opening pitch material. Less convincing is Budde's argument that the pitch material of the B section is derived from the flyaway motif in the A section. He proposes that the cello E and viola G♭ can be understood as linked to the E and F♯ of the flyaway motif by virtue of appearing at the same pitch level. Meanwhile, the viola B♭ and D, the second violin B♮, and the first violin B♮ and G♯ are linked, respectively, to the flyaway motif's B♮, C♯ C♮, B♭ and G♯ by virtue

7. The pitch level given here is that of the first statement in b. 6. Lewin's major third extension (giving the initial G♯) is applied to this transposition level, while his minor third extension (giving the final C♯) is only applied to the second statement in b. 10 but is presented here (transposed) for ease of comparison.

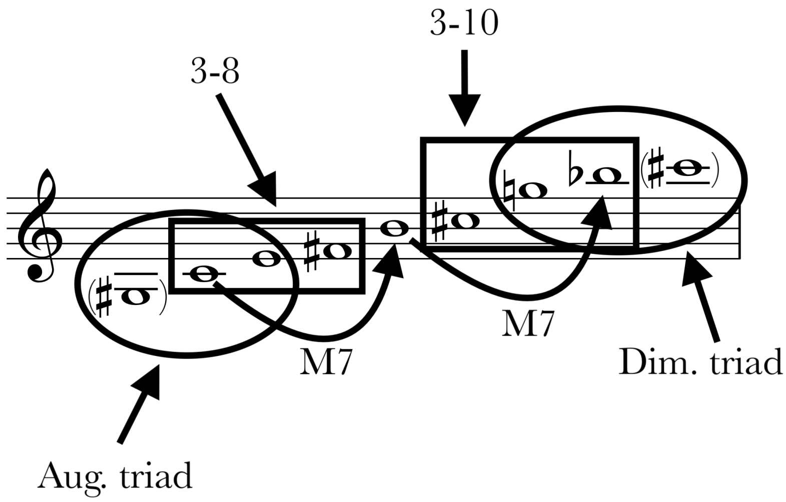

Figure 4.2: Flyaway motif, including Lewin's hypothetical 'extensions' in brackets.

of having moved only one semitone. Budde acknowledges that the C and E♭ in the first violin are harder to relate to the flyaway motif, but nonetheless tries, arguing that the minor third is an inversion of the preceding G♯-B (and thus G-B♭) in the first violin, and that the C6 is only a minor second from the previous B5 peak. Frankly, I find this unconvincing. He makes a sweeping statement that the B section material is derived from the flyaway motif, but, in order to make this work, has to apply several linking approaches, of various levels of plausibility, and two of the nine pitches under consideration remain essentially unexplained.

Lewin does not engage with Budde's work, but instead opens by pointing to Beach's comment that, with regard to the final statement of the motif, 'it is not entirely clear why Webern chose this particular transposition' (Beach 1979, 21): it is Lewin's aim to inject some clarity into the discussion. Interestingly, Perle's analysis, to which Lewin makes brief reference, tackles this very question: he argues that the transposition level is chosen so that the final pitch is F♯6, the same as was played in the first violin in bb. 2–3. For Lewin, this observation (unacknowledged) is relegated to a footnote (Lewin 1982, 41 note 3): clearly it does not provide enough explanatory force in his view. Instead, he finds a two-step process that, applied in a quasi-symmetrical manner, generates the final statement of the flyaway motif from, respectively, the first and middle statements.

Whether one is 'intellectually convinced', as he puts it, by Lewin's demonstration of serial technique, a particularly helpful implication of his analysis is the focus on the two ends of the motif as displaying, respectively, an augmented and diminished triad. To me, this broader argument for a whole-tone/octatonic polarity is better supported not through Lewin's hypothetical extension notes to form these triads, but simply by segmenting the first and last triads of the motif: 3-8 (prime form (0 2 6)) and 3-10 (prime form (0 3 6)), which clearly lean towards opposite poles. This segmentation is perhaps supported by the symmetry of the motif itself, bounded by two major sevenths emanating from the central B. Indeed, I find it something of a strange omission that in a section of his analysis in which Lewin is specifically talking about harmonic qualities present on the surface of the music ('the "final diminished triad" is a characteristic type of sound in the piece', 'the sound of an "incipit augmented triad" … is certainly a highly characteristic sound within the piece' (Lewin 1982, 43, italics his)) he would cling to the hypothetical extensions he applies to the motif and forgo the reality of Webern's music, rather than integrating the two perspectives.

This varied debate thus throws up several points of interest, but most significantly the relationship of the harmonic structure to the ternary form that is clearly articulated by textural and melodic features. It is into this morass that Yust briefly dipped a DFT-clad toe (Yust 2015a, 212–14). Yust's premise is to take various pc sets featured in the movement and apply the DFT to them. He is largely aiming to follow Forte's categorisation of pc sets into those that a) maximise ic6 and minimise ic4, b) minimise ic6 and maximise ic4, and c) other. Deploying the DFT aims to demonstrate that it identifies these properties and does so in a more sophisticated and detailed manner than simply comparing interval vectors.[8] An inevitable topic of discussion with this approach, however, is that eternal question of segmentation. As ever, Yust, in his list of 'significant pcsets' (Yust 2015a, 213),

8. Yust labels his various sets with letters (′ indicating pc set equivalence at a new transposition level) which is perfectly adequate when reading his analysis in isolation, but adds tedious friction to comparison. In case it's helpful in comparing analyses, his labels refer to: A: 4-9; B: 4-8; C: 6-z6; D: 4-19; E: 4-17; F: 5-30; V: 4-16; Flyaway: 7-19.

neglects to assess some of the sets that Forte describes as ‘the most prominent sets’ (Forte 1964, 174) (e.g. 3-2 in b. 2), and offers his own segmentations instead (e.g. 6-z6 across bb. 3–5). As in Beach’s analysis, I find the lack of consideration for 3-2 to be a particularly odd omission. On the whole, the point that Yust seeks to make is clear: his Universe A (a group of sets that includes A, A′, A″, and various combinations thereof)[9] is clearly defined by high $|f_2|$ and low values for other squared magnitudes; Universe B, meanwhile (sets D, E, F, and combinations of them) is characterised by high $|f_3|$ and low $|f_2|$. Less firmly, he identifies a transitional pair of sets (B and C) with high $|f_2|$ and moderate $|f_3|$, whilst V and flyaway are somewhere between Universes A and B. An immediate comment to make is that whilst Universes A and B have clear temporal distinction (respectively, the A and B sections of the form), the transitional sets (B and C) all temporally fit into the A section, though without harmonic features of the sets in Universe A, which seems like it would somewhat muddy the implied harmonic consistency of that section. If 3-2 were included it would confuse things further, as its squared magnitudes read: 5.73, 1, 1, 3, 2.27, 1. This fails to fit into any of the groupings Yust has identified. Indeed, it is something of an oddity of the analysis that the first sound in the piece, set-class 4-8, is labelled ‘B’. Nonetheless, this variation in $|f_2|$ and $|f_3|$ values implies a polarity. Interestingly, this is not between a diminished/octatonic and augmented/whole-tone realm as posited above, but rather between quartal ($|f_2|$) and augmented ($|f_3|$) harmonic worlds.

4.4.2 A New Approach

Evidently, a lot of weight hangs on segmentation. All the analysts considered thus far have differed in their segmentations, and so despite lots of overlap (and realistically, in a movement that lasts less than two minutes, there can only be so

9. In his analysis, Yust includes both the union and addition of two sets. The former aggregates the pitch class content of the two sets and weights them each as one; the latter in effect weights each pitch class according to its frequency (although he doesn’t use this term). As an example with the pair of sets (0 1 5) and (0 2 6): the union in conventional set notation has the content (0 1 2 5 6), or as a point in twelve-dimensional space, (1 1 1 0 0 1 1 0 0 0 0 0); the addition cannot be represented meaningfully in conventional notation, but in twelve-dimension space is (2 1 1 0 0 1 1 0 0 0 0 0).

much difference in segmentation), a fundamentally similar methodology has proposed various results. Of course, this is not a problem; indeed, these conflicting perspectives are a fruitful inspiration for hearing the music through multiple pairs of ears (even if through some the music seems oddly distorted). I hope, therefore, that to add another approach will not muddy the water so much as expand the pool. The viewpoint I suggest is, as this chapter focusses on, to consider the macroharmony of the work rather than these smaller harmonic units. The premise of a 'conventional' musically informed segmentation is to assume that the analyst can discern those features of the music that a listener hears as distinct units. A macroharmonic approach, by contrast, flattens the entire texture: pitches are grouped only according to whether they occur within a preordained temporal span, and are weighted only according to the principles laid out by the analyst (which might, as discussed above, include other, more 'musical' parameters such as motivic importance). These are therefore complementary approaches. In fact, at times these authors come close to considering what are essentially macroharmonies. As just one example, Yust's segmentation of C runs for $\frac{3}{4}$ beats from the final quaver of b. 4 to the seventh semiquaver of b. 6 and alters harmonic content midway through with the addition of first a G♮ and then a D♮. It seems to me that this could reasonably be partitioned into anywhere between two (break either with the entry of the G♮ or the D♮) and eight (break each quaver) segments. There are two levels on which to consider the macroharmony of the work: first, and simpler, is the overall pc distribution, as introduced above. Second, the macroharmony of subdivisions of the movement. This could be done in many ways: the three sections of the ternary form, for example, would make obvious subdivisions. The method I propose in this paper follows Chiu (2021) and takes a windowing approach. That is, it segments the work into successive 5-second passages, each a second apart (and therefore overlapping), and creates a pc distribution for each passage. This can then be analysed with the DFT, and the resulting squared component magnitudes can then be plotted on a line graph. Before delving into the results, there are a few comments to make about the method. The first is to note that these segments are made according to time as expressed in seconds rather than notational features like bars or beats. As mentioned above in the discussion about weighting, this is to take account of changes in tempo. Secondly, the decision to partition the work in 5-second windows follows from John A. Michon's review of the chronopsychological understanding of the present, in which he suggests that a 'dynamic present' typically lasts 2–5 seconds (Michon 2001, 52). Finally, just to reiterate the weighting process, distributions are first weighted by duration and then by the $log_2(x+1)$ function.

4.4.2.1 The Movement's Macroharmony

To begin, I consider the overall macroharmony of the movement; after all, it was the spike of squared DFT magnitudes in Figure 4.1 that inspired the choice of this movement as a case study. The movement's overall pc distribution is given in Figure 4.3; the squared DFT magnitudes that derive from this are given in Figure 4.4. Quite obviously, the spectrum is dominated by $|f_2|$, with lower spikes on $|f_1|$, $|f_5|$, and $|f_6|$, while $|f_3|$ and $|f_4|$ remain very low. A high $|f_2|$ was found above to be typical of Webern's music, so this is unsurprising. The low values for $|f_3|$ and $|f_4|$, however, are more unusual. Inspecting Figure 3 immediately makes sense of these findings, with particularly high peaks at 0, 4, 6, and 11. Both Budde (1989) and Charles Burkhart (1980) point out that E and F♯ are particularly important pitches in the movement, from the opening 4-8 to 4-9 succession (in the first violin), immediately followed by the viola, to the combination of cello and viola in the B section (now respelled as G♭). Meanwhile, even a cursory glance at the score suggests the significance of the dyad B-C. It forms a crucial part of many instances of 4-8 and 4-9 right from the opening but is also the central dyad in the symmetrical set 4-17 that forms the tender first violin melody in the B section. It is fascinating, therefore, to see the significance of this pair of dyads, emphasised by Webern through other means, reflected in the pc distribution. Of course, what the high $|f_2|$ points to is their dispersion on the pc circle, where the only repeating ic is 5, hence the quartal quality of the distribution. These findings are particularly interesting when put into conversation with Yust's work: his Universe A and B were defined, respectively, by high $|f_2|$ and high $|f_3|$. Whilst the former is reflected here, the latter is not. Of course, as has been pointed out several times by now the overall distribution is distinct from the smaller harmonic units that comprise it when aggregated, and there is no reason to assume *a priori* that they would reflect the same harmonic tendencies. To provide an extreme example, a piece could be constructed solely from three diminished sevenths, each of which on their own has squared magnitudes 0, 0, 0, 16, 0, 0. Now, suppose that these chords were rooted, respectively, on C, C♯, and D, and all had a duration of 1 second. In that case, the overall distribution squared magnitudes would all be 0. This is of course an extreme example, but while in tonal music Yust has found that distributions selected from the starts of works tend to be very similar to those of the whole work (Yust 2019, 13–14), at varying window sizes $|f_4|$ changes size while $|f_2|$ and $|f_3|$ do not (Yust 2020). Nonetheless, there is no reason to presume that the same correspondence would necessarily carry over to post-tonal music, where distributions tend not to follow the common patterns seen in tonal music. As a result, that the harmonic tendencies of Universe A are reflected in Figure 4.4 is a notable finding.

4.4.2.2 Changing Macroharmonies: A View Through Windows

A more detailed perspective on the macroharmonic worlds that this movement passes through is provided by the windowing approach summarised above. Figure 4.5 charts the squared magnitudes of these 5-second windows across the movement. Immediately, a tripartite form with introduction seems to stand out: approximately windows 0–11 (start to b. 3, fourth quaver)[10]; 11–37 (b. 2, fifth quaver to b. 7, seventh triplet quaver); 37–62 (b. 6, twelfth semiquaver to b. 11, fourth quaver); and 62 to the end (b. 10, final quaver to the end). More

10. Because windows are measured in durations rather than beats, when I refer to a window comprising a span of beats, the first and last beat are unlikely to be included in their entirety, but to save unnecessary verbiage I will not give this level of detail (for example, window 11 technically only includes 0.48 seconds of the fourth quaver of b. 3, or 47% of the quaver duration).

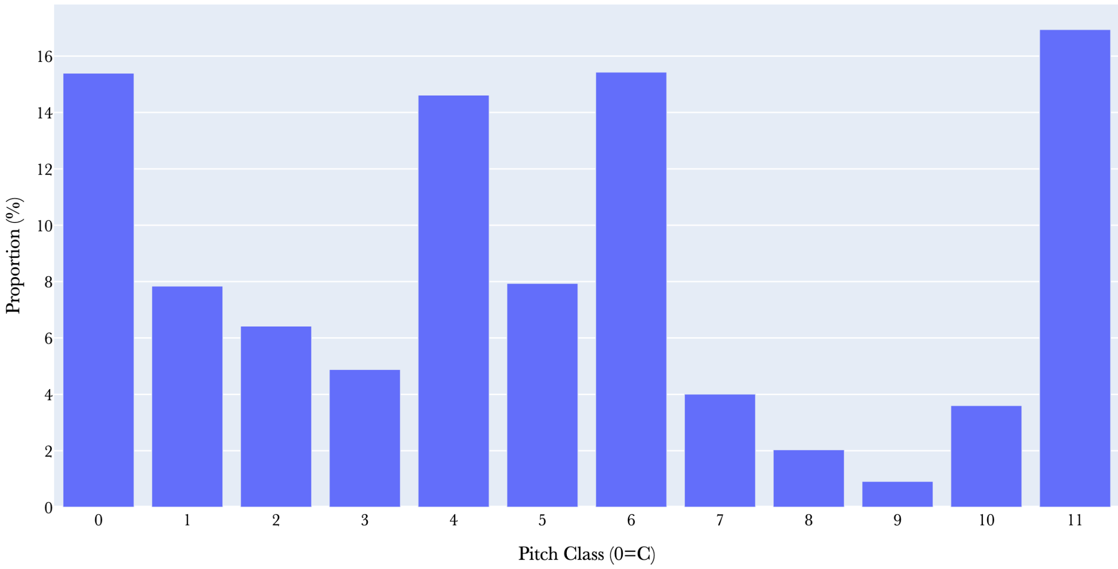

Figure 4.3: Op. 5/iv pc distribution.

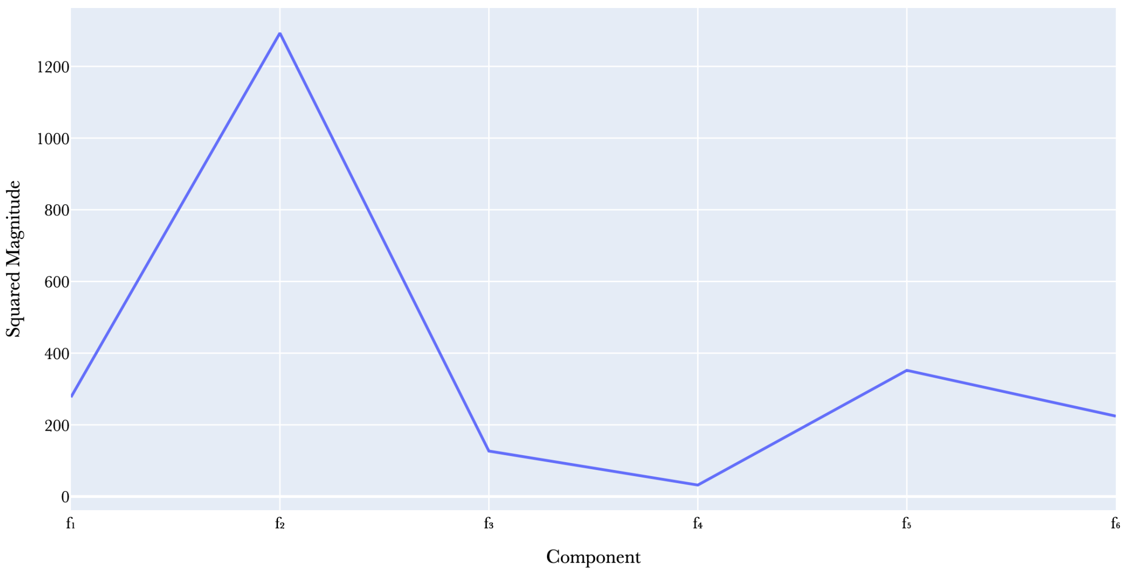

Figure 4.4: Op. 5/iv pc distribution squared DFT magnitudes.

specifically, the ternary form is obvious, with both A sections defined by an elevated $|f_2|$ above any other squared magnitudes, whilst the B section is characterised by $|f_2|$ and $|f_5|$ at a roughly equal level (following an initial spike in $|f_5|$), shadowed marginally lower by $|f_3|$. The prioritisation of $|f_2|$ from Figure 4.4 is clearly reflected on the local level, as was observed in Yust's analysis. This does show, however, when certain harmonic features become strongest: in the A section, the peak of $|f_2|$ is at window 27 (b. 5 third quaver to b. 6 first quaver); in

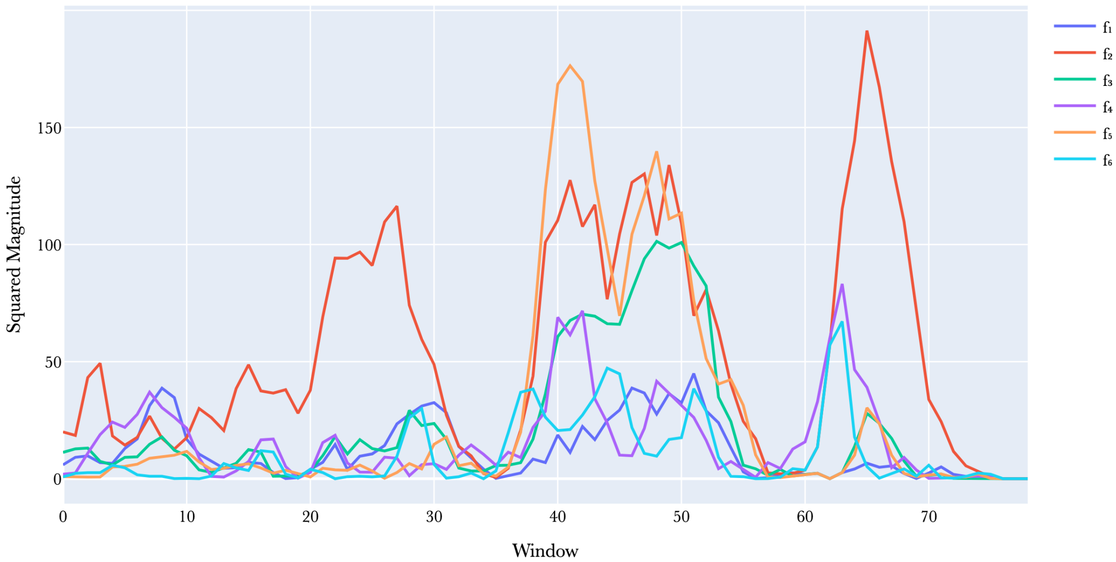

Figure 4.5: Op. 5/iv squared DFT component magnitudes of 5-second windowed pc distributions.

A′ it is window 65 (b. 11 third quaver to b. 12 second quaver). In both cases this is the canonic passage (in A between first violin and cello; in A′ between second violin/viola and cello) which is particularly harmonically concise. I do find it notable that the concentration of A in A′ is reflected in the greater $|f_2|$ strength of A′ compared to A. This is all down to durations and the octave doubling in violin 2/viola: the total pc content of both passages is set 5-7, but in A′ over a third of the distribution is concentrated in pc 0, which thus increases the total squared magnitude.

Things become more surprising in the B section, however, where $|f_3|$ and $|f_5|$ become stronger: this is implied neither by Yust's analysis, nor by Figure 4.4, but it demonstrates the strength of this approach. Whereas the total pc distribution assesses the overall macroharmony and so flattens the detail, an analysis of specific sets relies on a very detailed segmentation of the work. In this case, Yust's Universe B is comprised of sets D, E, and F (respectively, the viola line, the total violin II/viola/cello accompaniment, and the first violin melody): this stratified segmentation therefore fails to take account of the interaction of these parts in creating a successively evolving macroharmony, as demonstrated by this analysis. The temptation for his segmentation is clear: the accompaniment is—on the whole—harmonically and texturally static, and so it is easy to imagine that a listener might partition the texture horizontally, as Yust does. Nonetheless, a macroharmonic analysis shows a different side to the character. In this case, whilst $|f_3|$ points to the importance of the augmented triad in the viola, $|f_5|$ reminds us of the importance of the implied cycle of fifths present in the accompaniment: D(–A)–E–B–F♯/G♭. Yust argues that his Universe B 'might be heard as a reference to tonal harmony, made hazy by a lack of diatonicity' (Yust 2015a, 213); in fact, in the context of the movement there is more diatonicity than he suggests. Meanwhile, the melody prompts the spikes in $|f_6|$ as it moves in and out of the whole-tone collection implied by the accompaniment (E, G♭, B♭, D in the accompaniment; G♯ and C in the melody).

Another high-level observation is that all the squared magnitudes collapse in value at the A/B and B/A′ sections (around windows 35 and 58). What this points to is that in both cases the transitional flyaway motif is rapidly increasing the number of pcs in circulation, and so creating a flatter distribution and weaker squared DFT magnitudes. For example, window 27 (b. 5 third quaver to b. 6 first quaver), which is the first significant $|f_2|$ peak, has only five pcs, whilst window 35 (b. 6 eighth semiquaver to b. 7 fifth triplet quaver), which is the lowest point of the first transition, has eight pcs in circulation. The former also has much greater variation in usage amongst the pcs: the percentage difference between most common and least common pc (excluding those that are totally absent) for window 27 is 13.3% whilst for window 35 it is 6.6%. More broadly, this also points to the difference in pc emphasis between the A and B sections, which is exemplified by Figure 4.6.[11] This charts the phases of each window for Ph_2 and Ph_5, which, as found above, are the most significant components in this movement, and so speaks to their specific pc content. Figure 6 clearly demonstrates the difference between the B section and the rest of the material, as its windows are all clustered away from the rest of the material. With median

11. The windows associated with each section are only those that are fully contained within a section, rather than including those that bridge the divide between two sections. They are therefore as follows: Intro 0–8; A 13–32; B 38–58; A′ 63–end.

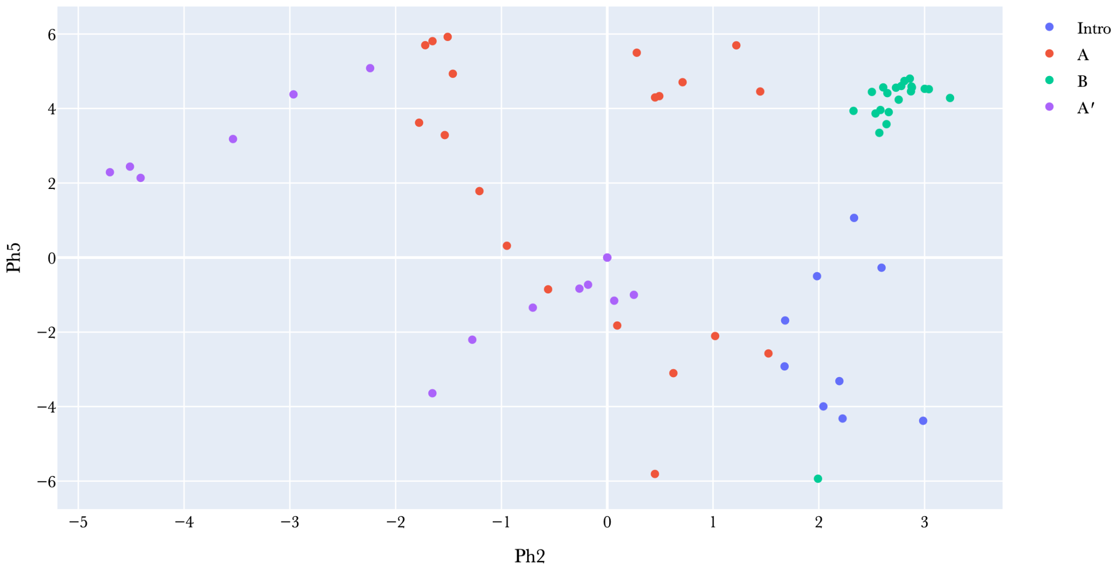

Figure 4.6: Op. 5/iv DFT component phases for Ph_2 *and* Ph_5 *of 5-second windowed pc distributions.*

values of 2.7 for Ph_2 and 4.4 for Ph_5, for both phases it bisects the gap between E and B in phase space, as can be seen from Figure 4.7, emphasising these pcs. By contrast, A and A′ are much more spread out on both axes. Beach describes 'slow turnover of pitch material' (Beach 1979, 19) as characterising the movement as a whole, but it should be clear from this analysis that this is a particularly strong feature of the B section, more so than the others. This draws attention to the flatter distribution with more variation across these sections. Meanwhile, the introduction is again demarcated in quite a different region of phase space. With a median Ph_2 value of 2.2, this highlights the prominence of the B-F tritone in this first passage (Ph_5 is much less prominent in this section, so is less relevant). Perle has described this as a movement—and Webern's music more broadly as constituting an idiom—in which 'harmonic and melodic homogeneity tend to obliterate distinctions' (Perle 1968, 18). I do hope that this analysis has shown that from both the perspective of harmonic quality and pc position, this could hardly be further from the truth.

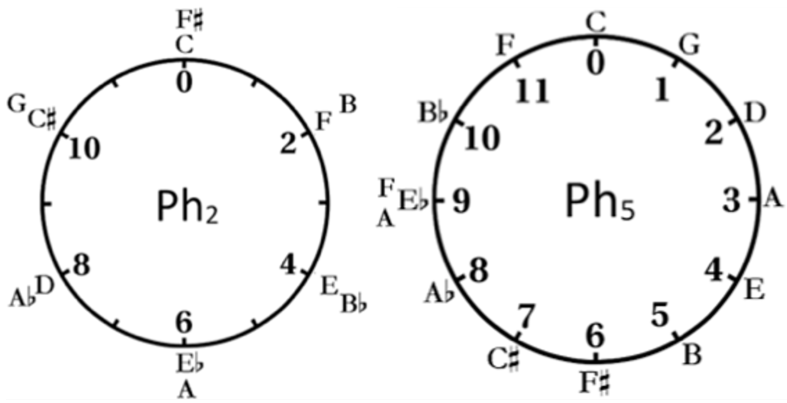

Figure 4.7: Ph_2 and Ph_5 phase space (from Yust 2016, 216).

4.5 Continuity or Change?

One of the major advantages of the corpus study as an analytical approach is its ability to contextualise an individual movement: to tell us whether something is typical or unusual. This was clearly helpful in the preceding section of this chapter and having spent some time 'on the ground' with an individual movement, the final section of this chapter will return to the broader perspective of the whole corpus. The question that I want to ask here is how stable the movements are with regard to harmonic quality (i.e. squared DFT magnitude). In Op. 5/iv there is a clear change between the squared magnitudes of the A and B sections, which in this movement had an important structural function. Although Figure 4.1 certainly suggests that the strength of the squared magnitudes in this movement are unusual, it says nothing about the degree of intra-movement variation. I suggest that there are two comparatively simple ways to measure this, which offer complementary perspectives on the same question.

The first is to calculate the spread, typically measured by the range of values, for each squared magnitude ($|f_1|$–$|f_6|$) in the windows across a movement, and then calculate the median of these six spread values. This provides a rough sense of how variable the magnitudes are: a movement where all squared magnitudes remain in a very narrow span would have a low median; one with a lot of variety would have a much larger median. As mentioned, the standard measure to use would be the range, however, there is an issue with this approach: if a window is silent (and so all squared magnitudes are 0) then the range becomes a measure merely of the highest strength of a squared magnitude in the movement, rather than an indication of the difference between its greatest and lowest strength in the harmonic collections used in the movement (let us assume, for now, that silence does not count as having any harmonic content). To take an extreme example, ignoring the hypothetical silent window one squared component magnitude might always have a value lying between 40 and 50 while another might fluctuate between 10 and 50, and yet they would appear to have the same range of 50. More broadly, this speaks to a general issue with ranges which is that they are significantly influenced by any extreme values, which may well be outliers and are not of interest in this enquiry. As in Chapter 3, a more helpful measure is therefore the interquartile range (IQR), which gives a sense of the central tendency of the spread and is less influenced by outlier values, and so that will be used here.

The second method of assessing variation is to calculate the correlation between squared DFT magnitudes of successive windows across a movement. A high correlation indicates that the two windows are very similar, and so there is continuity in harmonic quality; a low correlation suggests the opposite. Of course, this does not quantify the *nature* of the change, nor does it provide information about relationships between larger sections of a movement: for example, a movement that gradually progresses from a whole-tone collection to an octatonic collection and then returns to a whole-tone collection would look very similar to a movement that moved from a whole-tone to an octatonic to a diatonic collection. Nonetheless, it does provide a longitudinal measure of continuity, which can then be assessed in more detail—if desired—by separating out the individual DFT components. Further, calculating the correlations between overlapping windows has a tendency to artificially inflate values, as there is inevitably significant similarity between windows that share a large proportion of the same pitches. As a result, in practice I calculate the correlation between every fifth window, which thus minimises the overlap while ensuring every note is considered. I use Pearson correlation for this as it is less sensitive to swaps in rank order, which often occur as the squared magnitudes are often close in size, though the impact is minimal. This is informative on a movement-by-movement basis, but in the spirit of the corpus study approach, I can also calculate the median correlation value, which thus provides another measure of the degree of intra-movement variation.

There are two final points to make. The first is that for the IQR measure it is preferable to normalise the squared magnitude values by calculating them as a percentage of the maximum value any squared magnitude reaches. This discards the differences between movements in total squared magnitude strengths, and instead concentrates solely on the degree of variation within the movement. This normalisation is not necessary for correlations, as they are the same either way. The other thing to say is that it is important to note that the focus here is on squared magnitudes, and thus phase data is absent. This limitation means that intra-movement motion around the pc cycle is ignored, so a movement that, for example, is wholly octatonic, but frequently alternates between the two octatonic collections would show as very static.[12] Obviously, the same spread and correlation measures could be applied to assess changes in phase data, and thus in pitch motion, but this subject strikes me as less informative and less interesting than questions of harmonic quality. I suspect also that a more sophisticated measure of continuity could be developed using entropy models, but that will have to wait for another time.[13] Together, therefore, as mentioned these two methods provide different perspectives: in short, the spread describes how much variation there is across a movement, the correlation values indicate whether this variation happens suddenly or gradually, whether short-term variation is common or unusual.

12. I am indebted to Matt Chiu for a helpful and clarifying conversation on this topic.
13. I am very grateful to Fabian C. Moss for this suggestion in relation to some of my other work.

4.5.1 How much variation is there?

First, I will consider the IQR results. Figure 4.8 plots these and suggests that the overall level of variation is quite changeable across the corpus, though with a gradual decline. Indeed, there are 11 anomalous movements, including Op. 5/iv, although as it has a value of 3.91 it is only marginally over the threshold to be classed as an anomaly. The median value of Figure 4.8 is 1.14, so Op. 5/iv has markedly more internal variation than is typical. The picture of gradual decline is confirmed by the correlation between median IQR and corpus position, which is -.37.[14] This negative correlation is rather an interesting finding. On first glance this would seem to be explained by the relatively flatter distributions later in the corpus (a result of both the dodecaphonic technique, and the wider aesthetic that prioritised equality of pitches in this period). However, normalising for the squared magnitude strength already controls for this, which suggests that the remaining disparity between the earlier and later sections of the corpus is solely down to lower squared magnitude variation: Webern's later music is more static in its harmonic quality than the earlier music. This may be due to the unifying harmonic effect of a row in a movement. The research presented in the next chapter finds that there is a strong predictive relationship between the intervallic content of a row and the resulting music. As such, if a movement is governed by a particular row with a particular intervallic character (as Webern's rows famously are), then one would expect this intervallic quality to pervade the movement as a whole and thus every window in the work, therefore creating the comparative stasis that Figure 4.8 suggests.[15] To my eye, Figure 4.8 seems to lack much of a change around Opp. 11 and 12, which is more obvious in Figure 4.1; however, if the corpus is split between Opp. 11 and 12, these two subsections have

14. For this and the equivalent calculation with regard to Figure 4.10 I use Pearson's Correlation, as both figures suggest a linear relationship between the variables.

15. This could, of course, be tested empirically. A regression analysis could take the variety in intervallic content of the row as the independent variable and the median IQR from this chapter as the dependent variable. Statistical power might be an issue, as the sample size would be fairly small, and a method would have to be developed to portray the intervals in a row accurately (a basic frequency count of different interval classes would obviously be insufficiently sensitive; some weighting protocol would need to be introduced).

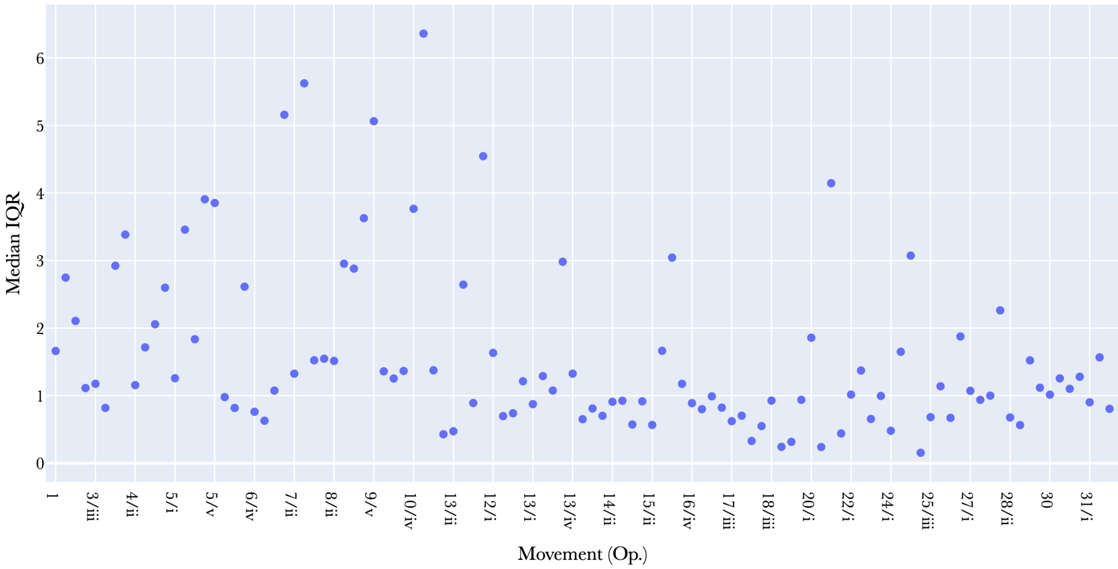

Figure 4.8: Median of IQR for each squared component magnitude.

correlations between IQR and chronological position of 0.14 and 0.03, which implies that actually they are relatively internally consistent. The overall decline instead comes from the disparity between the two. Digging into the latter subsection (Opp. 12–31), we can further subdivide this between the freely atonal music (Opp. 12–16) and dodecaphonic music (Opp. 17–31), to try and establish whether the dodecaphonic technique created any difference. These two segments of the corpus have correlations of 0.00 and 0.16, respectively. Fundamentally, all of these correlation values are very low, suggesting that while there is meaningful change across the corpus as a whole, this mainly comes from differences between subgroups, rather than a more gradual change.

4.5.2 Suddenly or gradually?

Turning to the consecutive window correlations, as a single example Figure 4.9 is a graph for Op. 5/iv. Any similarities to Figure 4.5 are not immediately obvious: although a ternary form can perhaps be discerned, this may be a case of motivated reasoning. This graph does, however, indicate more obviously the differences between the A and A′ sections with a much more harmonically volatile A′ section. Interestingly, this graph describes the B section as less static than the A. Looking back at Figure 4.5 makes sense of this: whereas the A section is consistently dominated by large $|f_2|$ peaks, the B section oscillates between $|f_2|$ and $|f_5|$. Figure 4.10 generalises this by charting the median correlation value for all movements. Here Op. 5/iv has a value of 0.95 while the overall median of medians is 0.99. Looking at the larger picture, there is a positive correlation with corpus position, though it is obviously stronger this time (0.28). This therefore supports the implications of the correlation in Figure 4.8. A lower median correlation value in Figure 4.10 implies a greater degree of intra-movement variation, and so the overall positive correlation implies that intra-movement variation decreases later in the corpus. Along these lines,Op. 27/ii, which is comparatively internally static (correlation median of 1.00), is a good example from later in the corpus. To get a sense of this, Figure 4.11 is the equivalent graph of Figure 4.5 for Op 27/ii. It is clear that while the window-to-window hierarchy is in flux, with the squared magnitudes constantly overlapping and switching positions, each component traverses a very narrow span (compare the y-axes between Figure 4.5 and Figure 4.11), and so there is a relatively low level of window-to-window variation. In musical terms, this implies that later in the corpus not only do the squared magnitudes themselves, the different harmonic qualities, become less variable across a movement, but the moment-to-moment harmonic successions are more continuous and less changeable. This obviously throws up a question of causality: with three variables all expressing largely the same chronological pattern, one is left asking whether perhaps smaller ranges prompt less window-to-window variation.

Alternatively, this may merely be a 'musical' phenomenon; the coincidence of the two non-temporal variables may be features of Webern's changing musical language. Perhaps he sought less moment-to-moment variation in his later works alongside the increased macro-level harmonic restriction that was controlled by the row? I will return to the latter below in assessing the impact of the intervallic qualities of the row on the music. As for the works in question, extremes in this case may be due to tempo. Both Op. 25/iii and Op. 27/ii are very fast: the former is marked *Sehr rasch* with 𝅗𝅥=96, the latter *Sehr schnell* with ♩=160. Indeed, in

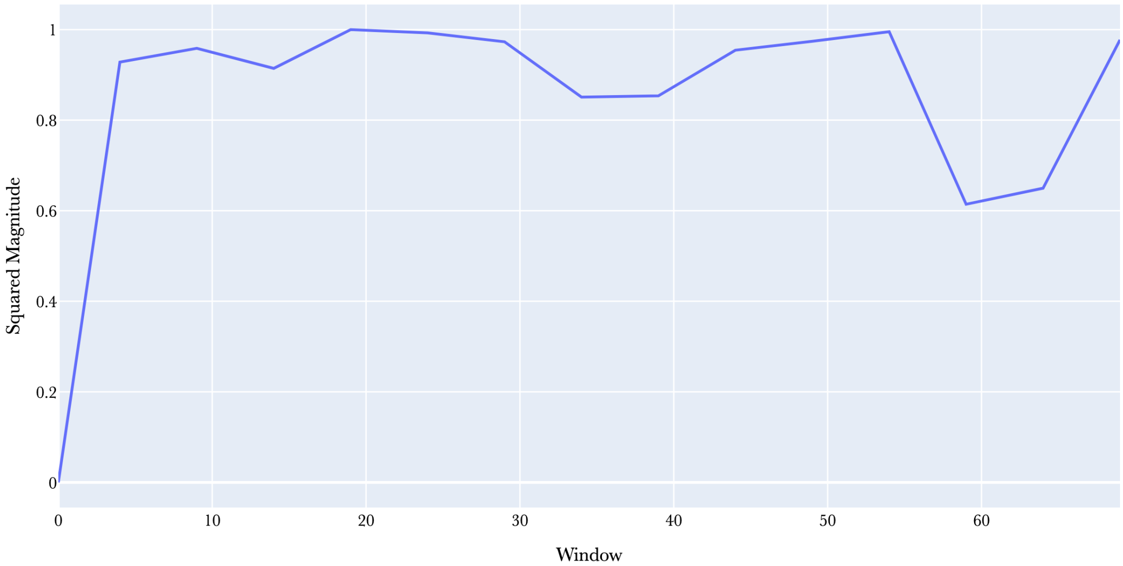

Figure 4.9: Op. 5/iv correlations between consecutive window squared DFT magnitudes.

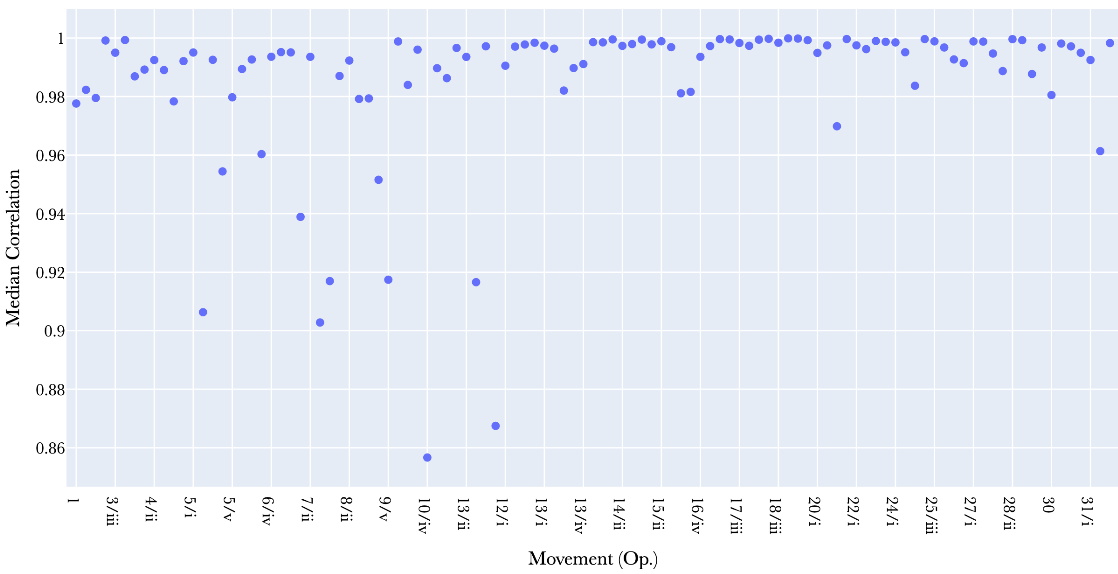

Figure 4.10: Median of correlations between consecutive windows.

an average 5-second window, Op. 27/ii has 21.5 pitches (this count of pitches is slightly different to notes in the score, as these may be subdivided in the analytical process, but the point stands that it is a high number) and two-thirds of the windows include at least one statement of each pc.

To explore the other end of the corpus I will now look at Op. 27/ii in some more detail. This will not be as extensive an analysis as that of Op. 5/iv but can be read comparatively as the process is similar. In many ways these movements are

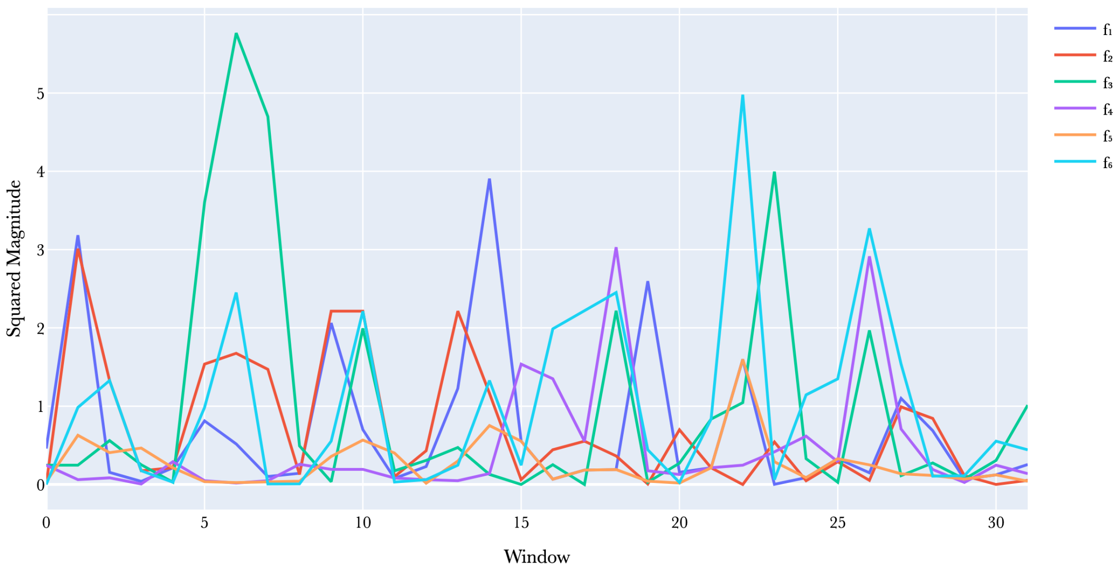

Figure 4.11: Op. 27/ii squared DFT component magnitudes of 5-second windows.

polar opposites: they are at opposite ends of the scale in Fig. 4.10, indicating that while in the earlier movement there is significant window-to-window change, this is not the case in the dodecaphonic movement; likewise, they are clearly (though not as extremely) differentiated in Figure 4.9, suggesting that in the quartet movement squared magnitude values for each window tend to be far more varied than in the piano movement. Op. 27 is another work that has received a vast quantity of scholarly attention, indeed as I mentioned in the introductory chapter the piano variations have more publications about them in Hoskisson's (2017) bibliography of post-1927 publications than any other work of Webern's. An exhaustive summary of this literature is therefore simply beyond the scope, so I will merely point out a few trends in the movement's analytical reception. Studies have taken a variety of angles: the enigmatic title has attracted a variety of formal interpretations (Bailey 1988; Busch 1988); the solo nature of the work has also inspired relatively many recordings, which has then prompted various performance-motivated studies (Cook 2017; Mead 1999; O'Hagan 2023); and the curious metre inspired Lewin twice (1962, 1993). The pitch material has been explored by a plethora of authors, in terms of the ingenious row deployment and integrated canons (Babbitt 1960; Bailey 1991; Ogdon 1962), but also in terms of resulting surface features apart from the rows (Barraqué reconstructed in Riotte and Mesnage 1989; Westergaard 1963), particularly with respect to the dyadic organisation (Buccheri 1975; Koivisto 1996), an attitude that Catherine Nolan (1995) terms 'revisionist'. A perceptive comment comes from Peter Westergaard (1963, 114), who points out that in hearing the music, the most prominent intervals are those formed by the interaction of the two canonic (and thus, dodecaphonic) voices, rather than the internal intervals of the rows themselves. As an example, the first two bars most obviously articulate a B♭-G♯ tone and the A-A unison that dominates the movement, rather than the two semitones B♭-A and G♯-A that comprise the opening intervals of the two row forms. Indeed, John Stephen Buccheri goes so far as to suggest that 'set articulation is an elusive if at all real phenomenon' (Buccheri 1975, 144). This truly is a movement in which the disposition of rows forms a background structure, rather than a thematic foreground. These comments lend weight to the importance of an analytical approach like mine that eschews the background structure in favour of the surface phenomena.

I will now offer a few analytical thoughts, deriving from the windowed DFT analysis presented in Fig 4.11. It is vital at all times to view these comments in the context of the overall movement: the rate of chromatic turnover is always very high, and as a result even in those passages where harmonic colours are established comparatively strongly, these are much weaker than in other music by Webern. Looking first at the form, the binary form of Op. 27/ii is much less clear in Figure 4.11 than the ternary form was in Figure 4.5. This is due to two features: firstly, because the tempo indication is not an exact division of the second, the windows do not overlap perfectly with the repeats, which means that the pc distributions are not the same either. This is predicated on the assumptions laid out in the previous chapter: that repeats are not optional and therefore should be spelled out, and that they do not create a tabula rasa in the harmonic windows. Just because the music has returned to the opening does not mean the previous pcs do not linger on. Secondly, because the music is so intensely chromatic, and thus the squared magnitude values are low, even minor changes in the pc distributions can have a significant effect. Windows 1 and 9 present good examples of this: window 1 spans the C♯ in b. 2 to the D in b. 8; window 9 runs from the second A in b. 1 to the G in b. 8. Almost identical, then, and with this prompt the similarity in Figure 4.11 is easier to pluck out, with high values for $|f_1|$ and $|f_2|$, and low values for the others. Nonetheless, $|f_3|$ and $|f_4|$ have swapped ranks, and $|f_5|$ and $|f_6|$ have both decreased in value.

I mentioned above that the majority of windows include all twelve pcs, those that do not are as follows: 1, 5, 6, 7, 10, 14, 23, 26, 31, 32, 33. As can be seen by a simple inspection of Figure 4.11, most of these windows therefore provoke a strong value for at least one squared magnitude. Window 6 provides a good example of this at work. This window runs from the final quaver of b. 8 to the first quaver of b. 4, a passage that omits the pc D♯, and emphasises the pcs C♯ and F in the distribution.[16] The relative insignificance of D♯ is a feature of the movement as a whole: in the overall movement's pc distribution it has by far the lowest frequency as not only is it most commonly presented as acciaccaturas (b. 15 is the exception) but it is also often used as a single-note elision between row forms, thus reducing its count by one in comparison to every other pc. The prominence of C♯ and F is mainly achieved by the status of these pcs as the only ones in this passage with the duration of crotchets, but other musical phenomena also highlight their significance. This crotchet motif is emphasised by dynamic and articulation, and these pcs are often highlighted texturally elsewhere, as in b. 10. The third most prominent pc in the distribution of Window 6 is A. This pitch is a real fixture in analysis of this movement: as many authors have pointed out (see for example Ogdon 1962, 135), the canons of the row forms are arranged symmetrically around the central A4, which is then given further weight by its four-fold repetition in couplet pairs. In line with her other row-key analogies, Bailey goes so far as to describe this pitch as taking on 'a tonic function' (Bailey

16. The values of the pc distribution are as follows (0=C): (0.67, 1.16, 0.25, 0, 0.25, 1.24, 0.67, 0.50, 0.50, 0.99, 0.50, 0.50.

1991, 262–63), with a large-scale relation to the E♭s that are the focus of the third movement (and, as described above, comparatively irrelevant here). Taken together, these three pcs, articulate an augmented triad, a symmetrical division of the octave that therefore elevates triadicity ($|f_3|$) in the distribution, as shown unmistakably by Figure 4.11. That this should be considered a significant finding is supported by the strength of $|f_3|$ in the adjacent windows 5 and 7. As mentioned above, where present, the harmonic qualities of this music are often fleeting, but this suggests that, at least briefly, the music clearly establishes this harmonic colour. Indeed, Lewin latches onto these three pcs as significant across the piece, as they form most of what he labels the 'TUNE' (Lewin 1993, 348).

Window 6, is, however, the exception, and an honest analysis of this movement has to take account of the vast majority of windows, which display no such harmonic patterning. Window 4, which runs from the final quaver of b. 5 to the second quaver of b. 1, can act as a good example here. The values of the pc distribution span [0.50, 0.74], providing a paltry range of 0.24. By comparison, those of window 6 have a range of 0.99, not huge, but over three times the size. Looking at Figure 4.11, the results of this are obvious: there is no clear differentiation between squared magnitudes, and all of the values are minimal. Evidently there is quite significant overlap between windows 4 and 6, with 3.75 bars shared between them, and this therefore further illustrates the point I made at the start of this discussion: because the music is so chromatic there are only minor changes in the distributions. As Figure 4.11 shows, harmonic colours in the movement tend not to be established and sustained for significant periods, but fleetingly appear and then disappear. The strong $|f_3|$ value of window 6, supported by high values in windows 5 and 7 is unusual here: the general picture is one in which low squared magnitude windows like window 4 form a background against which windows with higher squared magnitudes occasionally pop out. It is this low level of window-to-window change that Figure 4.10 points to, in comparison to Op. 5/iv.

4.6 Conclusion

Amiot has described the DFT as something of a 'Holy Grail' (Amiot 2017, 165) for scholars seeking to apply a quantitative approach to music analysis. Whilst I wouldn't want to comment on whether studying the maths behind it left me feeling eternally youthful, it clearly has enormous potential. Applying the DFT to Op. 5/iv in this windowed manner has revealed a wealth of information about the movement and provided a textured and multi-faceted way to consider the music; meanwhile, the comparative analysis of Op. 27/ii reveals the difference in strength of harmonic colour between these two movements. More broadly, the DFT's applicability to everything from individual pitch-specific chords to abstracted pc sets to detailed pc distributions marks it out as a remarkably versatile tool for analysis.

From the perspective of detailed analysis, the success of the windowed macroharmony approach that this chapter applies is very exciting. Particularly when compared to previous methods of, on the one hand, pc-set analysis, and on the other, large-scale macroharmonic analysis, this new approach seems to solve issues that plague its forbears. In traditional pc-set analysis, segmentation is a continually thorny matter. As demonstrated in the discussion of historical analyses of Op. 5/iv, different analysts inevitably hear different segmentations of any given work. Whilst this obviously does not disqualify pc-set analysis *per se*, it does prevent any replicable analysis, and indeed poses serious problems for large-scale computational analysis. Large-scale macroharmonic analysis, by contrast, has no replication or computational issues, but it clearly flattens a lot of information, and loses a lot of detail. By contrast, windowed macroharmonic analysis finds something of a happy middle between the two. Of course, beyond temporal matters it does not take account of different surface features in the music (although in different repertories this might be comparatively easier to include), but nonetheless it successfully finds a way to synthesise the strengths of the two other analytical methods.

As for the case studies presented above, both of these movements need to be placed in their context in Webern's corpus. I considered them in detail in this chapter partially because there was plenty of scholarly discussion already, and while Op. 5/iv seems to be an exception in comparison to its peers, Op. 27/ii demonstrates well the typical situation towards the end of the corpus. In the case of Op. 5/iv, it seems plausible that these two phenomena are connected: perhaps there is extensive analysis already *because* it is unusual. This movement lends itself particularly well to harmonic analysis of this sort, thanks in particular to its carefully controlled use of pitch material, and concentrated intervallic characteristics. In Op. 27/ii, although there is a rigorous background construction (both serial and not) which attracts formalist analysis, the surface effect is quite different: the form is *not* clearly articulated by the harmony in the manner of Op. 5/iv. That these movements have withstood so much analytical prodding is a testament to the strength of the music and the levels of interest and intrigue they provoke for generations of analysts (quite apart from their sheer beauty). Nonetheless, this analysis has happened in something of a de-contextualised manner. On the whole, analysts have tended not to relate either movement to Webern's wider practice, presumably because without a larger body of analysis it is impossible to make defensible claims about broader stylistic phenomena (though this does not always stop scholars from making such comments!). Indeed, for whatever reason, the only analyst to have a large-scale study of Webern's freely atonal music, Forte (1998), does not consider Op. 5/iv. By contrast, from the perspective of the size of squared DFT magnitudes, it is unquestionable that the work is unusual.

By way of comparison, it is worth pointing out that idiosyncrasies certainly do not always attract analytical scrutiny. Time and again Op. 10/iii stands out as a frankly bizarre movement compared to the rest of the corpus, but the scholarly work on this movement is very limited. Indeed, a topic of future research must be a DFT-based analysis of Op. 10/iii, offered in comparison to the above analyses. Op. 10/iii is more obviously unusual with its long ostinati and repetitive textures, which perhaps explains its relative obscurity, but evidently—in a different way—so is Op. 5/iv. It is important to note that the uniqueness of Op. 5/iv comes from its distillation and magnification of features apparent in Webern's corpus, particularly early on, more broadly; in particular, the strength of $|f_2|$. Rather than being a peculiarity, like Op. 10/iii, it is more of an extreme case. The same might be said of Op. 27/ii. I have demonstrated that as Webern's music becomes more chromatic, with flatter pc distributions, the harmonic colours created by the distributions are less clear and, although there is more window-to-window variation, this is typically all within a more constrained gamut. Op. 27/ii is therefore an extreme example of this phenomenon, demonstrating the features of the latter part of the corpus. As for the corpus as a whole, this analysis particularly points to the importance of a quartal basis to harmony and minimises the focus on the octatonic that Forte has proposed, instead reminding us of the important of triadic, augmented harmonies. It also offers support for Shreffler's contention that the major shift in Webern's practice took place after Op. 11, rather than with the advent of dodecaphony.

Chapter 5

The Relations Between The Pitch Classes: Intervals

5.1 Introduction

The analysis of intervals is a well-established topic in the analysis of the music of Anton Webern. It is a preoccupation that has been applied in various ways to music across his corpus from Eimert's (1959) summary of interval proportions in Op. 28/i to Hanson's (1983) analysis of various 'freely atonal' works. Indeed, Webern is often fêted for his delicate control of the aggregate intervallic content of his music, and authors often refer to particular intervals (especially the semitone) as typical: Bailey, for example, suggests that 'a focus on the minor second is particularly Webernesque' (Bailey 1991, 18); Richard Chrisman that 'semitones are generally characteristic of Webern's style' (Chrisman 1979, 83). Such broad comments about a composer's general style are ripe for computational investigation, as there is no other realistic way to explore a composer's intervallic tendencies empirically (manually counting intervals on that scale would be such a Herculean task as to be essentially impossible).

The primary research question adopted in this chapter is a simple extension of the idea posited above: across the corpus of his works, did Webern prioritise certain intervals over others? This initial question provides the jumping-off point for a number of related concerns regarding basic distributions: did Webern's intervallic preferences change across the corpus? To what extent did he deploy intervals unequally in individual movements? Did this change across the corpus? Having explored these initial distributional concerns, an understanding of Webern's intervallic practice can be supplemented by considering several other phenomena. The first draws directly on this research and concerns the relationship between the intervallic content of the dodecaphonic rows and the resulting music: how strictly does the former predict the latter? (This is not conceptually dissimilar to Lewin 1968, although it expands the scope significantly.) Recent work applying quantitative methods to dodecaphonic music has tended to focus on the statistical properties of the rows themselves, particularly in the context of the total set of all possible rows (Bisciglia 2017; Von Hippel and Huron 2020). Without a meaningful corpus of related music, little more can be achieved (programming a computer to create a corpus of all possible rows is relatively easy). The contribution of my work, by contrast, is its ability to link the rows with the resulting music.

The next topic under discussion in this chapter is that of harmonic coherence or integration. The relationship between the two 'domains' of harmony, linear and vertical, is another topic that has received significant attention in the analysis of Webern's music, and that of the Second Viennese School more broadly. Boulez, for example, suggests that Webern achieved 'the abolition of the contradiction that formerly existed between the horizontal and vertical phenomena of tonal music' (Boulez 1968, 383). Again, however, this discussion suffers from a lack of empirical investigation, particularly on any multi-movement scale. Analytical investigation instead tends to be confined to particular movements or even singular phenomena (Chrisman 1979; Whittall 1987). This chapter will instead apply a comprehensive empirical approach to consider the degree of similarity between the interval content of the two harmonic dimensions.

Finally, this chapter will expand on the concept of 'frozen' intervals, an idea that was casually introduced by Lewin (1968) and more formally explored by Jenine

Brown (2020). It concerns the degree to which particular ics are deployed as only a subset of the possible intervals (Brown, for example, is interested in the presentation of ic1 in Op. 24/iii as 'almost exclusively' 11 or 13 semitones). (Bailey (1983) has also explored this kind of 'freezing' with regard to pitch material in Op. 21, where she observes that throughout the exposition Webern freezes pcs at particular pitch levels, creating a symmetrical disposition of pitches around A4.) The questions here are similar to those above: where does Webern freeze intervals? Is there any regular change over time? Are some intervals more regularly frozen than others?

5.2 Method

As I have explored in previous chapters, the primary characteristics for the segmentation approach offered in my work are empirical rigour and replicability, as well appropriateness to the repertoire at hand. In particular, it is crucial that the segmentation process take account of intervals and harmonies that are articulated in the linear dimension, as well as the more traditional vertical understanding of harmony. It is often the case that analysts will shift freely between these different planes, depending on the passage in question, and not infrequently will often deploy a 'diagonal' approach too. Roughly speaking, I define a vertical approach as one that groups notes sounding simultaneously across different instrumental parts, a linear approach as one that groups notes sounding consecutively in the same instrumental part, and a diagonal approach as one that groups notes sounding consecutively and possibly but not necessarily simultaneously in different instrumental parts. Figure 5.1 shows these three approaches in a plausible segmentation of the final two bars from Op. 9/ii, with different segmentation planes given in different colours. Of course, this is only one possible segmentation: obvious critiques might be the inclusion of the violin I C♮ in the final segment or the inclusion of the violin II C♯ in the penultimate segment.

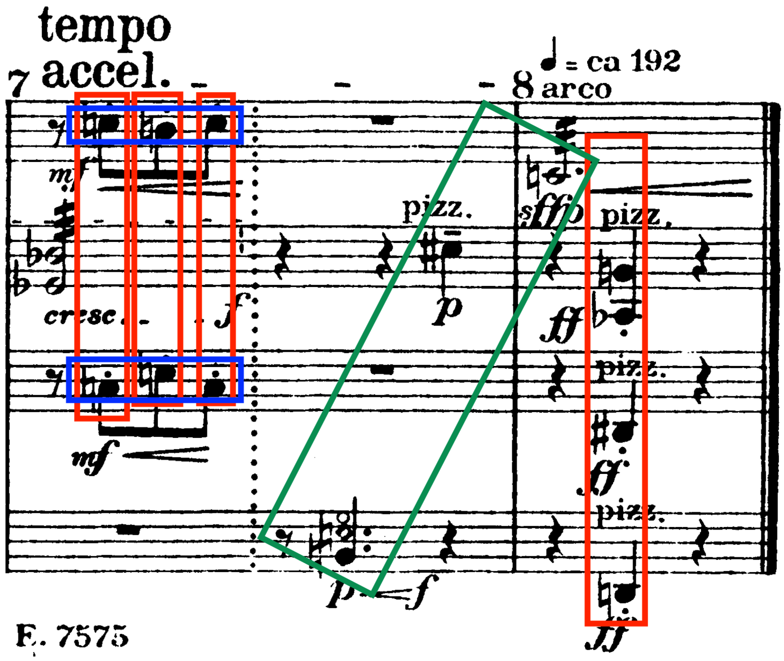

Figure 5.1: Op. 9/ii, bb. 7–8.

In devising a computational approach to segmentation, one cannot respond to the intricacies of the material. Instead, one has to develop a 'one size fits all' method that can be applied across the corpus under consideration. As a result, the segmentation strategies I deploy in this chapter use the vertical and the linear, but not the diagonal. Whilst the former two are relatively easy to apply, diagonal segmentations are much more contingent upon the specificities of the material, as they are justified only when the analyst can make the case that two notes are linked neither by instrumentation nor by simultaneous sounding, but instead by some other parameter(s) like registral proximity or motivic connection. A plausible case for the significance of a vertical or linear segmentation can almost always be made; the same is certainly not true for a diagonal one.

This chapter thus uses two related types of data. The first is the distribution of intervals in the vertical harmonic dimension, measured by duration; the second is the distribution of intervals in the linear harmonic dimension, measured by frequency. The measurement of vertical intervals follows the 'n-gram' principle of segmentation (Conklin 2002), which I touched on in Chapter 2. This has become something of a default strategy in computational music analysis, whereby the successive verticalities of a given passage are treated to a full expansion (termed 'chordify' in music21), which 'duplicates overlapping note events at every unique onset time' (Sears et al. 2017, 333). Figure 5.2 and Figure 5.3 show this process in action. Having chordified all the parts in each movement, the interval content and duration of each verticality is then recorded. Interval content is measured as the interval between each note in the verticality and the bass note of the verticality; duration is calculated in seconds, according to the tempo markings in the score. For example, the eighth verticality in Figure 5.3 (marked) contains two intervals, respectively 11 semitones and 25 semitones, and a duration of $\frac{1}{6}$ of a second. Measuring intervals from the bass note is certainly not the only possible method. Jonathan Harvey (1982), for example, has argued forcefully that post-tonal harmony should often be heard from the middle rather than the bass, an argument that Jonathan Dunsby and Whittall take up (1988, 123–130). Indeed, Harvey cites Webern as his inspiration in this, quoting from a 1940 letter from the composer to the poet Hildegard Jone: 'There is not a single centre of gravity in this piece [Op. 29]. The harmonic construction (resultant of the individual voices) is such that everything remains in a floating state.' (Webern, Humplik, and Jone 1967, 40). Whether this statement is capable of supporting quite the authoritative weight that Harvey hangs on it is questionable. In a letter from the previous year, Webern had made a similar point, suggesting, in relation to the song that would become Op. 29/ii that 'however freely it seems to float around—possibly music has never before known anything so *loose*—it is the product of a *regular procedure more strict*, possibly, than anything that has formed the basis of a musical conception before' (Webern, Humplik, and Jone 1967, 37). Whilst the similarity of language surely links the two, this hardly seems a comment about the specific construction of individual verticalities—as Harvey would have us believe. Rather, Webern seems to be deploying the word 'float' as a vague aesthetic description in contrast to the rigorous compositional procedures he employed. Perhaps even more significantly, a year later in 1941, Webern wrote

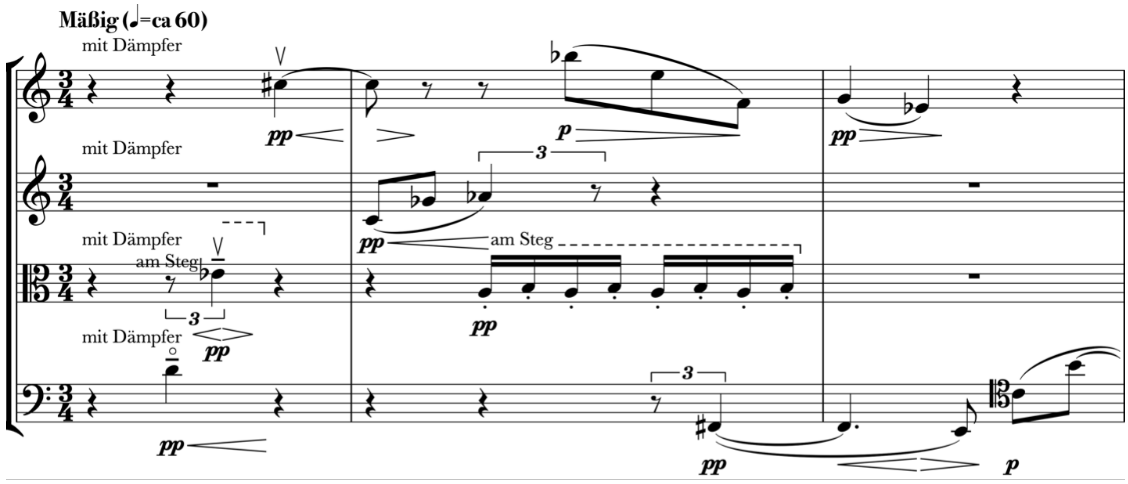

Figure 5.2: Op. 9/i, bb. 1–3. © Reproduced by kind permission of Universal Edition A.G., Wien.

in a letter to Willi Reich that his music, whilst using dodecaphony, ‘doesn’t on that account ignore the rules of order provided by the nature of sound *namely the relationship of the overtones to the fundamental*. However, it is impossible to ignore them, if there is still to be meaningful expression in sound! But nobody, really, is going to assert that we don’t want that!’ (Webern 1963, 61 (emphasis added)). Whilst this is hardly a comprehensive refutation of Harvey’s thesis, this does at least question the extent to which we should abandon bass-oriented interval analysis. In any case, Webern’s opinions need hardly proscribe (or indeed prescribe!) our analytical approaches, particularly as expressed throughout his letters, which Bailey and Barbara Schingnitz describe as ‘chaotic and repetitious’ (Puffett and Schingnitz 2020, see 24–26 for a fuller account of the volatile confusion that often characterises Webern’s letter writing). Evidently there is value in assessing Webern’s harmonic practice ‘from the middle’, yet I suggest that for a comprehensive approach, replicable across the entire corpus, it makes more sense to hear intervals from the bass. This is, after all, the traditional approach through which we tend to consider harmony.[1]

1. I use ‘traditional’ rather than Webern’s ‘natural’ with reason. Whilst that latter may be accurate, his suggestion that this adds value to the music commits the same logical fallacy that Julian Anderson (2000, 10) identifies in a raft of pre-spectral composers: given the inherently constructed nature of music, some sort of derivation from natural procedures does not produce or guarantee any degree of musical quality.

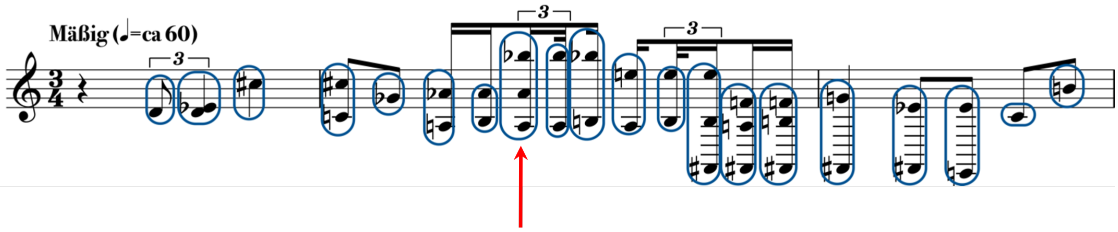

Figure 5.3: Op. 9/i, bb. 1–3, chordified.

Another possible approach would be to use Forte's interval vector (Forte 1973, 13–18), which charts every possible interval in a given verticality. Whilst this is certainly a powerful tool, for this analysis, focussed as it is on the musical surface, it is simply too reductive. Whilst on a deep structural level ic analysis is helpful, and indeed will be deployed in this chapter, to start from this position eschews an enormous amount of detail about the nature of the intervallic presentation that it is simply insufficient for dealing with the concerns of this research.

Turning to linear analysis, this is a similarly thorny issue. As soon as the composer introduces a second voice, and the music moves beyond a monophonic texture, the analyst has to decide which notes are connected and so which intervals are to be counted. In polyphonic music with clearly defined contrapuntal lines this is relatively easy: in Op. 16, for example, the parts are segmented enough to be easily identifiable. In other contexts, however, this is much less clear. Consider, for example, the two horns at the start of Op. 21/i (Figure 5.4). Should the G2-A3 in b. 3 be understood as a relevant linear interval? And what about the A? Should it be linked to the C3 or the A♭3 in b. 4? In this case, the disposition of parts, 'aeration' with a crotchet rest, and slur seems to make it clear that G-A♭ and A-C should be linked, but not every extract has such a clear resolution. Only to add to the confusion, how about the row structure in a work of linear topography such as this? Again, this example has a fortuitous resolution: the row structure tracks the contrapuntal structure suggested above, but raises further questions. The first statement of the row follows the second horn part to the end of b. 4 and then moves to the E5 in b. 6. Should this interval be counted? The approach adopted in this chapter is largely to ignore the row structure. The emphasis of this chapter is on the surface of the music, those intervals that can be (fairly) easily identified and plausibly heard (and which may therefore have some aggregate—if individually unperceived—effect). The rows, by contrast, form a background structure. Without getting into an extensive, and probably unresolvable discussion of Webern's ideological understanding of the row, he was clear that the row is 'not a "theme"' but achieved a sort of background unity, 'even if one's unaware of it' (Webern 1963, 55). This focus on the reasonably perceptible is also what underpins my analytical strategy for linear intervals. To expand the scope of the analysis comprehensively, I have produced a melodic expansion of each work that creates a series of monophonic lines representing the contrapuntal parts in the piece (e.g. Figures 5.5 and 5.6). The exact procedure is described below, but in essence it restricts contrapuntal lines to a single instrument, prioritises minimising interval size when a choice is to be made, and does not record an interval if the rest between notes is too long. The only other point to note is that duration of pitches has no bearing on the data collection, the analysis merely counts frequencies of intervals. This is because there is no clear or reliable relationship between the significance of an interval and the durations of the notes that define it; an interval that lies between, say, a minim and a crotchet, is not necessarily twice as important as once that lies between a minim and a semibreve. Of course, other parameters like duration, timbre, texture could lend greater significance to the interval, but equally could not, and these are difficult to predict in a replicable, computational manner (for some early speculative work on salience conditions in post-tonal music, see Lerdahl 1989, 73–74).

In order to carry out any large-scale linear analysis, whether of harmonic units or intervallic content, a process of segmentation needs to be established. Given the scale of the corpus study, this must be replicable in a vast array of contexts, and therefore simplicity is likely to be much more successful than a complex multi-part algorithm that tries, and likely fails, to take account of a large range of individual circumstances. Various segmentation approaches can be deployed, depending on the aspect of harmony under consideration: for example, intervals

Figure 5.4: Op. 21/i, bb. 1–7. © Reproduced by kind permission of Universal Edition A.G., Wien.

between pairs of notes (so a segmentation of every consecutive pair of notes), or the harmonic content of three consecutive notes (a possible motif). In each case, however, the particular notes under consideration have to be strictly defined: after all, in a contrapuntal texture the intervals between notes in different parts are unlikely to be relevant. The clarity of these contrapuntal voices across the 31 works in the Webern corpus varies hugely. In some of the later chamber music, for example, contrapuntal voices are made very clear through a combination of instrumentation, texture, and register; conversely, in piano writing particularly,

Figure 5.5: Op. 5/i, bb. 1–3. © Reproduced by kind permission of Universal Edition A.G., Wien.

Figure 5.6: Op. 5/i, bb. 1–3, melodic expansion.

consistent individual voices can be much harder to establish. The following outlines the general approach I have taken in producing such a contrapuntal expansion that is suitable for computational analysis. The goal is to produce a version of each movement that has some number of individual monophonic parts that, together, give the fully-realised work, and so take account of all individual voices without unnecessary doublings.

The fundamental contention of my approach is that linear segmentation is most obviously defined by instrumentation; that is, individual voices are restricted to individual instrumental parts, and do not cross. This may seem naïve, particularly compared to some of the larger scores of Webern's contemporaries (Schoenberg's Op. 16 immediately comes to mind, as do all of Mahler's symphonies); nonetheless, with no obvious instructions to the contrary (he eschews *Hauptstimmen* in his notation) it is difficult to assert otherwise consistently, and in an atonal context instrumentation provides such a clear structural guide. Unison doublings in pairs of instruments (e.g. two flute parts playing the same music) have been removed, but octave doublings are retained, as are unison doublings in different instruments, although these are fairly rare. When pairs of instruments play different lines, these have been treated as consistently independent voices (e.g. the aforementioned horns at the opening of Op. 21/i). There is a plausible argument that in some cases the aggregate of two parts, particularly given wide registral leaps, should be understood to produce one composite voice, or indeed two independent voices distinct from the notated parts, however this is such a subjective assertion as to be unfeasible to model comprehensively, depending as it does on the variable performance of individual players and perception of different listeners. I therefore assume that Webern meant to write individual parts as producing meaningful independent contrapuntal lines, not an unfair assertion given his obsession with counterpoint.

For keyboard instruments I have extended this approach, treating each stave as a separate line, unless the result is very obviously monophonic and split staves are

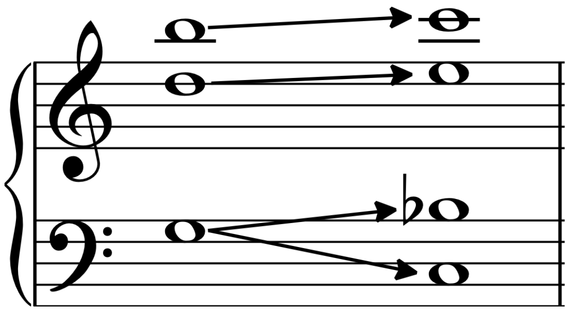

Figure 5.7: Voice-leading relations between a triad and tetrad.

simply to facilitate playing. In fact, particularly in the later work, Webern often uses cross-stave beams to indicate contrapuntal groupings, validating this approach. In chordal passages, whether on a keyboard instrument or elsewhere (e.g. string double-stops, guitar chords), if the chords have a consistent number of voices I have presumed that voice leading follows register (so, the top voice remains top, second-down remains second-down, etc.). If the number of voices changes, I have constructed the extra note as coming from or leading to another note in the other harmony, which has been calculated according to the axiom of minimal movement (so, if a triad becomes a tetrad, as in Figure 5.7, that new note is deemed to be connected to a note in the previous chord).[2]

The other point of contention regards rests: how long does a rest need to be until the notes either side should be deemed disconnected? This could be debated extensively, for example with regard to registration, what else happens in that rest, and other factors. Nonetheless, in the context of these works it seems unlikely that listeners are expected to prolong a note mentally for any extensive period of time. Thus, if the rest is longer than two seconds (the minimum of Michon's range), I view the notes on either side as disconnected.

2. I use the term triad merely to refer to a harmony consisting of three pcs, not to convey any tonal or other implications.

5.3 Results

5.3.1 Overall Tendencies

The following summary will outline some of the key results from the data analysis in this chapter. The full data is given in Appendix D. For legibility's sake I have provided the results in terms of simple intervals rather than interval classes or compound intervals. Comments about their implications for an understanding of Webern's music, and answers to the research questions, will follow in the Discussion.

Figure 5.8 presents the overall distribution of intervals in Webern's practice. Intervals greater than an octave have been reduced to simple intervals. In the vertical domain these intervals are registral; in the linear, temporal. The results of each movement have been converted to individual percentages to control for movement duration, and then a mean average has been taken. Amalgamating Webern's total corpus in this way, it must be acknowledged, an intensely reductive act, and so forms only the initiating point for this chapter. Nonetheless, this is exactly what authors imply when they remark that certain intervals are particularly 'Webernesque'. For each interval, the vertical proportion is shown on the left and the linear on the right. Correlations have also been calculated with chronological corpus position. These are listed in each bar, and are also represented in the colours of the bar (orange indicates positive correlation; blue, negative correlation; increasing intensity a stronger correlation). Calculating anomalous values indicates which intervals, if any, have a proportion which is unusually high or low. The only such anomaly is the linear semitone (20.9%). For context, if all intervals were deployed equally, each would represent 8.3%. The same result is found if the linear and vertical results are combined and averaged: the semitone is the only anomaly, with 15.9%. Meanwhile, perhaps the most important single metric for assessing individual movements is the range of values (the difference between the proportions of the most common and least common intervals in a movement), which indicates the spread, and thus how evenly

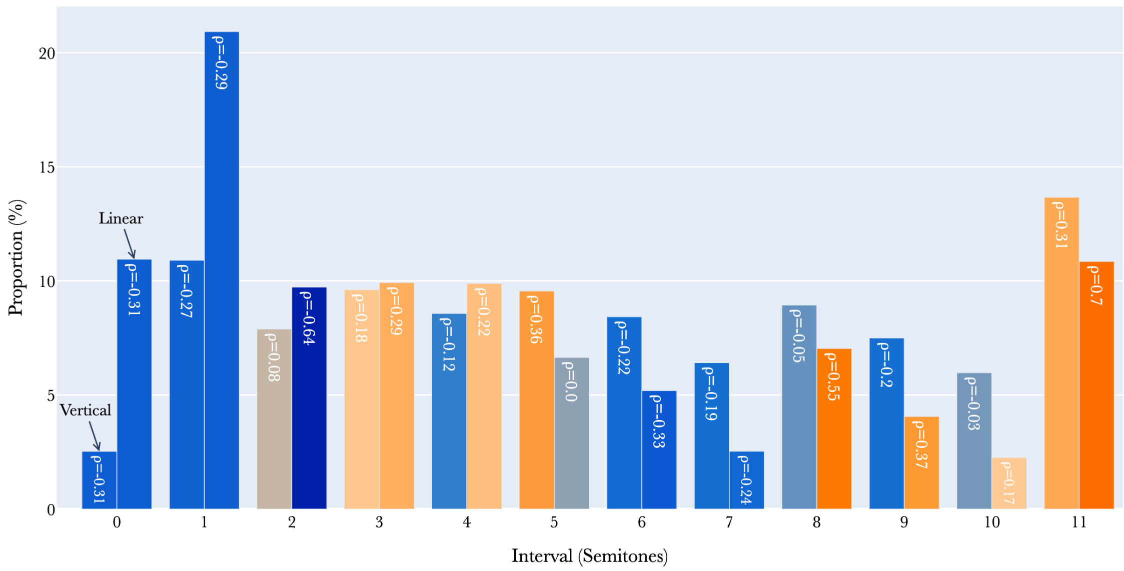

Figure 5.8: Overall interval proportions and correlation with corpus position.

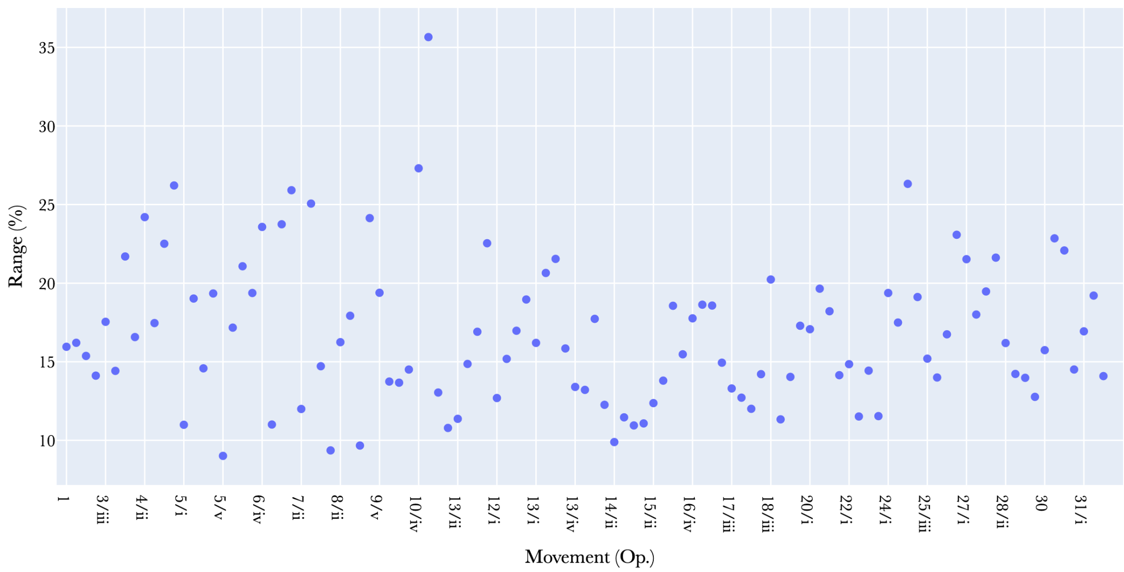

Figure 5.9: Ranges of interval distributions.

different intervals are deployed. Figure 5.9 presents the range of interval distributions in each movement, organised chronologically. The correlation of range with corpus position is -0.03.

5.3.2 Row Relationships

Having outlined the wider situation, I now turn to the second half of the corpus, the dodecaphonic works (Op. 17 onwards). The question here is to what extent the intervallic content of the row predicts the intervallic content of the movement. This is considered in terms of ics, as intra-row intervals can be deployed as any related ic. The prime forms of the rows have been adopted from Bailey's summary (Bailey 1991, 13–29), and include the interval 'round the back' (i.e. from the final note to the first note). Ic0 was removed from the analysis as it obviously does not occur in any of the rows. These counts are given in Appendix E. To investigate this, I ran an OLS Linear Regression Analysis for each ic, with the ic's frequency in a row as the independent variable, and the ic's proportion of a movement's total interval content as the dependent variable (note the slight difference here between the row's ic frequency (i.e. an integer in the range (0,12)) and the movement's ic proportion (i.e. a percentage)). Linear and vertical relationships were analysed separately. The results of the analysis are presented in Table 5.1.[3]

Ic	Linear R^2	Linear Coefficient	Vertical R^2	Vertical Coefficient
1	0.90	6.85	0.80	5.20
2	0.72	8.86	0.27	8.34
3	0.88	6.13	0.77	5.30
4	0.92	7.39	0.74	4.89
5	0.57	5.74	0.59	10.73
6	0.47	2.83	0.48	5.17

Table 5.1: OLS Regression Analysis for row and movement ic content, separated by ic value.

This analysis has some value in indicating how Webern deployed different intervals in his dodecaphonic work, but the primary question here is more abstract and merely concerns whether the prominence of an ic in a row predicts its prominence in a movement. As such, the precise ic value is irrelevant. What is more, statistical power is gained by amalgamating the results from six variables to

3. A brief guide to reading this table. First I assess the significance criteria (α). In every case, $p < 0.001$. The value here is the probability that the result is greater than chance. As it always is, these can be deemed statistically significant results. After this, the R^2 column indicates the percentage of the effect in the dependent variable that can be ascribed to the independent variable. Finally, the coefficient columns show the percentage change in the dependent variable for an increase of 1 in the independent variable. So, considering the results for linear ic1: $p < \alpha$ and so the result is more likely due to a relationship than chance; R^2 is 0.90, so most of the change can be ascribed to the independent variable; and the coefficient is 6.85, so for every additional ic1 in a movement's row, its proportion of the ics in the movement increases by almost 7%.

one. I therefore ran a further Regression Analysis, this time with the independent variable as an ic's frequency in a row and the dependent variable as an ic's proportion of a movement's total interval content. The results of this analysis are given in Table 5.2. Again $p < \alpha$ in both cases.

Dimension	R^2	Coefficient
Linear	0.80	6.74
Vertical	0.65	5.40

Table 5.2: OLS Regression Analysis for general row and movement ic content.

5.3.3 Harmonic Integration

Returning to the entire corpus, the third topic of discussion is harmonic integration. Given the empirical nature of this thesis, this is interpreted here quite simply as the degree of similarity between the vertical and linear domains of harmony across the corpus, therefore putting aside (for now) any broader philosophical concerns. Figure 5.10 shows the correlation between linear and vertical interval proportions in each movement, ordered chronologically. Moving one level higher, the correlation between corpus position and each movement's linear/vertical correlation is 0.23. The graph suggests that there is another shift at Op. 12, which can be investigated by comparing the correlations for the two portions of the corpus on either side of that line. For Opp. 1–11 the correlation is -0.20, not dissimilar in size from the overall correlation; for the latter portion, however, it is -0.11, which suggests that that section of the corpus is much more stable.

5.3.4 Frozen Intervals

The final topic of enquiry in this chapter is that of frozen intervals. As discussed above, frozen intervals refer to the deployment of an ic in only a limited subset of all its possible intervals. The extent of this limitation requires careful definition. Indeed, in a dogmatic sense considering all possibilities, most music displays frozen intervals: intuition would suggest that linear intervals in particular tend to

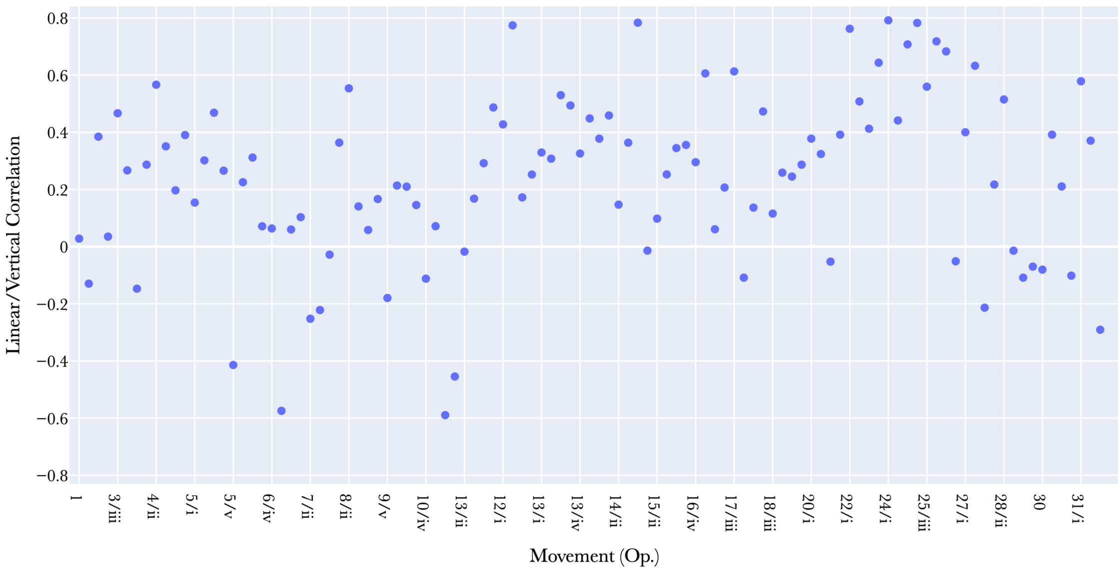

Figure 5.10: Linear/vertical interval proportion correlations.

lie within one or two octaves, when, even allowing for the ranges of most instruments, they could exceed this and be distributed evenly across 3–4 octaves (for a discussion of the impact of musical memory on this phenomenon, see Snyder 2016, 172–73). Rather, extreme decay between successive octaves might be expected, perhaps approximating a Zipf function.[4] Nonetheless, to my knowledge, there has been no work exploring frozen intervals in any other context (tonal or post-tonal). As such, broader context is lacking with which to theorise what might be deemed typical or unusual behaviour. This thesis is restricted to considering Webern's music: developing and introducing an alternative reference corpus for comparison could certainly yield interesting results, but is far beyond the scope of this research. As such, consideration of frozen intervals here will remain relativistic, assessing which intervals are *more* frozen than is typical in Webern's own practice. Whether his practice is itself unusual remains unanswered.

To measure the degree of freezing of an ic in a movement is akin to measuring

4. A Zipf function is a power law probability distribution which states there is an inverse relationship between rank and frequency distribution (for his introduction of the phenomenon with regard to linguistics, see Zipf 1949, 19–55). It has also been identified in musical contexts (Rohrmeier and Cross 2008, 5–6).

concentration or competition. As a result, to quantify the freezing of an ic, I borrow a measure of market concentration from economics, the Herfindahl-Hirschman Index (HHI).[5] This gives a value in the range $(\frac{1}{n}, 1)$ where n is the number of participants: $\frac{1}{n}$ represents perfect competition (i.e. each participant has an equal market-share); 1 represents a monopoly. Interpreting this in the context of 'competition' between intervals of an ic is quite simple: low values indicate a relatively even dispersion across the possible intervals; high values suggest that only a few of the possible number of intervals account for most of the usage of an ic. Considering the presentation of each ic in each movement is obviously beyond the scope of this chapter, although a few specific examples will be considered in the Discussion. At this stage, it is beneficial instead to present some summarising data. In this vein, Figure 5.11 shows the mean average value for each ic across the corpus, and Figure 5.12 shows the mean average value for all ics in each movement. With regard to change over time, the correlation between a movement's average value and corpus position is 0.24 for vertical ics and -0.58 for linear ics. In almost all cases, the most popular presentation is from the first two octaves (i.e. for ic 1 in a movement this would be any of 1, 11, 13, or 23 semitones). There are 749 ics to consider (7 for each of 107 movements) in both domains. In the vertical, there are 56 ics with a most popular statement outside the first two octaves; in the linear, merely 6. Within those first two octaves, the frequency counts of most popular ic presentations are, respectively, 188 (first octave smaller presentation, e.g. for ic1 this would be a minor second), 272 (first octave larger presentation, e.g. a major seventh), 145 (second octave smaller presentation, e.g. a minor ninth), and 71 (second octave larger presentation, e.g. a major fourteenth) for the vertical domain, and 601, 125, 26, 7 for the linear. In both cases the vast majority of the most popular presentations are within the first octave, although they differ between domains as to whether it is the smaller or larger of these two interval presentations that is most popular.

5. This is defined as the sum of the squares of each participant's market share, or in this case, the sum of the squares of each interval's percentage of its ic family.

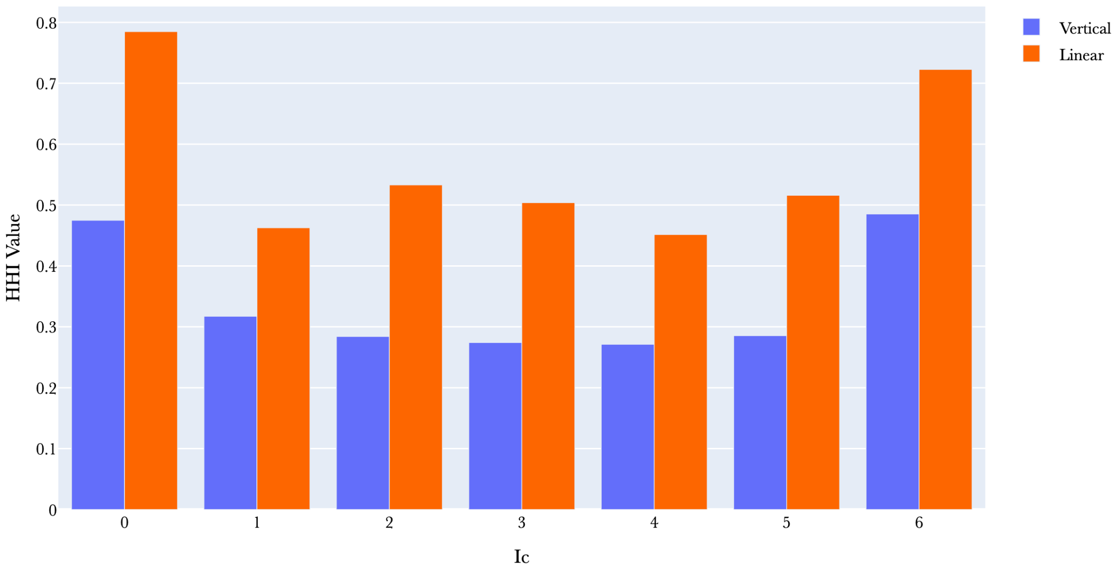

Figure 5.11: Mean average HHI values for each ic.

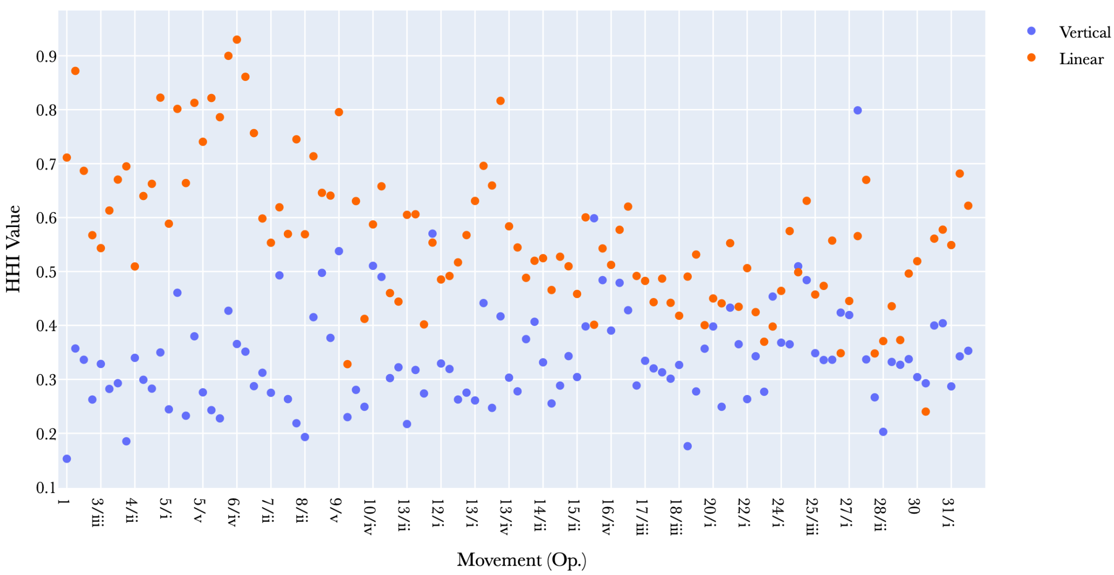

Figure 5.12: Mean average HHI values for each movement.

5.4 Discussion

The results presented above suggest a wide variety of insights into Webern's practice, across a range of different areas. Opening with the most general observations, I start by considering to what extent the minor second does indeed characterise his practice, as was suggested above. The results presented in Figure 5.8 certainly go some way to confirming this: as was mentioned, the linear semitone is the only anomalous value, indicating unusual prominence across the corpus. It is notable, however, that both the linear and vertical major seventh are more frequent than the vertical semitone, as indeed is a linear unison/octave. Bailey's interest is primarily on the linear, and so it is perhaps unsurprising that her general observation refers most accurately to this dimension. Whilst direct precedents for this line of research are limited, there are some forbears in the literature. Eimert (1959) carried out a statistical survey (manually, presumably) of Op. 28/i, in which he found that Webern displayed a preference, again in the linear domain, for major sevenths and minor ninths. I will return to Eimert below when I discuss the implications of the dodecaphonic technique; for now, it is sufficient simply to observe that his finding generally holds up across the corpus as a whole.

Another forerunner of this chapter is an early corpus study by Roland Jackson (1970) comparing some works from the late Romantic and early atonal period to some later and dodecaphonic atonal music. As is often the case with older corpus studies, Jackson's work suffers from a tiny sample size, nine movements in total, some of which are merely represented by fragments. Nonetheless, it stands as an interesting premonition of what would later be feasible. To consider vertical content, for example, Jackson deploys the same 'full expansion' technique that is now standard practice (though he does not call it this). Regarding Webern's music, the earlier period is represented by the first 80 bars from Op. 5 (this peculiar selection comprises the first two movements in full, and approximately two thirds of the third movement); the later music by Op. 28 (seemingly in its entirety). As for his findings, these can be difficult to compare with the present research as his representation in the essay tends to include only partial data and 'edited highlights' of his results. The one result that is somewhat comparable with this research is his suggestion that in the early period 'the minor second was slighted (and was almost totally absent in Webern's piece)' (Jackson 1970, 138). Unlike in my work, to measure intervals Jackson uses interval vectors, and so by 'the minor second' he means ic1. Nonetheless, this is a surprising comment given the typical view of Webern's practice, as expressed above, as saturated with ic1, a finding confirmed by this research. Direct comparison is unhelpful given the methodological differences between Jackson's research and my work. Nonetheless, as interesting context, turning to the vertical results in this research for Op. 5/i–iii we find that the semitone comprises, respectively, 13%, 11%, and 11% of interval durations, and the major seventh 12%, 7% and 11%. In most cases this is above what might be expected (recall that an even distribution would assign an interval 8%).

Whilst it is not possible to extrapolate much further from Jackson's research, his interest in observing trends does point to the next topic in this chapter. The correlation figures in Figure 5.8 show a wide variety of changes. Though most correlations are below ±0.5, and indeed a majority lower than ±0.3, there are a few intervals with strong correlations, displaying significant change over time: linear tone (-0.64); linear minor sixth (0.55); and linear major seventh (0.70). The first of these intervals might intuitively be associated more readily with tonal harmony, and so its decline is unsurprising. The rise of the linear major seventh can perhaps be understood in a similar vein, though in reverse, as a relatively dissonant interval, and therefore one that is more attractive in an atonal context. As for the linear minor sixth, an explanation for this increase is not obvious, beyond changing personal taste. With regard to the other scholars, Boulez provides a comparison between Webern's different periods. He suggests that whilst Opp. 5 and 6 are 'composed of conjunct intervals or, if they are disjunct, intervals disposed in a register sufficiently narrowed so that the ear is able to perceive the continuity at once' (Boulez 1968, 379), in Op. 21 'the disjunct intervals are positioned so as to avoid even fortuitous establishment of any tonal rapport' (Boulez 1968, 384). In sum, Boulez seems to be suggesting that Webern's intervallic preferences grew wider over time. This, of course, is something that can be assessed quantitatively using the data in this research. To do so, I interpret Boulez as meaning that Webern increasingly tends to prefer presenting an ic as a wider interval, rather than wider intervals on their own terms (i.e. Boulez is suggesting that later in the corpus Webern is more likely to present ic1 as a minor

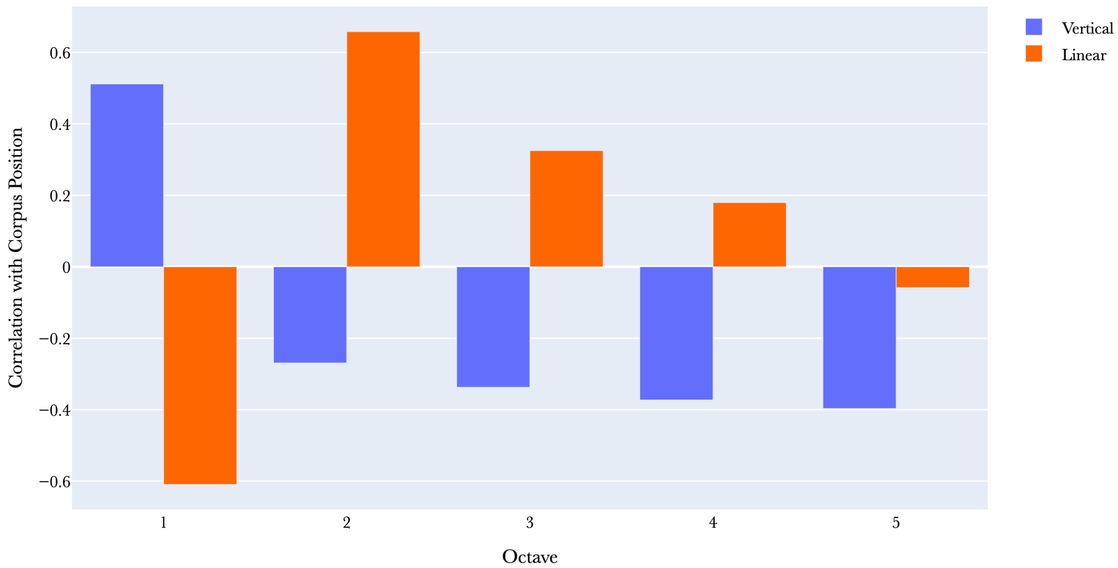

Figure 5.13: Correlations between proportion of intervals across each number of octaves and corpus position.

ninth than a semitone, rather than prefer, say, a minor sixth to a major third). As such, I can assess what proportion of intervals extend over each number of octaves in each movement, and then calculate the correlation of that proportion with corpus position. The results are given in Figure 5.13. Strikingly, the linear and the vertical are almost mirror-images of each other. Whilst vertical ics are increasingly concentrated as smaller intervals across the corpus, for linear ics the reverse is true. I suspect from Boulez's phrasing (for example, the reference to 'the continuity') that he is talking primarily about linear intervals, and so is correct. Nonetheless, that Webern clearly tended towards increasingly closely voiced verticalities over his corpus is also an interesting finding.

Another helpful metric in considering overall tendencies are the ranges of interval distributions. Figure 5.9 makes it very clear that in this regard there is no overall pattern across the corpus. The bulk of the movements have a range between 10% and 25%, and the tiny correlation value makes it clear that there is no diachronic change in this regard. Op. 10/iii stands out as particularly unusual with a range of 36%, perhaps unsurprising given the idiosyncratic repetitive textures of the movement, which I explored in Chapter 3. Indeed, 72% of linear intervals in this movement are unison/octave. Otherwise, however, it does not appear that Webern characterised individual movements by particular intervals particularly strongly at either chronological end of the corpus.

The final overall perspective on the trends in Webern's intervallic language is provided by dispersion. Linguists have recently started to explore dispersion as a supplemental metric alongside frequency measures (Egbert and Biber 2019; Gries 2020): this is a measure of the distribution of a proposed item (typically a word) across a corpus, which therefore provides an indication of temporal concentration. As an example, translated to the context of Webern's corpus, it allows the analyst to distinguish between two intervals that have similar frequencies, but in which one dominates a few movements but is otherwise absent, and the other occurs infrequently across comparatively many movements. The precise measure of dispersion adopted in this chapter follows Gries (2021) and is the Kullback-Leibler divergence (D_{KL}), with values normalised to fit in the range [0,1] where a higher value indicates greater concentration in a smaller number of movements of the corpus; a value closer to 0 proposes that the interval is more evenly distributed in proportion with the lengths of different movements (0 itself indicates that the verticality is not present in this movement).[6] For a discussion of the strengths of this approach compared to others, see Gries (2021). Jesse Egbert and Douglas Biber (2019, 84) claim that in their analysis there is such a strong correlation between frequency and dispersion that frequency can be dispensed with altogether. Gries (2021, 8–9) caveats this by suggesting that this is only really true for very frequent and very rare words, which inevitably must be distributed, respectively, very evenly and very unevenly. Meanwhile, the words that are likely to be most interesting will fall in neither of these camps. Even this is something of a theoretical fallacy, however, notwithstanding empirical evidence that suggests it is an accurate description in the linguistic corpora these scholars consider. Blatantly it is not necessarily true for very frequent words, which may remain extremely concentrated, and except in truly extreme cases where the

6. Normalisation by the following equation: $D'_{KL} = 1 - e^{-D_{KL}}$.

frequency value is not high enough to feature in every text in the corpus, this need not be true for the rare words either. Either way, there is no guarantee that this empirical finding would translate to a musical context.

Figure 5.14 is a chart that plots the inter-movement dispersion values for each interval against their frequency proportion. I identify four quadrants in this chart, representing intervals that are common-concentrated, uncommon-concentrated, common-spread out, and uncommon-spread out. The question to ask is where the dividing lines lie. On the y-axis this is easy to determine: 0.5. The x-axis, however, is slightly more complicated. I propose that the dividing line should be positioned at 8.3 (or more precisely, 100 ÷ 12), the proportion each interval would occupy in a hypothetical equal interval distribution. Of course, the rigidity these dividing lines suggest is deceptive, but as a basic method of grouping intervals it is a helpful heuristic. Given that there are intervals in all four quadrants, it seems reasonable to dispense with Egbert and Biber's contention that dispersion encompasses frequency information: there is two-dimensional variety here. A few intervals immediately stand out as being unusual. Starting with those intervals that are comparatively rare across the corpus, these are 0 (vertical), 7 (linear), and 10 (linear). Although they have very similar frequency proportions, their dispersion values differ by almost 0.3. The vertical octave is primarily concentrated in a few movements, and comparing Figure 5.14 with Figure 5.8 indicates that these are likely to be early in the corpus, given the negative correlation in the earlier chart; linear 7, by contrast, is more spread out. Further along, linear 0 and vertical 1 have dispersion values that differ by over 0.4: while linear 0 is again concentrated (and, again, Figure 5.8 indicates that this is early in the corpus), vertical 1 is much more evenly spread out. That both the vertical and linear octave/unison are concentrated earlier in the corpus is relatively unsurprising. In the case of the vertical octave, there are only three movements with values greater than 8.3: Op. 1, Op. 4/i, and Op. 6/ii. All of these are clustered towards the start of the corpus, and two are orchestral works, where vertical octave doublings are often commonplace, although, of course, it is

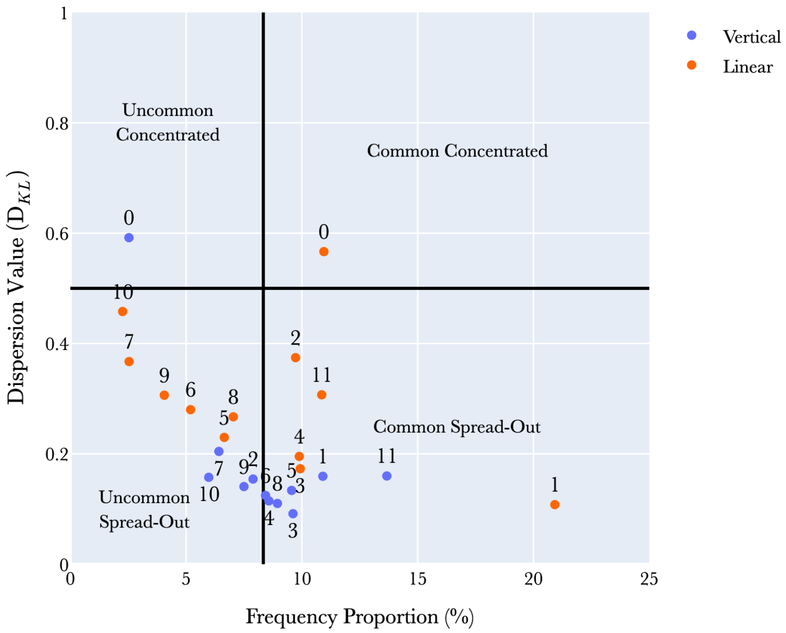

Figure 5.14: Inter-movement dispersion and frequency proportion for each interval.

notable that these vanish from the later orchestral works. Op. 30, for example, has a value of 2.6% for octaves. Linear octave/unisons are much more common, as Figure 5.14 demonstrates, but again they are clustered in certain movements dominated by static harmonies, such as Op. 10/iii or Op. 9/iv, rather than being evenly dispersed. Returning to ic1, it is notable that these 'Webernesque' intervals all have dispersion values in the lowest third of the possible distribution, indicating that they are all evenly distributed across the corpus. It is surely the combination of this even distribution with a relatively high frequency that has created the association of this ic with this composer: even if other ics were as frequent (which Figure 5.8 shows they are not) it is the recurrent use of this ic across the corpus as a whole that has surely given it its significance in our understanding of the corpus.

Up to this point, I have used dispersion to assess the distribution of intervals across the corpus as a whole, but if a movement is itself segmented temporally, dispersion can be applied on an intra-movement basis too. This allows the analyst to identify, in the same way, whether a given interval or indeed harmony is largely concentrated in one region, or across the movement as a whole. In order to facilitate inter-movement comparison, each movement is split into the same number of segments, and within each movement, the segments are of equal length in seconds. As with the windowing method for pc circulation, described in Chapter 3, this means that windows often overlap slightly, so as to ensure everything is included. The goal is to assess how equally particular intervals are dispersed across the temporal span of the movement, and so this does not take into account structural phenomena particular to each movement. This could certainly be an interesting point for future research, perhaps most obviously in the works articulating a traditional form (sonata, rondo, etc.). This strategy also assumes that dispersion can be compared between movements of different lengths, which range from, using the Boulez recordings, the 26 seconds of Op. 9/iii to the 10 minutes of Op. 1. In reality, Op. 1 is something of an outlier: the vast majority of movements fall between 30 seconds and 4 minutes in length and thus comparison is relatively uncontroversial, and in fact there are only four other movements longer than five minutes (Op. 20/ii, Op. 21/i, Op. 26, and Op. 30).

Clearly, the number of segments into which each movement is divided is an important determining feature; akin to a tuning parameter in machine learning, it cannot be learned from the data and must be set by the analyst. Intuition suggests that extreme values (e.g. two, or hundreds) would be unhelpful: the former would only identify works that are saturated with intervals, the latter would approximate a frequency analysis as segments would contain very few different intervals. That said, a greater number of segments provides a higher level of granularity. Therefore, there is a trade-off between the number of segments and the number of intervals in each segment. To find a midpoint that optimises for these values, I use the common 'elbow method' heuristic: plot a line graph of these two values and identify the kink in the curve (the 'elbow') where the two values are optimised. Figure 5.15 is such a graph, plotting different segment sizes against the mean number of pc sets in each segment (calculated according to the method outlined in Chapter 6). In order to facilitate linear/vertical comparison later in this chapter, the same number of segments

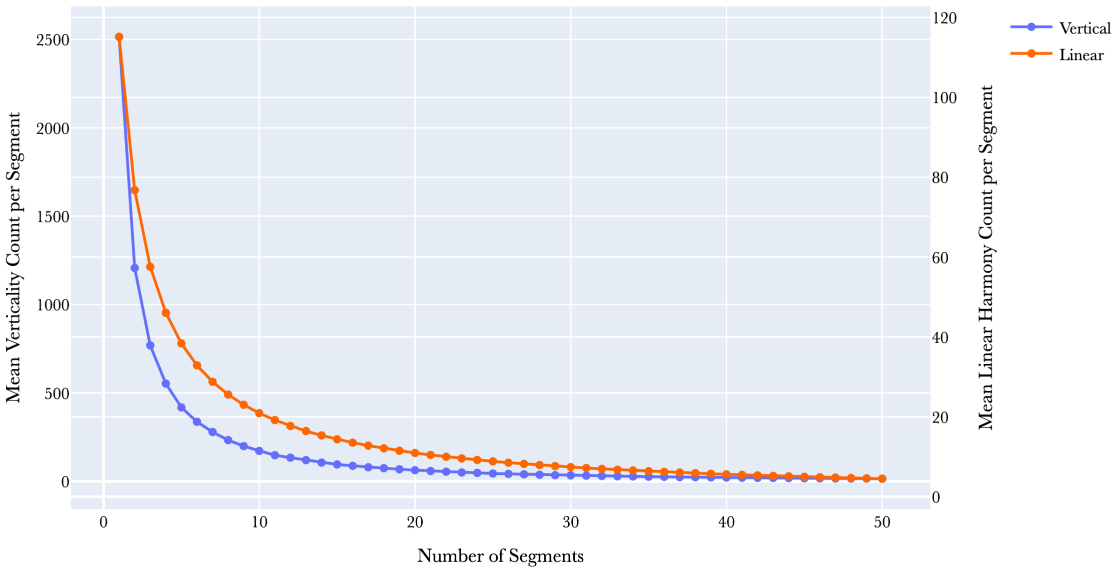

Figure 5.15: Average number of harmonies per segment.

will be used for both analyses, and so both lines need to be taken into account. There is no unambiguous bend in either curve; however, choosing 6 segments seems to balance the options best.

For each movement, 11 dispersion values are calculated (one for each interval). As this gives a rather unwieldy amount of information to deal with, I will use some summary statistics to better get a handle on the information. Two statistics are useful here: the range and median. As in previous uses, the range indicates how different the dispersion of intervals across a movement is. A high range value suggests that while one or more intervals are very concentrated in particular segments of the movement, at least one other is not. Meanwhile, median values indicate what the general use of the intervals is in a movement: whether they tend to be concentrated or evenly dispersed. Figure 5.16 plots these median values. The graph suggests a decline in median values across the corpus, a phenomenon supported by the correlation values with corpus position (linear: -0.59; vertical: -0.33). As in previous chapters, there seems to be a clear inflection point around Op. 12, further supporting the view that Webern shifted his practice at this stage. This is a noteworthy observation particularly because intra-movement interval dispersion has practically nothing to do with the pitches

Figure 5.16: Median intra-movement dispersion values.

themselves. In Chapter 3, I described a shift towards flatter pc distributions as a precursor to the dodecaphonic technique. This finding suggests an even wider reinvention of his style.

By contrast, Figure 5.17, which plots the range values, has no discernible chronological pattern, and the correlations are very weak (linear: -0.06; vertical: 0.22). Taken together, these values suggest that, overall, the general concentration of particular intervals in areas of a movement declines across the corpus (much more significantly in the linear domain than the vertical), but that the amount of variation between intervals in a movement stays similar across the corpus. Thus, although a typical movement at the start of the corpus is likely to have as much difference between its most and least concentrated intervals, a later movement is likely to have a greater preponderance of the intervals being spread out more evenly across the movement.

To illustrate this phenomenon in effect, a pair of case studies is useful. Beginning at the end, from the very end of the corpus, Op. 31/vi, the concluding chorus from the second cantata, presents a good example of a movement with intervals distributed extremely evenly. Table 5.3 presents the vertical and linear dispersion

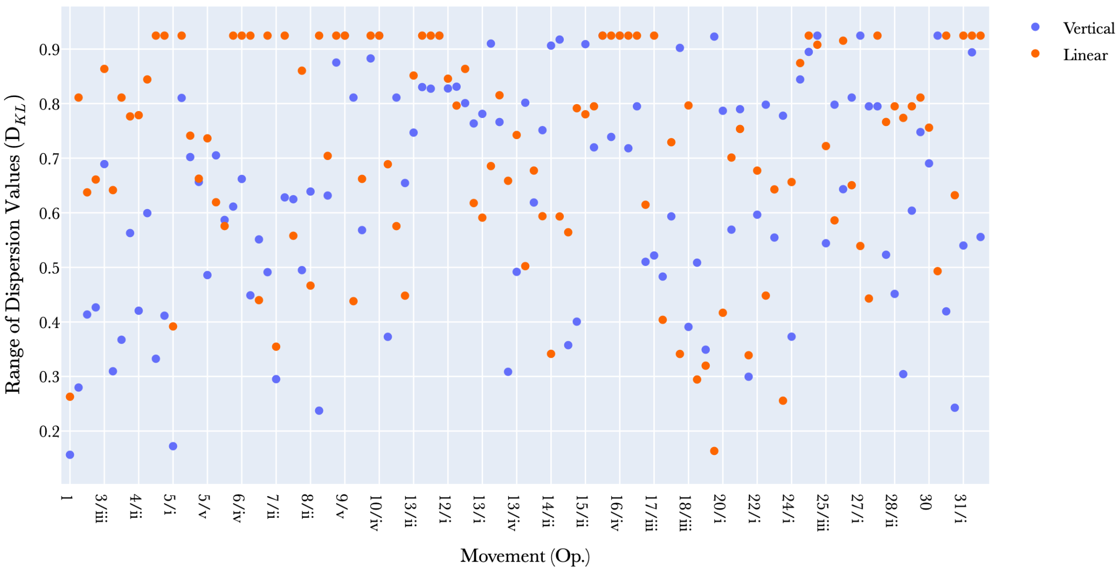

Figure 5.17: Intra-movement dispersion value ranges.

values for each interval, and with a few exceptions, these are all very low values.[7] Op. 31/vi has a curious construction, at least in comparison with Webern's other dodecaphonic works. Each vocal part works through three row forms, doubled at the unison by various combinations of instrumental parts. The row statements are never shared between vocal parts in the manner typical of Webern's works written with linear topography, and as such the vocal parts display no linear intervals other than those in the row. The row itself is dominated by minor seconds and major thirds; the only other intervals present are a single minor third and a single perfect fourth (although ic3 is deployed as both a third and sixth, ic5 is used solely as a fourth). As such, ic2 and ic6 are missing from the row and, as indicated by the asterisks in Table 5.3, almost entirely missing from the movement. The only exception to this is the minor 7th. Although this never occurs in the vocal parts, there are a few occasions where the segmentation of the part in an instrumental doubling creates inadvertent other intervals. In fact, this only occurs in the oboe part, in b. 16 between G5 and A4 (because of the repeats, this phenomenon thus sounds three times). Although the soprano voice fills in some of the intervening rest with F♯4 and B♭3, the oboe part omits these notes,

7. Asterisks indicate that this interval has a frequency of 0.00, and thus either never appears, or appears so fleetingly that rounding removes it.

but as it rejoins the soprano so quickly, after less than a second and a half, the segmentation algorithm groups these two notes. The same phenomenon also acts to create occasional unison pairs of notes (e.g. the oboe in bb. 18–19), which explains the high value for the linear 0 interval. Putting aside these oddities, the other intervals all have uniformly low values, the highest being 0.24, but almost three quarters of these intervals have values below 0.10. Again, the strictly procedural construction of this movement explains this phenomenon in the linear domain, but it is notable that the same is true vertically, a relationship I will return to below.

Interval (Semitones)	Vertical Frequency (%)	Linear Frequency (%)	Vertical D_{KL}	Linear D_{KL}
0	4.97	2.10	0.00*	0.63
1	4.97	20.17	0.01	0.02
2	13.81	0.00	0.03	0.00*
3	11.60	6.30	0.02	0.20
4	4.97	30.67	0.08	0.01
5	15.47	10.08	0.05	0.00
6	6.63	0.00	0.20	0.00*
7	9.39	0.00	0.24	0.00*
8	6.63	18.07	0.10	0.01
9	4.97	2.94	0.00	0.13
10	10.50	0.42	0.02	0.63
11	6.08	9.24	0.04	0.01

Table 5.3: Op. 31/vi frequency and intra-movement interval dispersion values.

As for the alternative, Op. 9/v is a movement that displays very high levels of intervallic concentration. Table 5.4 presents the dispersion values, along with the frequency counts. This movement is what we might think of as ‘classic Webern’: short, punctuated by silences, consistently quiet (alternating between ***pp*** and ***ppp***). With the exception of the vertical semitone and those intervals that do not feature at all, the dispersion values are all high, indicating significant temporal concentration. Noting the implications of the low value for the vertical semitone is an important step: this is an interval that pervades this movement, from the very first bar to the very last. Indeed, almost every vertical collection in the movement includes a semitone above its bass note, the only exceptions being b. 3, and passages in bb. 4, 7, and 12. This is a harmonic quality Chrisman (1979) cites as typical of the *Bagatellen* as a whole, tracing a variety of vertical collections characterised by chromatic adjacencies, and which is often deployed in the form of the wedge principle identified by Benjamin K. Davies (2007). Nonetheless, the crucial feature in Op. 9/v is that while these vertical semitones create this intervallic background, other intervals are then deployed in particular areas of the form. In many cases this is because these intervals only appear once (e.g. the linear semitone, the vertical minor seventh). This certainly has an important structural effect: much as in these miniatures Webern gradually unfolds the total chromatic (see my previous discussion in Chapter 3), in this movement at least he applies the same level of intense control to intervallic revelation. There are some intervals, however, that combine a comparatively high frequency value with a high dispersion, most obviously the linear and vertical tone. In the linear domain the tone is sounded in bb. 1, 3, 5–6, and 7; in the vertical domain, bb. 5, 6, 7, 8, and 13. The picture, therefore, is one in which the use of this interval seems to be carefully deployed: introduced at the start of the work as the very first linear interval, it then infuses the central third of the work, before recurring in the very final verticality, the penultimate sound in the movement. Indeed, it is particularly notable that the linear tone is restricted to the first seven bars. Although the form of this movement has undergone various interpretations, there is no question that the end of b. 7 forms a crucial structural point. It is the first silence since the 'movement proper' gets going in the second bar, and as Forte (1998, 193) points out, the A introduced in b. 7 completes the exposition of the total chromatic. Comparing this usage to that of the aforementioned vertical semitone demonstrates the subtle intervallic control that Webern executes. This movement has been a relatively easy example to discuss: it is short and slow, and many intervals recur infrequently if at all. Nonetheless, the point of Figure 5.16 is to demonstrate this this is not unusual practice: many of Webern's movements display similar levels of controlled intervallic dispersion. Op. 9/v is surrounded by a raft of other movements with similarly high median D_{KL} values. As just one brief example, Op. 6/iii has vertical semitones in 7 of its 11 bars, while both the minor third and the minor sixth appear in only three (respectively, bb. 2, 5, 6, and bb. 4, 5, 6) despite a higher overall frequency in both cases. It is curious that in both movements the vertical semitone is comparatively evenly distributed; indeed, the median D_{KL} value for the vertical semitone across all movements is the second-lowest, above only the vertical minor seventh, and in the linear domain it has the lowest D_{KL} value of all. In a similar manner to the inter-movement dispersion discussed above, then, it seems that the semitone forms a constant presence in Webern's music, evenly distributed both within movements and between them, creating a stable intervallic backdrop on top of which other intervals are deployed in more chronologically focussed ways.

Interval (Semitones)	Vertical Frequency (%)	Linear Frequency (%)	Vertical D_{KL}	Linear D_{KL}
0	0.00	21.05	0.00	0.80
1	33.51	5.26	0.36	0.92
2	13.62	21.05	0.60	0.63
3	6.54	5.26	0.66	0.92
4	9.81	10.53	0.81	0.80
5	9.81	10.53	0.92	0.80
6	4.90	5.26	0.92	0.92
7	9.81	5.26	0.75	0.92
8	8.17	0.00	0.92	0.00
9	0.00	0.00	0.00	0.00
10	1.63	5.26	0.92	0.92
11	2.18	10.53	0.92	0.92

Table 5.4: Op. 9/v frequency and intra-movement interval dispersion values.

Turning to the dodecaphonic music, the finding in Table 5.2 that an increased frequency of an ic in a row predicts an increased frequency in a movement is notable, if unsurprising. Most fascinating, however, is that the effect is only marginally greater in the linear domain than the vertical, although both the R^2 and coefficient values are higher. Scholars have often suggested that Webern was far more interested in the linear than the vertical. Bailey, for example, suggests that 'it is difficult to determine … whether the vertical effect of the coincidence of parts was a matter of much concern' (Bailey 1991, 334). Intuitively, it might be expected that the deployment of dodecaphonic rows would have a more profound linear effect than a vertical one, and yet that is hardly the case. Considering the manner of construction leads to topography (Bailey 1991, 30–93): one might reasonably expect a different effect between works constructed from different topographical techniques, and in particular, the row to have a greater vertical effect in block topography, and a greater linear effect in linear topography.[8] The results for these subsets of the corpus, shown in Table 5.5, generally confirm this. An increase in ic frequency has a greater vertical effect if the topography is block (though combined is even more significant), and a greater linear effect if the topography is linear. These differences are, on the whole, small, approximately 1%, but that too is surprising: the effect of the manner of construction on the resulting music is tiny. The one exception here is the R^2 value for vertical harmony and linear topography, which suggests that although there is a predictive relationship, it is quite weak. Taken together, this indicates that although the row has a significant impact, there is still a predictive relationship even if it has little mechanical effect, which must surely be due to Webern, consciously or subconsciously. In the light of this research it seems difficult to support Bailey's contention that Webern was unconcerned with the vertical effect: the evidence is that there is a connection between row content and musical content, even when the row has limited impact on the mechanical construction of the music.

Topography	Linear R^2	Linear Coefficient	Vertical R^2	Vertical Coefficient
Block	0.80	5.85	0.61	6.17
Linear	0.90	7.42	0.32	4.87
Combined	0.86	6.23	0.57	6.63

Table 5.5: OLS Regression Analysis for different topographies.

8. As outlined in Chapter 2 block topography is an application of the dodecaphonic technique in which the row is unfurled across the whole musical texture; in linear topography the row is expounded in various structural voices sounding simultaneously.

Clearly the implications of this finding concern harmonic integration, and indeed suggest that Webern achieved this (consciously or subconsciously) irrespective of the method of construction. This tallies with the general picture in the literature, which is of Webern following Schoenberg in an obsession with *Zusammenhang* or coherence (Gerhard 1951, 27–28; Webern 1963, 60–61) and *Faßlichkeit*, or comprehensibility (Webern 1963, 17), which for Webern existed in a relationship in which the former guarantees the latter (Webern 1963, 18). Indeed, his lectures demonstrate an almost pathological obsession with coherence: again and again he returns to hammer home the point that for unity to be achieved, everything has to be derived from 'one basic idea', whether in a Bach fugue, a Beethoven sonata, or Schoenbergian dodecaphony (Webern 1963, 34–35; 40–41; 53–54). For the purposes of this chapter, I defined harmonic integration above as very simply the similarity between vertical and linear interval distributions. Whether Webern would have agreed with this is impossible to know, although his lectures demonstrate a limited interest in the vertical relations between parts. He is keen to emphasise the importance of the relationship between parts in the abstract, indeed, he argues that in polyphonic music, 'The idea is distributed in space, it isn't only in one part—one part can't express the idea any longer, only the union of parts can completely express the idea' (Webern 1963, 19). Nonetheless, in his analyses this rarely takes on any formal characteristics, beyond an aside about the music of the fifteenth-century Netherlanders, 'where the theme was introduced by each individual part … with different entries and in different registers' (Webern 1963, 35). Nonetheless, Webern was explicit about both his interest in interval content, and the background coherence provided by a row. His language suggests that this latter phenomenon was both metaphysical and literal: constant reference to Goethe's fictional primeval plant is juxtaposed with the suggestion that 'we've often found that a singer involuntarily continues the row even when … it's been interrupted in the vocal part' (Webern 1963, 55), and indeed he explicitly argues that even if the row is not perceptible, it will have a subconscious effect on the listener. Given that, in early dodecaphony constructed with block topography, the row controls the vertical coincidence of pitches as well as their linear succession, it seems odd to propose that this multi-dimensional relationship becomes irrelevant in the other dodecaphonic work. Of course, in formal analysis such as my own, avoidance of the intentional fallacy (Haimo 1996) dictates that Webern's intentions are of no importance beyond historical curiosity, and so on those grounds alone, my rather circumscribed definition of integration can stand. Naturally it should not be taken as a comprehensive pronouncement on such a complex issue, but nonetheless it is a revealing way of considering this age-old topic.

Figure 5.10 allows the exploration of harmonic integration more extensively than merely in relation to the dodecaphonic works. The general weighting is towards positive correlations, and therefore alignment between linear and vertical interval proportions. A majority of movements (85/107) have positive correlations and the median correlation is 0.27, a fairly weak positive correlation that suggests some degree of integration but with plenty of variety. That the correlation between movement correlations and corpus position is 0.23 suggests general increase in integration over time. Nonetheless, there is no marked change with the onset of dodecaphony, and several of the later works have negative correlations indicating relative disjuncture between harmonic dimensions. Similarly, there are no movements with an anomalously high correlation, although there are two with an anomalously low value, Opp. 6/v and 9/ii, both from the middle of the corpus. That the change across the corpus is limited is something of a surprising finding: it suggests that the new techniques, which might be expected to increase the vertical-linear relationship significantly, did not greatly change Webern's aesthetic output.

Finally, I will consider frozen intervals. Looking first at the overall trends in Figure 5.11, perhaps the most obvious observation is that both vertical and linear present something of a U-shape. This is largely meaningless, as ic0 and ic6 have only one possible interval per octave, so are inherently more concentrated in a smaller number of intervals. Other than that, ics1–5 display remarkably similar HHI values. More notable are the higher linear values for every ic and almost every movement in Figure 5.12. This is perhaps to be expected: larger intervals are more logistically feasible in a vertical context than a linear one, particularly when measured from the bass of a verticality, as they are not limited by an individual instrument's range, and so allow more easily for a greater variety of available intervals. As for chronological change, the linear correlation suggests that over time Webern felt more able to deploy a wider variety of intervals for each ic, an observation which is consistent with the above finding that he used intervals from a greater set of octaves later in the corpus. In sum: as time went on, he moved from using a relatively constricted set of comparatively small linear intervals in each movement to a more varied selection that also encompassed larger-octave intervals.

Returning to Brown, she cites various authors who identify Webern as focussing on representing ic1 as either a major seventh or a minor ninth (J. Brown 2014, 45–46). As this is apparently a commonly held position in the literature, it is worth digging into, as is her focus on Op. 24/iii. Starting with this movement, compared to the rest of the corpus it has the 30th highest linear HHI value and the 11th highest vertical value. This is certainly on the more concentrated end, but it is not remarkable in this regard. More exceptional are the anomalous movements: Op. 6/iv (linear HHI value 0.92), Op. 27/ii (vertical HHI value 0.80), Op. 16/ii (vertical HHI value 0.60), and Op. 11/iii (vertical HHI value 0.57). Op. 27/ii perhaps presents a particularly clear example: ics 2, 3, 4, 5, and 6 are all frozen at one interval level (respectively, minor seventh, minor third, major tenth, perfect fourth, tritone), while it is only ic1 that recurs at two interval levels (major seventh and minor ninth).[9] Recall from the previous chapter that

9. An attentive reader will find this puzzling: the only intervals produced in the dyads and triads of Op. 27/ii are perfect fourths, tritones, major sevenths, and major tenths. Whence do the minor third and minor seventh come? The answer is that in following (Bailey 1991, 414) I have grouped the grace notes with their adjacent notes. Whether or not one agrees with Bailey's suggestion that all the notes should be struck together, and then the grace note immediately released, the speed of this movement means that these pairs tend to be heard as one intervallic

this is a movement with a very tight canonic construction, in which vertical content is limited to a smattering of dyads and triads. As such, the strict limitation of vertical intervals is no surprise. Looking at linear intervals, a similar trend is presented. Although Op. 6/iv is the anomalous movement, its predecessor, Op. 6/iii, has an HHI value only 0.03 lower, and is a slightly easier example to follow. In this movement, no interval is greater than a major sixth, and five ics occur in only one interval presentation: ic0 (unison), ic1 (semitone), ic2 (tone), ic5 (perfect fourth), and ic6 (tritone). Meanwhile, 77% of ic3 occur as minor thirds (the remainder all major sixths) and 78% of ic4 as major thirds (the remainder are all minor sixths). Following the dramatic conclusion of Op. 6/ii, this movement is defined by restraint: dynamics range from ***p*** to ***ppp***, it is the shortest movement of the cycle, and the orchestration emphasises delicacy with muted strings and brass, string harmonics, and prominent roles for harp, celesta, and glockenspiel. In this context, a very constrained intervallic palette is clearly part of the same aesthetic goal. Brown's analysis of her chosen movement therefore provides a demonstration of exactly the sort of possible pitfall that a corpus study can avoid. Without any context from other works, it is impossible to know whether this movement is unusual or typical in its behaviour; this context provides a better sense. Of course, Brown is particularly interested in the treatment of the set-class 3-3 (0 1 4) which is prominent in the movement, and so the choice of example can be understood from that perspective.[10] As for ic1, Figure 5.11 shows that it is unexceptional in the context of the total corpus. The idea that Webern is particularly focussed on presenting ic1 in frozen interval presentations, in a way he is not with other ics, is simply specious.

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entity, at least by me. Were the grace notes to be considered rhythmically independent of the notes they accompany, then this would increase the degree of 'freezing' (decrease the temperature?) of this movement even further: no ic would be presented vertically in any other than one interval position.

10. Though it is beyond the scope of this study, it would theoretically be feasible to assess from a corpus perspective the degree of freezing in the presentation of particular set-classes, which would contextualise Brown's paper further, though statistical power might start to prove problematic on a movement-by-movement basis.

5.5 Conclusion

This chapter presents a wide variety of observations regarding Webern's style. Obviously, several of them could be bolstered by further analysis, both on a local and macro scale. In particular, the development of a representative reference corpus would provide a fascinating contextual framework to which to compare Webern's music. Nonetheless, this research makes the case clearly that developments in intervallic content are an important and characteristic part of Webern's practice. In several areas, this chapter also outlines the need for corpus studies: investigating commonly held assumptions sometimes reveals that scholars' intuitions were correct (Boulez's assertions, for example, about widening intervals), but at other times the empirical reality does not match the assumptions. That there is a clear link between row content and the resulting music is notable, and yet the discussion of topography makes it clear that this relationship *is not wholly controlled by the application of the row*. This poses serious questions about the assumption of Webern as focussed on the linear, and on the importance of the row in the mechanical construction of the music. If the same effect could be achieved in both domains of harmony, even when only one is controlled by the row content, surely one must conclude that the row was less *mechanically* omnipotent than one might otherwise think. The narrative of the row as something that came as a gradual, almost inevitable aesthetic development seems apt (for Webern's telling of this history, see Webern 1963, 39–40). The discussion of the semitone also poses interesting questions about how scholars, and listeners more broadly, characterise style, and what comes to define it. This chapter has shown that this interval, traditionally associated with Webern, is certainly very common, but is also consistently evenly distributed between and within movements. Above, I suggested that this even distribution is crucial to its association with Webern's music. As for integration, the comments here are clearly partial, there is much more to be explored in this area, but they make an interesting suggestion that Webern's oft-expressed desire for integration was less effective in practice, and crucially that neither the onset of dodecaphonicism, nor his broader personal development as a composer, had any major effect in this regard. Finally, the discussion of frozen intervals demonstrated the danger of context-less analysis. Whilst Brown's example in Op. 24/iii is certainly stimulating, this research has not only put that movement in a wider context, but explored other movements that better demonstrate the claims Brown is making.

Chapter 6

The Closer View: Pitch-Class Sets

6.1 Introduction

The analysis of pc sets in the music of Anton Webern, and of his contemporaries, has long been a major concern in the academic literature, and so it is fitting that it comprises the final chapter of this thesis. Indeed, pc set theory was in large part developed precisely to offer a way of thinking about this repertoire, and in particular, the freely atonal music of the Second Viennese composers (Forte 1973, xi). (Their dodecaphonic music, by contrast, provided plenty of analytical material through the endless chasing of row forms.) As outlined in Chapter 2, traditional pc set analysis, therefore, segments units deemed musically interesting by the analyst, typically due to some combination of surface features such as texture, instrumentation, rhythm, or dynamic, and then identifies the pc set that the notes of this unit articulate. The analyst then compares those pc sets that appear in the music under consideration, typically searching for a variety of relations that profess to ascribe some underlying structural integrity to the work at hand (Forte 1973, 93–178). Beyond the conceptual critiques I offered above, as with much manual analysis, the limiting factor for the scale of this work is the analyst's patience in identifying sets. Calculating prime forms from inputs of unordered collections has been computationally assisted (though unacknowledged) since the beginning of this work in the 1960s (Schuijer 2008), but the process of choosing these inputs has remained largely manual. This is not necessarily a bad thing, though it makes for an analytical method that is, to some extent, unreplicable, and therefore intrinsically unscientific (despite the pseudo-scientific veneer provided by a bevy of numbers and an often abrasive writing style), as it rests on a single human analyst's unique hearing of a piece. This therefore allows for as many possible analyses as there are analysts willing to offer them, and a rich and endlessly changing set of perspectives on the pieces under consideration. It is in large part for this reason that I argued above that segmentation is an intrinsic part of the analysis, not the pre-analytical move it is often portrayed as.

As in the rest of my work, the approach I have adopted in this chapter therefore aims to establish a rigorous, transparent, and replicable approach to the segmentation of pc sets. Having done so, this allows me to explore Webern's localised harmonic language, as a complement to the more macroharmonic analysis offered in previous chapters. In particular, I ask whether Webern's music prioritises certain pc sets, taking the so-called 'Viennese Trichord' as a case-study, and how his use of pc sets changes over time. I then re-deploy the DFT to assess the harmonic qualities that these pc sets articulate and deploy an analysis of intra-movement dispersion to quantify the degree to which particular harmonic colours are concentrated in specific areas of the musical structure. Along the way, I use a selection of movements as brief case studies to exemplify the trends I identify. Finally, I home in on the dodecaphonic music, considering the relationship between the row and the music generated with it.

6.2 Method

At this stage in the thesis, there is little that I do that I have not outlined elsewhere: the segmentation approach draws on the intervallic segmentations in Chapter 5, the DFT has been introduced in Chapter 4, and I have discussed the strengths and pitfalls of pc set analysis in Chapter 2. The one area that does require some discussion before proceeding, however, is the field of keyword analysis, and the general adoption of techniques from literary studies into music analysis. I have looked glancingly at the parallels between these fields in my discussion of Moretti's 'distant reading' in Chapter 2, and borrowed analytical tools from the analysis of linguistic patterns in Chapters 3 (clustering) and 5 (dispersion), but this chapter too draws heavily on techniques pioneered in linguistics. In particular, the methods in this chapter are inspired by two primary techniques: the n-gram model, and keyword analysis.

I touched on the first of these in my discussion of segmentation in Chapter 2, and then covered it further in Chapter 5. The basic approach borrows from linguistics, particularly from string-based approaches. In linguistics, a string is defined as a 'finite sequence of letters' (Wintner 2010, xxxix) constructed from the relevant alphabet, and is typically equivalent to a word (for more detailed explication, see Wintner 2010). A sentence can thus be modelled as a series of strings, a paragraph as a series of sentences, and so on. In music, a work or passage can similarly be modelled as a series of strings. The classic example of this is to interpret vertical harmonies as strings, which can then be described in various ways. The segmentation approach then simply categorises each vertical harmony as a distinct event. Having carried out this initial segmentation, frequency distributions are an obvious first step, which reduces the model to a so-called 'bag-of-words'. In this conception, every verticality in the movement can be conceptualised as a token labelled with the relevant word/harmony content, tossed into a bag. Frequency distributions therefore describe the likelihood of pulling out from the bag a token with a specific label, and, as explored above, have implications for the relative significance of these words/harmonies, on which more later. This segmentation approach, however, also paves the way to the consideration of dispersion, as explored in Chapter 5, and harmonic sequences. The usual method here is to look at two-harmony progressions, so-called 'bi-grams', and adopt the Markov Chain model (for an introduction, see Ibe 2013). In basic terms, this argues that the probability of harmony Y following harmony X is informed solely by the nature of harmony X (as opposed to the harmonies preceding X: W, V, U, etc.). As a simple example in tonal music, one might reasonably expect than any dominant seventh harmony is most likely to be followed by tonic harmony (as a guess, perhaps fulfilling 75% of cases), with submediant harmony being the second most likely (again guesswork, but perhaps 15% of cases), and then other harmonies comprising the rest of the probability distribution (in my hypothesised distribution, the remaining 10%). In each case, it seems reasonable to suggest, at least as a basic model, that the harmony preceding the dominant seventh has little impact on the harmony following it, whether it is, in numeral terms, ii, IV, I^6_4, or something else, although each of those pre-dominant harmonies will have their own probability distribution as regards the harmony immediately following them (though I would imagine that in each case dominant harmony is the most likely successor). Unsurprisingly, this approach has been taken up in music analysis, where Rohrmeier and Cross's (2008) study of harmonic progressions in Bach chorales provides an excellent example of this method at work while the introductory chapter to White's (2022) monograph includes an example with scale-degrees. A further extension of this method has been with so-called 'skip-grams', which have again been explored both in linguistics (Guthrie et al. 2006) and music analysis (Finkensiep, Neuwirth, and Rohrmeier 2018; Sears et al. 2017). Here, the authors argue, again commonsensically, there is a predictive relationship between harmony W and harmony Y, irrespective of harmony X. This continues to satisfy the Markov Condition, therefore, because the nature of harmony Y depends solely on one preceding harmony, but this is simply the harmony that is two positions prior rather than one. The musical justification for this is to take account of ornamental phenomena like passing notes. In an n-gram segmentation, a single passing note between verticalities X and Y creates a new verticality (X_1) that therefore removes the predictive $X - Y$ relationship. Skip-grams therefore provide a way to consider those 'main' harmonies, without the intervening ornamentation.

Having outlined some of the extensions of the n-gram approach, I should now confess that applying these methods is simply beyond the scope of my own thesis. Although an obvious next step is to consider bi-gram and tri-gram progressions in Webern (complemented with skip-grams), limitations of space and time render it impossible here, but it is a follow-up for which I strongly advocate, and which I hope provides some interesting context for the work I will offer. Following Chapter 5, the segmentation method I deploy in this chapter is therefore derived wholly from the n-gram method. As before, I will segment vertical harmonies through chordified salami-slicing, and linear harmonies through the same melodic expansion I introduced in Chapter 5. Again, diagonal harmonies are not considered. The only further detail to offer is that I impose a maximum duration on linear harmonies of 5 seconds, and a minimum size of three distinct pcs, with no upper limit. In practical terms, this means that the summed duration of the internal items (notes or rests) of a candidate harmony has to have a value lower than 5 seconds, such that a successfully segmented harmony includes the entirety of those internal items and some portion (of whatever size) of the first and last items in the candidate harmony. As in Chapter 5, I use a limit of 2 seconds as the maximum duration of an internal rest within a harmony. It is important to note that while the vertical and linear approaches here have much in common, there is one significant difference regarding subsets. At least in the primary analysis, this vertical method does not consider subsets: a (0 1 4 6) tetrad is constituted simply as a four-pc entity. In the linear domain, however, every subset is implicitly included: a (0 1 4 6) progression, assuming the limits introduced above, will contribute three prime-form harmonies to the analysis: (0 1 4 6), (0 1 4), and (0 3 5).

At this point, I suspect a brief example might help clarify this method, so I will outline the segmentations applied to Figure 6.1. Vertically, this phrase opens with two monads, thus providing two instances of (0). Following that, the next verticality is constructed from the two cello notes (for vertical analysis acciaccaturas are grouped with the note they precede), G and A♭, and the E in

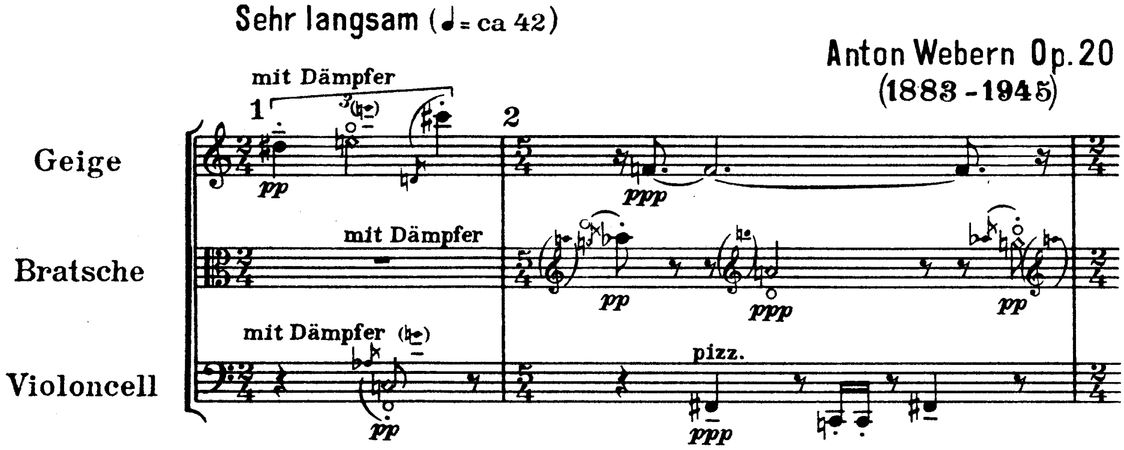

Figure 6.1: Op. 20/i bb. 1–2. © Reproduced by kind permission of Universal Edition A.G., Wien.

the violin, thus a statement of (0 1 4), and it continues in like fashion. In the linear, the first segment is constructed from the first three notes of the violin part: D♯, E, D♮ (0 1 2); the second segment adds the following C♯ (0 1 2 3); third adds the F (0 1 2 3 4). To add any more notes would breach the 5-second limit, so the algorithm now starts segmenting from the E, giving E, D, C♯ (0 1 3), E, D, C♯, F (0 1 3 4), and then from the D♮. Once the violin part has thus been segmented fully, the algorithm continues to segment the other parts in the same way. The viola part should be fairly self-explanatory, but the cello part helpfully indicates what happens with regard to internal rests. The first segment would be A♭, G, F♯, but as the rest between G and F♯ is greater than 2 seconds (2.14 seconds to be more precise) the notes in that first bar are not counted as part of any linear segment. The next segment to assess, therefore, would begin in the second bar, F♯ followed by two Cs. As this produces a dyad, however, this is not counted, nor is the harmony formed by the addition of the next F♯, and so it is not until b. 3 introduces two further pcs within a five-second window that a linear harmony is finally segmented.

I have thus outlined the segmentation approach that I use to choose the pc sets of interest in my analysis, and so now I offer a brief contextualisation of the approach that will ensue. As in previous chapter, following this segmentation I will present frequency distributions for the different harmonies in Webern's corpus, and look for anomalous harmonies in these distributions. The motivating question behind this is to ask whether there are certain pc sets that Webern prefers: was he developing a new harmonic language to replace the common practice one, akin to the intervallic tendencies described in Chapter 5, or, as the pc distributions of Chapter 3 showed that he sought to avoid prioritisation of pcs, did he similarly aim to avoid prioritising set classes?

The most obvious scholarly background for this enquiry is the literary field of Keyword Analysis. Egbert & Biber define this as the pursuit of 'any words that offer important insights into the "aboutness" of a text or corpus' (Egbert and Biber 2019, 78); they cite Paul Baker (2004), who proposes this function for keyword analysis: 'An examination of the keywords that occur when two corpora are compared together should reveal the most significant lexical differences between them, in terms of aboutness and style' (Baker 2004, 347). Early keyword analysis often hinged primarily on frequencies: keyness was calculated as those words that occurred to a statistically unusual degree in one corpus compared to another (for a more extensive discussion of possible methods see Egbert and Biber 2019, 79–81). Thus, one author's tendencies are defined in terms of a general background provided by the context of other authors. This reference corpus might be constructed from generic, chronological, national, or some combination of these and other phenomena. My own research differs on a fundamental level from this in avoiding the target corpus/reference corpus distinction. The interest is in Webern's music on its own terms, rather than as compared to other corpora. It certainly would be possible (and indeed interesting) to assess key harmonies in Webern's music compared to other corpora—interesting reference corpora might include European classical music from 1750-1900 or the music of Schoenberg, Berg, or other of Webern's direct contemporaries—but this is not the focus of my research. This is for two reasons: one practical and one conceptual. The practical reason is simply the amount of encoding required to develop such a reference corpus. Encoding all of Webern's works was an effort enough and given that a reference corpus is typically at least several times larger than the target corpus, this was simply not feasible (copyright restrictions and lack of interest from amateur encoders mean that the music that would most obviously make up a reference corpus is not publicly available in an encoded format in any large-scale way). The conceptual reason for considering Webern's music on its own terms is the same argument that underlies much of my research. While there has been a lot of analytical fire concentrated on this music, the methods and the empirical foundation that underlie my research are still relatively new to the field. As such, I propose that the academic knowledge of Webern is less comprehensive than might be presumed.

The usual justification for the necessity of the target corpus/reference corpus distinction is that without that comparative frame, a corpus study is dominated by ephemera. In linguistic analysis, the typical example of this is so-called 'stop words'. These are small words, typically articles or prepositions, that recur very frequently. Thus, a comparison of frequency distributions in a corpus study of both Beckett and Shakespeare is likely to be dominated by 'the', 'and', 'a', and 'an'. Indeed, the likelihood is that these corpora would display great levels of similarity simply due to these words, which say next-to-nothing about what characterises the languages of these authors. On one level it seems questionable to me to argue that there are musical equivalents of these stop-words, but even if there are, and these might seem more plausible in the common practice period than in Second Viennese music, my argument is, at least in Webern, we do not even have an empirical grasp of these supposed ephemera. Two frequency analyses of Wagner and of Bach might both throw up (0 3 7) as the most significant pc set, and that would reveal little new information. In Webern, I suggest, even that basic knowledge of the pc set language is lacking.

6.3 Results and Discussion

6.3.1 Any preference?

The first area to discuss concerns relative proportions and asks whether Webern appears to have any preference for particular harmonies. As previously, for verticalities these proportions are measured in seconds according to the tempo indications of the score: the cumulative duration of each harmony is determined and then calculated as a proportion of the duration of the movement. For linear harmonies these proportions are measured in frequency counts. These proportions are then compared for each movement, which also normalises by movement length so that longer movements with more harmonies do not outweigh shorter ones. Unless otherwise noted, I am dealing with prime forms. Full results are provided online, following the link in Appendix G.

Firstly, I will consider vertical harmonies. I have calculated mean average distributions across the total corpus, to provide a broad sense of Webern's general practice. In this frequency distribution, it is unsurprisingly (0) that is most common (26.1%), which speaks to the often pared-down texture of Webern's music.[1] All the dyads are also anomalously common (ranging from 6.7% to 1.4%), and so are some larger entities (9 triads and 10 tetrads). The cutoff value for a value to be anomalous is very low, however (merely 0.68% of a distribution) so this has limited information to offer. Indeed, while identifying these overall is a useful initial method for asking whether Webern prioritised certain harmonies, the analyst needs to dig in further for a greater level of sophistication. It is not a surprise that anomalies are identified here. Frankly, it would be more striking to find that out of the hundreds of available prime forms, Webern displays no preference. A quick glance at the raw data (given in the Appendix) suggests that there is a general preference for smaller harmonies. On one level, this is unsurprising: ensemble size naturally limits the size of possible harmonies. There

1. Notwithstanding this comment, I will stay away from exploring textural matters in this chapter as they are too dependent on factors I am not engaging with (dynamic, orchestration, register, etc.) for a full accounting.

are, obviously, also fewer smaller harmonies (12 trichords, 29 tetrachords, 38 pentachords), which means that if Webern were to use each harmony size an approximately equal amount, then each of the smaller-size harmonies would have a higher proportion. In fact, the correlation between cumulative frequency and harmony size is near-perfect: -0.93.[2] It is worth asking, then, whether Webern displays preferences amongst harmonies of the same size. With regard to dyads, there are two anomalies, (0 1) (6.7%) and (0 6) (1.7%); for triads, there is one anomaly, (0 1 6) (3.4%); and for tetrads there are none. All of these values are anomalously common, except for (0 6) which is anomalously uncommon. In looking at linear harmonies, I have opted not to segment monads or dyads, so there is a procedural difference that immediately differentiates this data from that of vertical harmonies. Nonetheless, applying the same anomaly analysis remains a helpful strategy. Again, the anomalies are mainly restricted to the smaller harmonies: a selection of triads, tetrads, and pentads. As before, this in itself says little; it is unsurprising that smaller harmonies are used much more, and the correlation between cumulative proportion and harmony size backs this up: -0.97. Curiously, however, comparing these harmony sizes to themselves there are no anomalous triads or tetrads, and only one pentad, (0 1 2 3 4), with a frequency value of 1.5%.

Comparison of numerical values between the two domains is problematic due to the difference in method that underlies the data collection, but I can compare select higher-level trends, in particular those harmonies that are anomalous in both domains. There are twelve such harmonies, a selection of eight triads and six tetrads.[3] The prevalence of ic1 in these harmonies is notable, if not surprising. Out of the fourteen anomalous harmonies, eleven include at least one instance of ic1. On one level this is unsurprising, particularly given the findings of Chapter 5. The analytical methods are slightly different, for example the linear intervallic

2. For this correlation and the equivalent one in the discussion of linear harmonies I have used Pearson rather than Spearman correlations, as the relationship appears to be linear.

3. In full: (0 1 2), (0 1 3), (0 1 4), (0 1 5), (0 1 6), (0 2 5), (0 2 6), (0 3 7), (0 1 2 4), (0 1 2 5), (0 1 2 6), (0 1 4 5), (0 1 4 8), (0 2 3 6).

analysis takes account of all adjacent pairs of notes, whilst the linear harmonic analysis only considers pairs of notes that are part of a larger entity. Also, if ic1 is between two non-consecutive notes in a linear harmony then it would not show up in the intervallic analysis. Together, therefore, these analyses describe a harmonic language permeated by ic1 on both the note-to-note and larger harmonic level.

6.3.2 Where are they?

From a chronological perspective, there are only three vertical harmonies that have significant (≥ ±0.4) correlations with corpus position: (0 2) (0.46); (0 3) (0. 41); and (0 2 3 6 8) (-0.40). When unfurled, the last of these constitutes a dominant minor ninth, so it is perhaps unsurprising that it declines precipitously across the corpus. All six dyads have positive correlations with corpus position of varying strengths, so the two that are identified here with these moderate correlations reflect a general shift in Webern's practice. I would speculate that on the whole in common practice period music dyads are relatively uncommon other than on rare occasions or in music for ensembles comprised of two melody instruments (itself a comparatively unusual genre). Such ensembles only appear in Webern's practice in the Op. 16 canons, coincidentally positioned almost perfectly in the middle of the corpus, and so this clearly indicates that dyads become an increasing part of his harmonic vocabulary irrespective of genre. Turning to the linear harmonies, there are four harmonies with correlations ≥ ±0.4: (0 1 3 4), 0.52; (0 1 2 4 5), 0.46; (0 1 2 3 4 5 8), 0.40; and (0 1 2 3 5 6 8 9), 0.40. The increase in the tetrad is notable for its octatonic quality, a feature of Webern's music propounded by Forte, and which I discuss further below. 4 out of 5 of the movements for which this is anomalously frequent, however, come from the end of the corpus, not the freely atonal music under consideration for Forte.

Comparing these findings, that so few harmonies have even a moderately strong correlation with corpus position is rather a surprise. Indeed, the median vertical correlation is -0.11 and the median linear one is 0.04. There are several plausible hypotheses that could have been offered to explain change over time: either that as his harmonic language evolved, Webern developed a preference for certain harmony types, thus generating higher frequencies for certain harmonies; or alternatively, one might have proposed that as the ideology of non-repetition encouraged by serialism strengthened, his distributions would have become more even. Instead, there is no evidence of significant change. The most plausible candidate for this would probably have been (0 3 7), as one would assume that after deploying it liberally in Opp. 1 and 2 he would go on to minimise its usage. In fact, not only is the correlation low, -0.12 for the linear and 0.06 for the vertical, but the only movements with anomalously frequent proportions of these harmonies are from later in the corpus, not the two indisputably tonal ones, and indeed the same is true of (0 2 5 8). Brought together, these observations suggest that not only is there no major overall change in the degree of preference in Webern's harmony distributions, but that there is no significant change in his use of individual harmonies either; in fact, it suggests a situation in which Webern's pc set language is remarkably stable across a 40-year span.

As in Chapter 5, however, a more nuanced perspective on chronology can be found through the measurement of dispersion, which I again measure using the Kullback-Leibler divergence (D_{KL}). The overall picture is one of concentration: the median D_{KL} value for verticalities is 0.98 and for linear harmonies is 0.80. These results do display, however, the powerful effect that comes from the vast number of pc sets used very rarely, and therefore inevitably in a concentrated manner. Indeed, the correlations between frequency and dispersion are extremely strong: -0.87 for verticalities and -0.83 for linear harmonies. To avoid this issue, I will continue the discussion by focussing solely on the triads and tetrads. These are harmonies that all have comparatively high frequencies, they are possible in almost all the ensembles Webern deploys (Op. 16 would be the only counterexample: vertical tetrads are impossible here) and therefore they provide a helpful snapshot of his practice. I present the frequency and dispersion values for these harmonies in Figure 6.2. This is an equivalent chart to 5.14, but there are a few comments to make about its construction before discussing the results. The first is that frequency values have been scaled by plotting their proportion out of the total set of triads and tetrads, rather than their proportion out of the total set of all used pcs. Likewise, dispersion values have been calculated only in terms of triads and tetrads, rather than in relation to the total set of pc sets. As in Chapter 5, I have separated the chart into four quadrants. While the horizontal quadrant separator remains at 0.5, this scaling affects the vertical one. This represents the value were all (triadic and tetradic) pc sets to be used in equal proportion, and so in Figure 6.2 it is positioned at $100 \div 41 (\approx 2.4)$. There are intervals in each of the four quadrants in Chapter 5, and the same is just about true in Figure 6.2, as linear (0 3 7) is just about included in the Uncommon Spread-Out area of the chart. Nonetheless, it is clear that most pc sets fall in the Uncommon Concentrated area and that, on the whole, the same general relationship between increasing frequency and declining concentration holds (the correlations between frequency and dispersion are -0.81 for verticalities and -0.82 for linear harmonies; although these variables cannot have a linear relationship this does at least indicate montonicity). There are nonetheless some harmonies with similar values in only one plane: linear (0 1 6) and (0 2 6), for example, have a similar frequency of about 5% and yet the former harmony is the second-most evenly dispersed, while (0 2 6) is clearly quite concentrated; (0 1 6) has a range of frequency values of 10.9%, (0 2 6) ranges a full 50.0% between its most and least prominent movements.

As in Chapter 5, dispersion can also be measured on an intra-movement basis. Again, I separate each movement into six segments, and then calculate the dispersion of the various harmonies across those segments. As in much of this chapter, the following discussion is restricted to triads and tetrachords, to avoid skewing the results with endless harmonies used very rarely. The median D_{KL} values for each movement are given in Figure 6.3 and the range values in Figure 6.4. Both sets of data have moderate correlations with corpus position: for medians, -0.57 vertically and -0.43 for linear harmonies; meanwhile, for ranges,

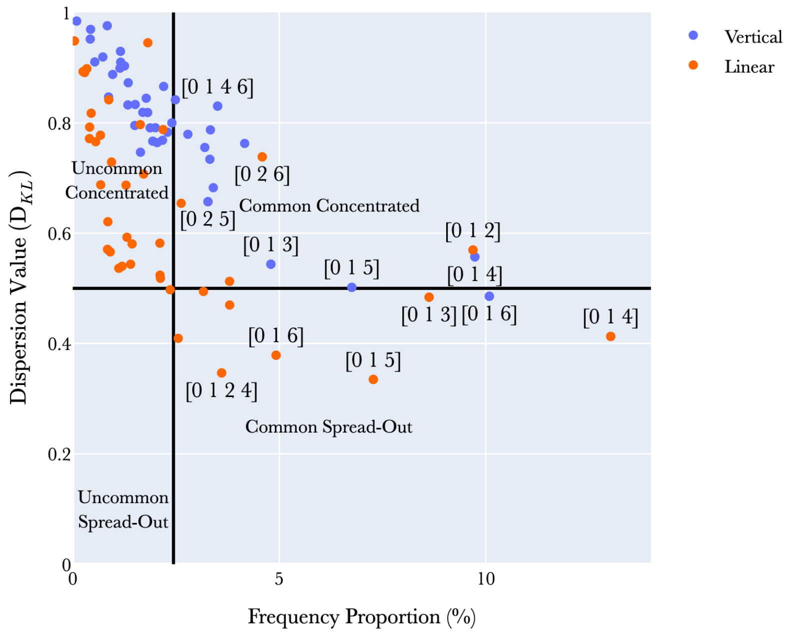

Figure 6.2: Inter-movement dispersion and frequency proportion for triadic and tetradic pc sets.

0.60 in the vertical domain and 0.51 in the linear. This suggests that as time went by in both domains Webern deployed his harmonic palette in such a manner as to create lower average values, suggesting that individual pc sets were deployed more evenly across a given movement, but the growth in range values indicates that while this was the general tendency, there was increasing variation in the deployment of pc sets in each movement with some more concentrated and some more spread out.

At this stage, these values can feel rather disorientating, as they are several degrees of abstraction away from the music itself. Even more than in other situations, a short example is therefore helpful to demonstrate what these phenomena mean in practice. Op. 31/ii portrays the situation at the end of Webern's corpus. In blunt terms, typical features of movements at this end of the corpus are middle-to-low median values and high range values. This movement has median values of 0.83 (vertical) and 0.35 (linear) and range values of 0.82 (vertical) and 0.89 (linear). Looking first at the vertical pc sets, the contrast between (0 1 5) and (0 3 7) is instructive: the first of these appears eight times in all, across all six segments; the latter appears three times, though in only two

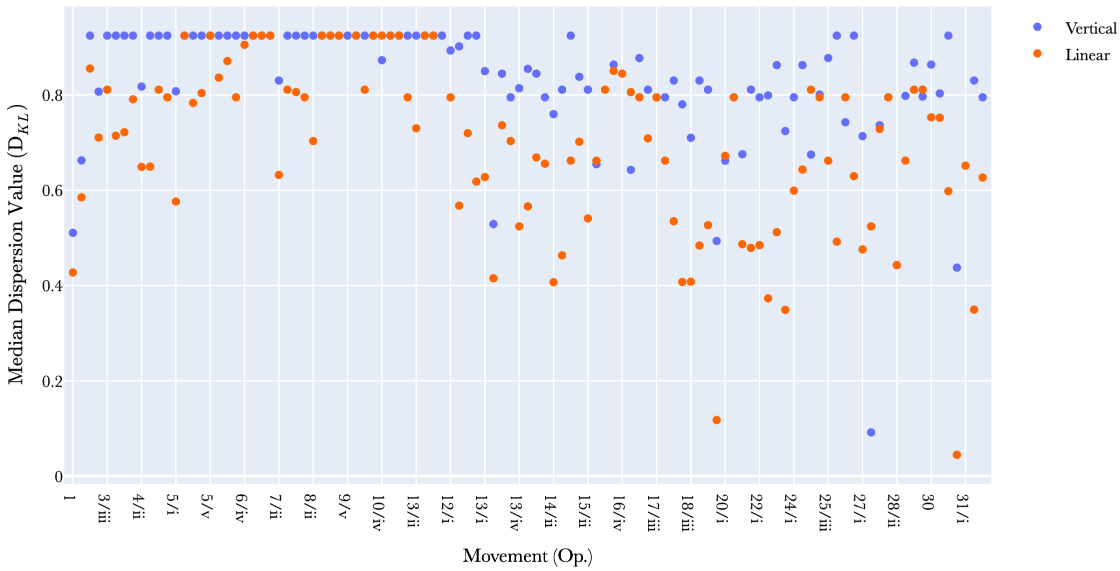

Figure 6.3: Intra-movement dispersion value medians.

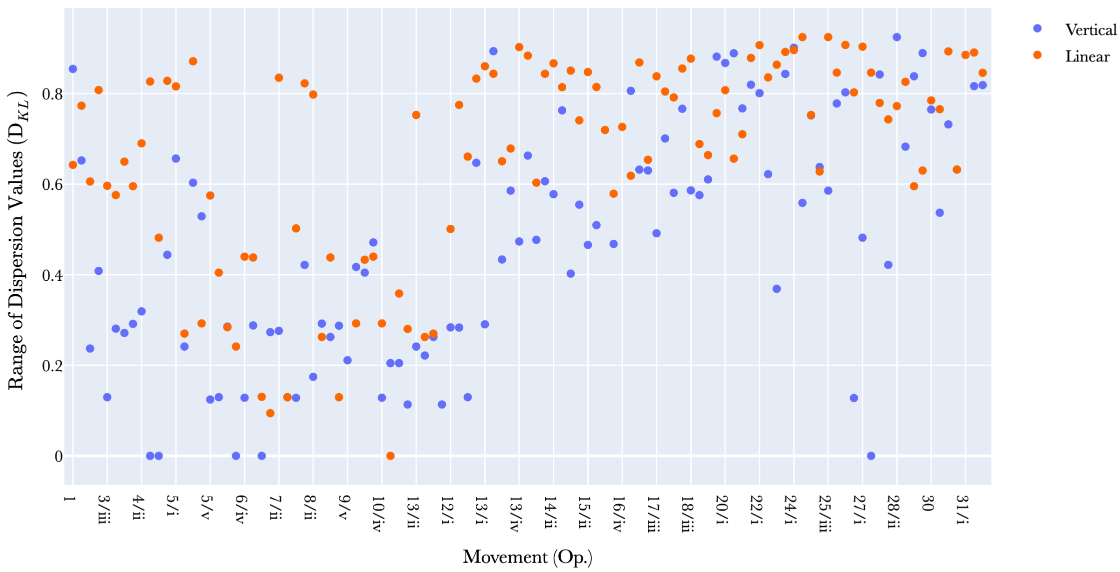

Figure 6.4: Intra-movement dispersion value ranges.

segments. Figure 6.5 shows this in practice by presenting a harmonic reduction of the first 48 bars, with (0 1 5) highlighted in red and (0 3 7) in blue. Although the spread of values for harmonies in this movement is wide, from 0.92 for the eleven pc sets that appear in only one segment through to 0.11 for (0 1 5), most pc sets are comparatively concentrated, somewhat unusual for this end of the corpus. For linear harmonies, however, the median value is much lower, although again there is a significant spread. At the bottom end, (0 1 2) has a dispersion

Figure 6.5: Op. 31/ii bb. 1–48 harmonic reduction. © Reproduced by kind permission of Universal Edition A.G., Wien.

value of 0.03, appearing, respectively, 4, 5, 5, 3, 5, and 3 times across the six segments. At the other end of the scale, alongside the various one-time only pc sets, (0 1 3) and (0 3 7) feature in only two segments. For linear pc sets, it is the former pc set that is more typical, although this is certainly an extreme example. Overall, later works in the corpus tend to display this sort of behaviour in both domains: pc sets are, on the whole, more evenly distributed across movements. It may be that this is a result of the dodecaphonic technique, as I explore below, or that Webern's practice simply changed. Either way, the results seem to suggest that he made more structural use of particular pc sets to characterise sections of a form earlier in his practice than later.

6.3.3 The Viennese Trichord?

Before continuing along this path from Webern's preferences and chronological tendencies to the implications of his choices, I am going to pause briefly to consider a single harmony in some closer detail. Another case-study, but not of a movement this time. The so-called 'Viennese Trichord', set-class 3-5 (0 1 6), is often cited as a common harmonic fingerprint of Second Viennese School music (Boss 2019; Roig-Francolí 2007). In my own mind, it is the luscious second group from Schoenberg's *Kammersymphonie* No. 1 that heralded the dawn of this triad's significance, although evidence for this will have to wait until somebody applies similar empirical analysis to Schoenberg's work. Either way, it is often seen as archetypal of these composers, a claim that bears some investigation in the music of Webern. On a related topic, with regard to Op. 9 Chrisman (1979) mentions it as one of a series of harmonies constructed from a semitone plus some other interval, in league with (0 1 2), (0 1 4), and (0 1 5) ((0 1 3) is relegated to secondary status). With regard to tetrads, Chrisman similarly highlights (0 1 4 5), (0 1 5 6), and (0 1 6 7), two of which therefore include (0 1 6) as a subset. The accuracy of these statements, and in particular Webern's usage of this triad, is thus ripe for investigation in light of the data presented here.

On a basic level, the results given above would seem to make this an open-and-shut case. (0 1 6) is not only an anomalous verticality compared to the other available verticalities but is the only anomalous triad when compared to its eleven compatriots. An overall value of 3.4% might initially appear small, but, in context, for one in every thirty verticalities to be an instance of (0 1 6) is quite remarkable. What is more, in context of the twelve triadic pc sets, (0 1 6) comprises 19% of the distribution (by comparison, reprising the hypothetical equal distribution idea of Chapter 3, in that imaginary case each triad would have a frequency of merely 8.3%). In the linear domain, however, this triad is rather less exceptional: it ranks fifth behind the other triads formed from a semitone and another interval.

With regard to dispersion, Figure 6.2 indicates that in both domains (0 1 6) is comparatively spread-out, indeed it is the second-most spread-out triad in the linear domain and the most spread-out pc set in the vertical, swapping places with (0 1 5). As I suggested in Chapter 5, as with the semitone it may well be these low dispersion values that contribute to the sense of this triad dominating the corpus: wherever one turns in Webern's oeuvre the Viennese Trichord is likely to appear. Comparing the frequencies of vertical (0 1 6) across each movement, there are five that are anomalous, from which Op. 16/v provides a good example. (0 1 6) occurs 14 times, to make up 11% of duration-weighted verticalities. It appears in every bar save three: the first and last (both of which only have two voices), and bar 6. As such, it has the lowest intra-movement dispersion value of any vertical pc set in this movement, 0.12. In fact, (0 1 6) also describes the pitch relationship between the opening notes of the three voices (D, E♭, G♯), which points attention towards the linear. Here it is the most common triad, taking up 2.5% of the frequency distribution. It has a middling intra-movement dispersion value, however, of 0.48. Across the six segments of the movement it appears, respectively, 0, 9, 10, 0, 8, and 4 times. The extreme differences between these segments derive in part from the canonic construction: if a statement appears in one voice, it is typically going to recur promptly in the other two (harmonies that cut across segment boundaries are not counted, hence this does not happen exactly each time). This sort of treatment is very typical: the median intra-movement dispersion value for linear (0 1 6) is 0.47. Meanwhile, the median intra-movement dispersion value for vertical (0 1 6) is 0.68, which suggests a somewhat more concentrated deployment. In both cases, these relatively central dispersion values could well contribute to the sense of (0 1 6) as pervading Webern's music: rather than being so evenly distributed that they form an unnoticeable background, or so concentrated that they are restricted to single portions of movements, instead they form repeating clusters across movements.

Overall, the picture is therefore mixed, and looks different depending on the harmonic domain. Although (0 1 6) is certainly very common both vertically and linearly, overall it does not seem radically different in its usage from other harmonies rooted in a semitone. More specific analysis of individual works could assess whether it is lent prominence in different ways: important thematic functions, for example (see the aforementioned Schoenberg!), or through conventional significance parameters like dynamic and orchestration. Nonetheless, from a frequency standpoint, it seems more important to highlight the general group of pc sets comprised from a semitone and another interval. In both domains, this set of five pc sets are the top five triads, and comprise over 65% of the distribution of triads.

6.3.4 What does it sound like?

Touching on the characteristics of these notable harmonies introduces the next section of this chapter, which concerns the variety of harmonic colours that Webern deploys in his music and their use across the corpus. To assess this, I will again deploy the Discrete Fourier Transform, as introduced in previous chapters. Calculating the DFT values of the different harmonies that Webern deploys therefore provides an empirical perspective on the harmonic colours that pervade his work at the granular level, a particularly important area for a composer who was as cognisant of the expressive qualities of strictly controlled intervals as Webern.

The first step is to look at the overall picture by calculating the squared DFT magnitudes of each harmony deployed in the corpus and weighting them by average duration or frequency as outlined above (thus also normalising by movement length). The results for each movement are given in Appendix F; the overall results for the entire corpus are shown in Table 6.1. For the vertical squared magnitudes there is one anomalously strong squared magnitude, $|f_4|$, and one that is anomalously weak, $|f_6|$. For the linear squared magnitudes $|f_6|$ is again anomalously weak, the only anomaly. That the vertical squared magnitudes tend to be lower than the linear squared magnitudes is likely due again to the difference in weighting (durations as opposed to frequency counts) so no information should be inferred from this disparity in scale. The prominence of $|f_4|$ here is very important, especially given the findings of Chapter 4 with regard to squared DFT magnitudes on the macroharmonic level. In that previous chapter I pointed out that although Forte ascribes to the octatonic supreme significance in Webern's freely atonal music, the results of a macroharmonic analysis downplay the significance of $|f_4|$ in favour of $|f_2|$ and, to a lesser extent, $|f_3|$. The results in the present chapter present a complementary picture: that $|f_4|$ is in fact unusually prominent in the vertical domain, although not in the linear, whereas $|f_2|$ is the most prominent linear squared magnitude, though not outstandingly different, while being positioned in the middle of the vertical pack. These findings are obviously not contradictory; rather, they demonstrate the possibilities of an analysis that takes account of different structural levels: while on the granular, chord-to-chord level the music leans in one harmonic direction, the cumulative effect of this is rather different. Meanwhile, it is notable that in the pc distribution DFT analyses present in Chapter 4, $|f_6|$ was also the weakest component. Clearly Webern had little affinity for whole-tone colours, whether on a micro or macro scale! This is probably explained by his aforementioned predilection for the semitone, the presence of which immediately negates any whole-tone possibilities.

Component	Vertical Squared Magnitude	Linear Squared Magnitude
$\|f_1\|$	0.18	0.46
$\|f_2\|$	0.18	0.57
$\|f_3\|$	0.16	0.51
$\|f_4\|$	**0.23**	0.52
$\|f_5\|$	0.18	0.52
$\|f_6\|$	**0.10**	**0.37**

Table 6.1: Median DFT squared magnitudes.

This phenomenon can be illustrated by a case study, for which Op. 18/i is a good candidate.[4] This movement's pc distribution is given in Figure 6.6, for which $|f_4|$

4. This was not an arbitrary choice: to choose a suitable case study I calculated the proportion

is the weakest squared magnitude, comprising only 2% of the total squared magnitude values. Given how flat the distribution is, compared to other works by Webern all the squared magnitudes have low values, but there is no *a priori* reason to expect the weakness of $|f_4|$ as compared to the other components. By comparison (and curiously, given the comments made above) $|f_6|$ takes up 40% of the total squared magnitude values. With this clue, Figure 6.6 can be seen to include a prominent whole-tone segment that therefore weakens any octatonic possibilities: in fact, the top six pcs run 10, 8, 6, 4, (3,) 2. Meanwhile, $|f_4|$ has the second highest value for median squared verticality magnitudes, making up 25% of the total squared magnitude values. To explain this, I need to consider the most prominent verticalities in the movement, but before that, I present a survey of the little academic engagement there has been with this movement. Bailey (1991) lists the row and then quickly dispatches the structure of the movement with a single paragraph, noting merely that the row is used solely as P_0, with no elision and perfect note order, with guitar chords formed from adjacencies. Johnson (1999) folds this simple row-form into a folklike interpretation of the song, arguing that the simplicity of the row treatment follows the folklike text. Mark Sallmen (2002) followed up with an analysis that, from my perspective at least, is notable principally for its focus on (0 1 6), though he supports Johnson's interpretation by highlighting the Marian, and Mahlerian instrumentation. He points to the five-fold recurrence of this set-class in the row, as well as seven other statements in nearly adjacent pc sets (as a good example, in the case of the tetrad formed from the eleventh pc and the following first three pcs, (6 0 11 5), all four possible triads, (6 0 11), (6 0 5), (6 11 5), and (0 11 5), are members of the set-class (0 1 6)). He also traces this set-class in the guitar part, noting that the vast majority of triads are statements of (0 1 6), each larger chord includes (0 1 6), and even the dyads are all subsets of (0 1 6). Adam E. Shanley (2016) develops aspects of both of these preceding analyses: he argues that the consistency of (0 1 6) is

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that $|f_4|$ takes up of pc distribution squared magnitudes, and verticality squared magnitudes, and then selected the movement with the greatest difference between these. Thus, Op. 18/i is in fact not merely a good candidate but the best candidate to demonstrate this phenomenon.

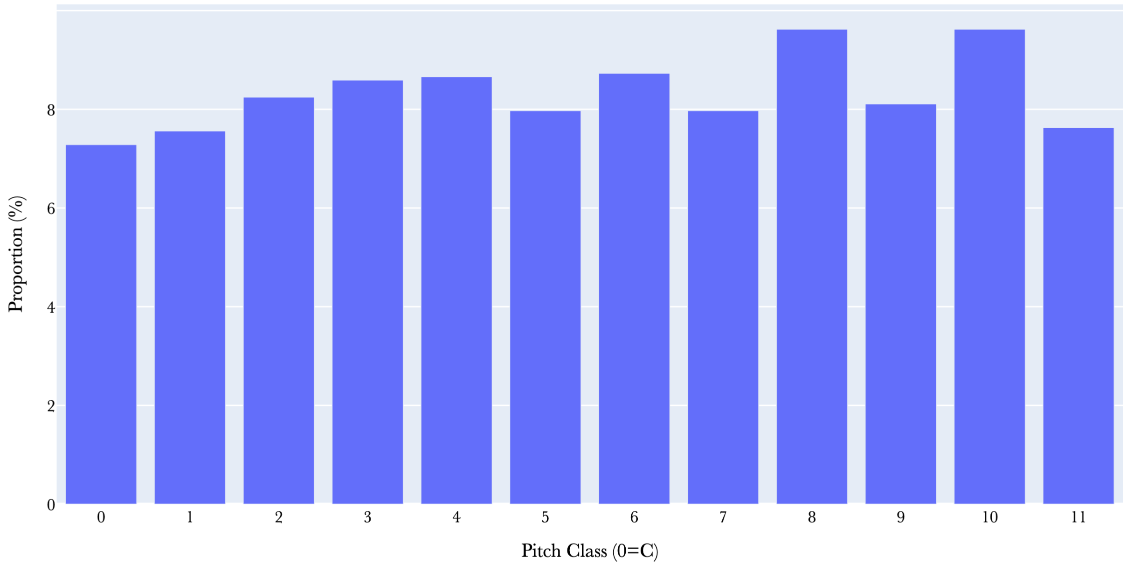

Figure 6.6: Op. 18/i pc distribution.

combined with the simplistic row treatment to express Webern's perception of folklike simplicity. In service of this, his harmonic analysis principally seeks to trace statements of (0 1 6) across the song. His segmentations principally take the form of simple verticalities (as in my own analysis), or segments formed from adjacent pairs of entities. Segments that, in my terms, are purely linear, are much rarer. As ever, a few of these segmentations seem somewhat suspect. I am particularly unconvinced by segments that separate out only a single note from a guitar chord, and group that note with pcs in the clarinet and voice, which feel like end-motivated cherrypicking. Two brief examples are provided by Ex. 3.12 (Shanley 2016, 100) which links the vocal F♯ and C♯ with the guitar's G, thus neglecting the D it is sounded with, and Ex. 3.14 (Shanley 2016, 102) in which the F♯ at the top of the guitar's second triad is grouped to the C♯ and the C in the voice. Nonetheless, it is undeniable that the movement is saturated by (0 1 6), and on the whole Shanley does a thorough job of pointing out many convincing appearances of the set-class.

Previous academic literature, then, has focused solely on (0 1 6), and has made a convincing case for its significance. Indeed, although my analytical method is clearly somewhat different to those of Sallmen and Shanley (if nothing else, because it indiscriminately groups cross-instrument verticalities), it finds that the two verticalities that are anomalously frequent are (0) and (0 1 6). While the monad is not relevant, for the triad $|f_4|$ is the second strongest squared magnitude after $|f_2|$. In fact, the same is true of many of the top verticalities. Over 50% of duration-weighted verticalities come from the top ten pc sets,[5] and a few of these have very obvious octatonic qualities: (0 1 6), (0 1 3 6), and (0 1 3 6 7). The other two non-dyadic harmonies, (0 1 2 6 7) and (0 1 5 7), have weaker values for $|f_4|$. In the case of the former, although (0 1 6) is a subset of the harmony, its octatonic implications are negated by the two chromatic pcs, 2 and 7, which lean toward the other octatonic collection. In the latter, the same sort of principle applies: although (1 5 7) is consistent within one octatonic collection, this is negated by (0). In the case of the dyads, both (0 6) and (0 3) have octatonic implications as they occur in both octatonic collections, while (0 1) and (0 2) are weaker. I mentioned that in this movement $|f_4|$ has the *second* highest median value for squared verticality magnitudes after $|f_2|$, and this characteristic is obviously displayed by the harmonies under consideration here: all the sets with three or more pcs include at least one tritone, and (0 6) is the most prominent of the dyads. Nonetheless, what this analysis points to is that not only is the movement saturated with (0 1 6) harmonies, but that, at least in the vertical domain, these create a broader octatonic colour.

The 5-second window analyses presented in Chapter 4 also provide another helpful counterpoint here. Figure 4.10 charts the degree of window-to-window variation, and here Op. 18/i has the third lowest value, indicating a high level of variation from window to window. Considering the six median values for each squared magnitude across these 5-second windows, $|f_4|$ comes in second, after only $|f_1|$. This indicates, therefore, that not only is an octatonic colour being expressed in the individual verticalities stated in the song, but that the cumulative effect in the medium term (i.e. 5 second passages) similarly expresses this harmonic colour. As I explored in Chapter 3, music written with a dodecaphonic

5. In full: (0), (0 1 6), (0 6), (0 1 2 6 7), (0 1 3 6), (0 2), (0 1), (0 1 5 7), (0 3), (0 1 3 6 7).

structure will typically, though not necessarily, display a comparatively flat overall pc distribution, as Figure 6.6 demonstrates is the case here. What is crucial, however, is that on a more micro level this movement expresses a consistent harmonic colour, in this case an octatonic one.

Having considered the overall tendencies in the corpus, I now turn to individual movement-level information. Here, the spread of the squared magnitude values for each movement is a useful metric for establishing how evenly or unevenly squared magnitudes are used, and thus how changeable the harmonic quality of a movement tends to be. As described in Chapter 3, spread can be measured by either range or IQR, and as I am interested in the general tendencies rather than edge cases I use IQR here. The exact process of measurement is to calculate first the six squared magnitude values for each verticality in the movement, again weighted by duration or frequency. For each of the six squared magnitudes I then calculate the IQR value across the weighted verticalities of the movement. This therefore provides six IQR values, one for each squared magnitude, and then I calculate the median of these six IQR values. This therefore gives a single-value metric per movement, the median IQR value, which represents the typical amount of variation for a squared magnitude across a movement, and thus indicates how consistent or varied the movement is in terms of harmonic colour. I only calculated these values using the pc sets deployed in a given movement, rather than including zero values for those pc sets that theoretically exist (and perhaps Webern used elsewhere) but were not used in the movement under consideration. Figure 6.7 is a graph of these values. Again, although I have plotted linear and vertical values on the same axis, absolute numerical comparisons should not be made; however, plotting them on the same chart allows me to point out similarities or differences between trends in the sets of data.

The overall picture in Figure 6.7 is quite clear: in the vertical domain the median IQR values are relatively scattered, with no clear chronological trend. The picture with linear harmonies is different: primarily low values, with a few

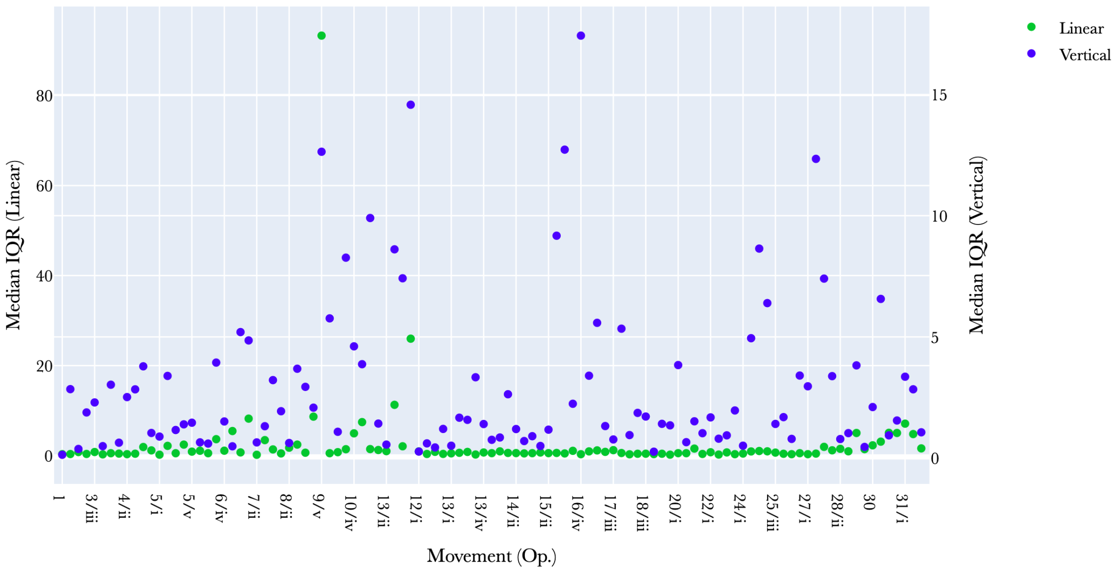

Figure 6.7: Median IQR of squared DFT magnitudes.

extreme outliers. The situation is therefore one in which in both domains there is little consistent change across the corpus, but whereas the linear harmonies tend to express harmonic qualities quite evenly, in the vertical domain there is much more intra-movement variety. This may be due to the difference in data collection as the inherent inclusion of linear subsets will shape the results here, prompting multiple harmonies with similar qualities.

6.3.5 What about rows?

A related question to pose about Webern's harmonic language concerns the relationship between the harmonic content of the rows, and the observed harmonic content of the music composed with those rows. This is an extension of the analysis presented in Chapter 5 of the relationship between the interval content of the rows and of the derived music. It is standard practice when considering rows to break them up into smaller units. Hexachordal relationships are perhaps the most famous example of this as exemplified in Schoenberg's music, but Bailey has demonstrated Webern's focus on symmetry, invariance, and motivic relations within his rows (Bailey 1991, 13–29). In this chapter, my first step is to produce a frequency count of all triads and tetrads in each row, as formed from consecutive pitches (including 'round the back'). I then apply OLS

Linear Regression to ask whether the frequency of a triad or tetrad in the row predicts its frequency in the resulting music. The independent variable is therefore the row count of a harmony; the dependent variable is the observed proportion of that harmony. So that triads and tetrads can be compared, the observed proportions only take into account other harmonies of the same size, not all harmonies in the movement.

As in Chapter 5, I consider vertical and linear harmonies separately. The results from the analysis are presented in Table 6.2. $p < 0.001$ in all cases for the regression and the f-statistic. As in Table 5.2, to increase statistical power I did not calculate different OLS values for each different harmony, but merely grouped each row-movement pair across the entire corpus. Likewise I treat the observed duration/frequency count as the dependent variable, and the frequency count in the row as the independent variable. Again, differences in the numerical scale between linear and vertical results should not be taken as significant: in the former they relate to frequency counts, in the latter durational proportions. In both domains there is clearly a predictive relationship, and the standard error column indicates that it is noticeably greater for triads than tetrads. Although comparing linear/vertical coefficients for the same harmony size is not possible, the R^2 values indicate what proportion of the variance of the dependent model is explained by the model, and thus the strength of the relationship between the model and the dependent variable. For both harmony sizes, the relationship is stronger for linear harmonies than vertical ones.

Harmony Size	Domain	Coefficient	R^2	Standard Error
Triad	Linear	1.46	0.66	0.05
	Vertical	1.00	0.34	0.06
Tetrad	Linear	0.95	0.46	0.03
	Vertical	0.45	0.09	0.07

Table 6.2: OLS Linear Regression between row segment and harmony frequency/duration counts.

Following on in the same way, it also makes sense to apply this analysis to the topographical subsets of the corpus, the results of which are presented in

Table 6.3 (again $p < 0.001$ in all cases for regression and f-statistic). With regard to intervals, Chapter 5 indicated that there were stronger relationships between linear topography and linear harmony, block topography and vertical harmony. Combined topography had a marginally stronger relationship with vertical harmony, but it was a minute difference. The same pattern is revealed here and confirmed by the standard error: whether for triads or tetrads different topographical organisation alters the effect size of the row on the observed harmonies. Not only is it the case that coefficients within the same domain are stronger for linear/linear block/vertical pairings, but the R^2 values also emphasise this relationship even more strongly.

Harmony Size	Domain	Topography	Coefficient	R^2	Standard Error
Triad	Linear	Block	0.93	0.54	0.07
		Combined	1.00	0.52	0.11
		Linear	1.87	0.77	0.07
	Vertical	Block	1.47	0.53	0.11
		Combined	1.40	0.71	0.10
		Linear	0.64	0.17	0.09
Tetrad	Linear	Block	0.67	0.39	0.04
		Combined	0.60	0.30	0.07
		Linear	1.22	0.56	0.05
	Vertical	Block	0.70	0.35	0.05
		Combined	0.46	0.27	0.05
		Linear	0.31	0.03	0.07

Table 6.3: OLS Linear Regression between row segment and harmony frequency/duration counts separated by topography.

The implications of this are several. That relationships are universally stronger for triads than tetrads is likely to be a result of the greater amount of possible variety in triadic entities than tetradic ones. The strong relationship between topography and harmonic domain indicates the importance of that compositional decision in governing the outcome of Webern's creative process. In that context, the shift towards the end of his corpus to writing solely with linear topography explains a shift in his harmonic language (from Op. 26 onwards only one movement is not written with linear topography; before this, 13/23 movements are written with block topography): as a result of this change in practice, there is a stronger relationship between row forms, and the harmonies Webern chose to encode in these, and the linear harmonies expressed in his work. Meanwhile, while there remains a predictive effect between the row and the vertical harmonies in the resulting music, it is much less important. This therefore highlights the importance of the topographical decision: an early choice in the compositional process that would have significant ramifications in what Webern went on to write. This is a topic that, to my knowledge, has received little sustained interest beyond Bailey's formal accounting of Webern's music. This is unfortunate given the importance my own research shows it to hold.

Considering row content in this way also inevitably poses the possible importance of invariance. Many of Webern's rows are famous for this property (Bailey 1991, 13–14), often with very limited triadic or tetradic content (indeed the rows of Opp. 28–30 each express only three distinct triadic pc sets, and, respectively, four, five, and three tetradic ones). It might be expected that a lower number of distinct pc sets would lead to a stronger relationship between the row and the observed harmonies, as this would concentrate the harmonies further. Table 6.4 presents the results of an OLS Multiple Linear Regression analysis, using the number of distinct pc sets in a movements row as a further variable to augment the simple analysis presented in Table 6.2 (here, $p < 0.001$ in all cases for regression and f-statistic bar one: the row variable for vertical tetrads, which only satisfies a significance criterion of 0.05, much weaker). The results suggest that, in fact, invariance has little additional impact. The R^2 values tend to be very similar, the only one for which there is a noticeable improvement is the linear tetrad. That there is little change here is curious given the attention this phenomenon has received in the literature and the broader reception of Webern. Even if he did perhaps expend great energy and attention on the construction of the rows, integrating invariant properties had less impact on the harmonic content of the resulting music than another early compositional step, that of the topographical decision.

Harmony Size	Domain	Variable	Coefficient	R^2	Standard Error
Triad	Linear	Row PC Sets	1.36 0.08	0.67	0.06 0.02
	Vertical	Row PC Sets	0.75 0.16	0.39	0.07 0.02
Tetrad	Linear	Row PC Sets	0.73 0.07	0.58	0.05 0.01
	Vertical	Row PC Sets	0.20 0.07	0.14	0.07 0.01

Table 6.4: OLS Multiple Linear Regression between row segment, pc set count, and harmony frequency/duration counts.

Finally, I suggested above that decreasing dispersion values in the latter part of the corpus may be due to the impact of the dodecaphonic technique. Given the evidence presented above, that the pc content of the row has a clear effect on the pc content of the music, this makes intuitive sense. One way to assess this empirically is to consider the correlation between, for each pc set, its frequency in the row and its dispersion value in the music composed with it. Thus I can assess whether prominence in the row leads to a decrease in dispersion as hypothesised. The correlation values indeed support this view, though neither are particularly strong: -0.40 for triads and -0.28 for tetrads.

6.4 Conclusion

As I said at the outset of this chapter, pc sets have been a long-term concern for analysts of Second Viennese music. A whole thesis could consider the pc content of only individual pieces, and so this chapter makes no claims to be comprehensive. Nonetheless, taking a corpus approach does reveal some new findings. Firstly, although the frequencies of pc sets are surprisingly stable across the corpus, I find the increasing frequencies of dyads to be notable, especially in a historical context in which, I suspect, they are somewhat unusual. This is yet another area which would be greatly improved by expanding the tonal portion of the corpus: I suspect that with a larger body of work to consider, a more profound difference might be identified between his tonal and non-tonal music. Nonetheless, that across the 107 movements here few pc sets change significantly in either direction is certainly a surprise. This is particularly relevant given that both Figures 6.3 and 6.4 do show changes: Webern deploys these pc sets differently across the movements of the corpus, even while the overall frequency distribution remains stable.

That his pc set vocabulary is dominated by sets including semitones is no surprise, and simply expands upon the findings of Chapter 5. Nonetheless, the frequency distributions shown in Figure 6.2 remain dramatic, with semitonal pc sets far outranking those without. This semitonal quality leads to the DFT results. Notably, the DFT results, and thus harmonic qualities, differ at this structural level compared to those presented in Chapters 4 and 5. I will return to this subject more extensively in Chapter 7, but the comparative significance of $|f_4|$ and then insignificance of $|f_6|$ is important for characterising Webern's pc set tendencies. This likely ties into the preference for semitones mentioned above: as soon as a semitone is used it minimises $|f_6|$. As for the dodecaphonic music, this largely continues the patterns seen in Chapter 5, although the relative insignificance of invariance in predicting the relationship between row and music is a new and rather unexpected result.

Chapter 7

Conclusion

This thesis began by conceptualising style. I proposed that a movement could be thought of as being described by a number of variables. In so doing, it can be positioned in space, and so comparisons can be drawn between it and its peers. What I have attempted to do over the last four chapters is explore a few such variables, in order to consider how Webern's music might be thought of stylistically. The reader might now expect me to mash these variables together, to create the sort of hypothetical mega-graph I described in the introduction. If that is the case, then I apologise for such a graph will remain firmly hypothetical. This is for a few reasons, but they stem from the reality that to do so is a much more complex endeavour than it might sound. Firstly, the variables need to be chosen. The chapters above have often used a variety of summary statistics to describe the data they discuss, and it is not obvious which of these should be included. Take pc distributions: should a piece's distribution be described by its range? Its IQR? Perhaps its median? Maybe all three, or even simply the twelve variables of the distribution. At least all those variables describe the same basic data. It is certainly not clear how or if more distant variables, like dispersion metrics, for example, should be included. As a secondary part of this, the analyst has to consider whether the different variables are truly independent from each other, or whether they are describing related phenomena. An obvious example of this comes with the combination of pc distribution and DFT: as a reversible transformation, in some sense the squared DFT magnitudes of a movement's distribution are indistinguishable from the twelve frequency values that describe the distribution in simpler terms. Had all this been done, then next it would be necessary to weight the variables. This poses serious analytical problems: intrinsically it implies that the analyst is able to assess the comparative significance of each of the different variables employed in the model and assign them a numerical value. At this point, a more statistically adept reader might wish to wipe away these concerns as mere excuses. After all, techniques like Principal Component Analysis exist precisely to reduce high numbers of variables into a more manageable set of dimensions. Likewise, the independence of variables can be established statistically. Weighting variables is standard practice in demographic surveys and other statistical projects. All of this is true, but it is a task beyond the scope of a conclusion. Instead, I will conclude by identifying a few of the large-scale patterns that these different chapters have identified, and then offering some thoughts on where this project might go next.

The most significant trend this thesis has identified is the importance of the mid-period Lieder as the major area of change in Webern's practice. This exemplifies the importance of the exhortation to avoid the 'Webern canon', which tends to ignore these works, for a more accurate telling of history. Chapter 3 demonstrated what a radical shift took place in Webern's music years before he adopted dodecaphonicism, both in terms of pc distributions and pc circulation. This supports Shreffler's contention that the instrumental miniatures constituted a 'crisis' that required radical change, and indeed that change is obvious in many of the figures presented above. This is a phenomenon particularly demonstrated in the whole-movement macroharmonies discussed in Chapter 3 and the start of Chapter 4 (Figures 3.6, 3.7, and 4.1). The shift here can likely be ascribed to a single (conscious or subconscious) motivation, the increasing desire for the equalisation of pcs. This motivation is also manifested in the structural use of the total chromatic, as I explored in Chapter 3 Section 3.4. Here, typical statements of the total chromatic make up a smaller proportion of a movement after the Op. 12 boundary than before. Although there is change with the advent of dodecaphony, this is minor when compared to the shift with the onset of Webern's mid-period Lieder. What is more, other areas of this thesis have also shown alterations at this point, even when seemingly unrelated to chromaticism. In Chapter 5, both Figures 5.10 and 5.16, which plot, respectively, correlations between vertical and linear interval distributions, and intra-movement interval dispersion, have this same sort of change. In the former, I pointed out previously that there is no marked change with Op. 17 (i.e. dodecaphony), but there is a gradual increase in inter-domain correlations, and there is something of a change with the start of the Lieder, though this mainly comes from negative correlations becoming much rarer, rather than an increase in the values of the positive correlations. Although the median value for that latter half of the corpus will be higher than the former, the range will also be much smaller, suggesting that Webern achieved this level of integration in earlier movements, he just did other things too, whereas later on his practice was more consistent. For the intra-movement interval dispersion, both linear and vertical domains showed a clear decline in concentration across the corpus, such that intervals in the later music tend to be more evenly spread out than in the earlier music. In Chapter 5 Section 5.3.2 I demonstrated that the intervallic content of the row predicts the intervallic content of the music, whichever topographical option Webern chose; coupled with the evidence from the pc circulation that statements of the total chromatic became briefer in the later music, it would be easy to ascribe this feature to the advent of dodecaphonicism in Webern's music. As with Figure 5.10, there certainly is a change at Op. 17, but there is also a shift earlier, at this aforementioned Op. 12 boundary. These results describe at least two distinct areas of Webern's practice: on the one hand interval distributions and their intra-movement dispersion, and on the other a constellation of variables describing his chromatic practice. I cannot see any way in which there can be a dependent relationship between these two; that there is a correlative relationship must stem from a third variable, Webern's changing preferences, and this comprehensive empirical evidence shows exactly when this happens.

At various points during this thesis I have referred to the 'Webern canon'. I gave a rough summary of this in Chapter 1, highlighting the much-discussed instrumental works at the cost of the Lieder. I suggested that there were two principal reasons for this: performance practice and the traditional hierarchy of genres. Across my own work I have singled out various movements for more detailed treatment. These case studies have often been chosen either because they are an unusual extreme, or because they are the best example of a particular trait in Webern's practice. Indeed, providing this sort of broader context is one of the great strengths of a corpus study. Although I have sometimes considered some of these neglected works, Op. 10/iii in Chapter 3, Op. 31/vi in Chapter 5, or Op. 18/i in Chapter 6, I have also assessed some of the classics of the canon: Op. 5/iv and Op. 27/ii in Chapter 4, and Op. 9/v in Chapter 5. It is notable that selection of all three of the latter movements was due to the first criterion, that they provide an unusual extreme. For Op. 5/iv it was unusually high levels for squared DFT magnitudes that prompted further analysis; Op. 27/ii was selected due to its high level of variation with regard to squared DFT magnitudes; as for Op. 9/v, it was very high levels of intra-movement interval dispersion that merited detailed investigation. With the exception of Yust's (2015a) sole DFT analysis of Op. 5/iv, in each of these cases I was applying the given analytical technique to this music for the first time. Nonetheless, that these movements have attracted so much analytical focus must surely be down to what I can now demonstrate is, in each case, their status in the corpus as unusual from the standpoint of at least one musical feature. I mentioned that Op. 9/v is what we think of as 'classic Webern', but in fact this demonstrates that, if anything, the inverse is true! This is not to write off the vast majority of Webern scholarship as lacking in value, but rather to make the point that without a broader context the analytical literature loses a huge amount. Understanding these works is not possible *only* in a comparative manner, but I do propose that the isolated fashion that analysts have used up to this point does leave a rather malnourished picture, which could certainly be fattened up by wider investigation of style *as is* rather than as it tends to be described.

On the whole, I have avoided singling out Webern's dodecaphonic music. Indeed, one of the major claims this thesis makes is for greater continuity in his practice, and this is demonstrated nowhere better than in the gradual transition he made from free organisation of atonal music in dodecaphonic composition discussed above. I referred to Shreffler's historical narrative of the haphazard way in which Webern approached adopting dodecaphony, and this is certainly borne out by the empirical evidence presented above. Nonetheless, exploring the predictive relationship between row and music is another novel contribution of this research. The basic finding here, that instances of intervals, triads, and tetrads in a row directly relate to their frequency in resulting music, is unsurprising, although it emphasises the importance of row construction. Indeed in this context it is notable that the frequencies of ics in the 19 rows Webern uses clearly reflects his usual preference for the semitone, taking up over 40% of the ics in the rows.[1] I am hesitant to linger on this for too long, as row construction has been explored thoroughly in the analytical literature. More originally, this research points to the significant impact that topography had, and which seems to have gone under-appreciated in previous analyses. Conversely, I briefly considered invariance in Chapter 6 Section 6.3.5, which seemed to have little impact on this predictive relationship: limiting the number of pc sets in the row did not appreciably increase the predictive relationship between row and music. Similarly, the example of intersection in Op. 21/i, explored in Chapter 3 showed that this had little effect on overall pc distributions. There are certainly other features of the rows in which the possibility of predictive relationships could be explored, canons and symmetries are just two minor examples, and as I discuss below, this could certainly be expanded to other serial music. Webern described the construction of a new row as happening 'when an idea occurred to us, linked

1. Frequencies as follows: ic1, 93; ic2, 11; ic3, 47; ic4, 45; ic5, 16; ic6, 16.

with an intuitive vision of the work as a whole; the idea was then subjected to careful thought' (Webern 1963, 54), and he went on to compare this process to the development of themes in Beethoven's sketchbooks. After constructing the row, in his words, composition became 'free fantasy' (Webern 1963, 55): it is this fantasy that needs further investigation.

Another major contribution of this thesis has been assessing the intervallic qualities of Webern's harmony, simply in terms of its intervallic preferences, but also in their resulting larger-scale effects, as articulated through pc sets and squared DFT magnitudes. That the semitone is important in Webern will come as a shock to nobody, but it is helpful to have some empirical backing for this long-held assumption. As I discuss below with regard to the corpus, it also sets up a method and an example that could now be the basis for comparative study. Not only is it interesting that Webern's music is so obviously characterised by the semitone, but it is a notable finding in general that his music is consistently characterised by a single ic. Whether this is true for other composers remains open for exploration. More novel information comes from considering the various harmonic levels this thesis has explored. In particular, it is rather unexpected that there are discrepancies between them, rather than necessarily always displaying the same phenomena. I have explored harmonic colours as expressed in the whole-movement macroharmonies, the five-second windows, and the pc sets. I demonstrated in Chapter 4 that the large-scale quality of his harmony was dominated by a quartal quality, far above the octatonicism that Forte hypothesised. As I explored in Chapter 6, this octatonicism is relevant to the pc sets that are Forte's concern, though not perhaps as all-encompassing as he would wish: while it is the strongest squared magnitude for medians of vertical pc sets, it is outflanked by the quartal colour in the linear pc sets, which also ranks strongly in the vertical domain. Picking outright winners is perhaps a bit crass, but integrating these different perspectives is certainly a valuable endeavour. To do so, I have four datasets, each with six values per movement: 1) a set of squared magnitudes for whole-movement pc distributions; 2) a set of the median squared magnitudes for five-second windows; 3) a set of squared magnitudes for linear pc sets; 4) a set of squared magnitudes for vertical pc sets. Having explained at length the difficulty of combining different datasets at the outset of this chapter, that is actually what I am going to do now, but because of the similarity of the data and its collection, this is a much easier task than integrating conceptually different types of data. For each movement in each dataset, I first calculate the proportion that each squared magnitude comprises, thus normalising for total squared magnitude power (as in Table 4.2). Although this discounts the difference in squared magnitude strength between movements, it better takes account of the listening experience of each movement as an individual entity: the, say, octatonic value of Op. 3/ii is not considered in relation to the octatonic value of the other 106 movements, but rather in relation to the five other squared magnitudes in that movement. The only weighting decision I make is to weight the linear and vertical pc sets at half the strength of the pc distributions and five-second windows as they describe the same level of harmonic structure. Although there is clearly a difference between the two pc set datasets, I conceptualise this aggregate description of each movement as encompassing three harmonic levels, pc distributions, five-second windows, and pc sets, and it is these that I want to compare, hence combining the third and fourth sets of data into one.[2] That therefore creates four new datasets, and for each movement I then sum the value for each squared magnitude, and then express it as a percentage, providing a final dataset with six values per movement. The median values of these squared magnitudes are given in Table 7.1. The results are unequivocal: on this basis, $|f_2|$ is clearly the most significant squared magnitude, and indeed $|f_4|$ pales into fifth place. While it may be the case that vertical pc sets

2. As I hinted at above, this sort of weighting procedure could be treated in a far more complex manner: an analyst might view the sort of macroharmony described by a whole-movement pc distribution as of secondary importance to the other harmonic levels, or even rank the three by their level of detail. Additional hierarchical levels could be included, most obviously by adding different window sizes, but perhaps also by considering bigram or trigram progressions, or if some method was formulated to take account of the diagonal segmentations discussed in Chapter 5 Section 5.2.

are coloured octatonically, other harmonic levels favour other components, and the overall effect is primarily quartal.

Component	Value
$\lvert f_1 \rvert$	10.00
$\lvert f_2 \rvert$	37.16
$\lvert f_3 \rvert$	29.82
$\lvert f_4 \rvert$	7.76
$\lvert f_5 \rvert$	8.80
$\lvert f_6 \rvert$	6.50

Table 7.1: Median multi-level squared DFT magnitudes.

Further, I can consider the correlations in each movement between these datasets, to establish how similar these different harmonic levels tend to be. As there are four groups of data for each movement, this generates six correlation values (between each possible pair). Taking the median of these six then indicates how similar, overall, the correlations are. These values are shown in Figure 7.1. Clearly the vast majority of movements have positive correlations, indicating similarity between squared magnitudes across harmonic levels. Although there is a group of 17 movements with strong positive correlations above 0.5, the overall median of the values in Figure 7.1 is only 0.2, surprisingly weak. The correlation between these median values and corpus position is -0.03. This suggests that on average there is little similarity between harmonic colours across different levels, and there is no change across the corpus. Indeed, comparing the six correlation values for each original pair of datasets, it is between the 5-second windows and the linear pc sets that there is the strongest correlation, but this remains merely 0.42, moderately strong. As usual, without a reference corpus there is no way to contextualise this empirically, but my suspicion is that this would prove to be an unusual finding when compared to earlier music (certainly we would expect high $\lvert f_5 \rvert$ across all domains), though perhaps not to other music of Webern's contemporaries. When I discussed archetypal pc sets from Common Practice Period music in Chapter 3, I pointed out that they are characterised by clear diatonic patterning. In the language of the DFT, this would therefore prompt

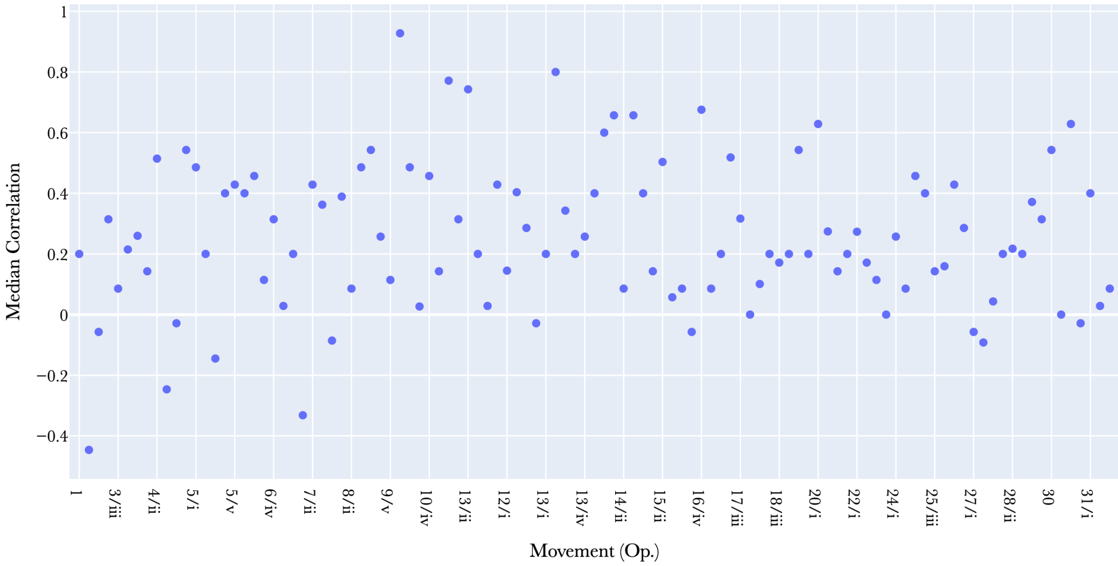

Figure 7.1: Median multi-level squared DFT magnitude correlations.

high values for $|f_3|$ and $|f_5|$. Yust's (2019) work on pc distributions has suggested that this tendency is generally true both for whole-movement pc distributions, and for selected extracts from the beginning and end of works in his corpus. Although he finds increases and declines in various components, the hierarchy tends not to change (one small exception to this is that in minor-mode piece endings, $|f_3|$ and $|f_5|$ swap ranks in c.1820 (Yust 2019, 9)). 20-beat segments from the beginnings and ends of pieces are certainly not equal in analytical value to comprehensive window analyses, but they do provide something of a rough equivalent to my middle level. In the case of pc sets, conventional diatonic vertical harmonies would typically be expected to emphasise these same components. If equivalent linear analysis was deemed applicable, a contestable contention, we might expect high $|f_5|$ to remain relevant, though similarities in other squared magnitudes might be less strong. I would hypothesise, then, that an equivalent graph to Figure 7.1 for a corpus of Common Practice Period music would show much higher and more consistently positive values.

Returning to Figure 7.1, there are movements with the median correlation value from across the corpus, ranging all the way from Op. 1 to Op. 29/ii. Op. 13/i (Table 7.2) sits right in the middle of these and demonstrates the typical situation:

little similarity between harmonic levels. $|f_3|$ tends to be comparatively important: with the exception of the 5-second windows it is always in the top three components; conversely $|f_6|$ is the weakest squared magnitude three times, and second weakest for whole-movement distributions. Nonetheless, there is significant variety both amongst the comparative strengths of squared magnitudes in each movement (from the massively varied 5-second windows to the comparatively concentrated linear pc sets) and between their rank orderings.

Component	PC Distribution	5-Second Windows	Vertical PC Sets	Linear PC Sets
$\|f_1\|$	25.54	8.89	16.45	19.17
$\|f_2\|$	13.11	85.21	16.81	15.41
$\|f_3\|$	48.36	1.30	17.13	18.84
$\|f_4\|$	6.63	2.24	21.93	15.41
$\|f_5\|$	1.67	1.24	17.40	19.17
$\|f_6\|$	4.68	1.11	10.27	11.99

Table 7.2: Op. 13/i squared DFT magnitude proportions.

The dispersion analysis, particularly on the intra-movement level, seems to me an area that merits much further enquiry, and could be applicable to a much wider repertoire. I explained above why I divorced the segmentation of movements from any structural boundaries particular to them, but this was a requirement of quite specific circumstances. For works with clear structural divisions there is no *a priori* reason not to segment them along these lines. This might need to be followed by sampling from within these segments rather than creating full distributions to avoid problems caused by different section lengths, although normalisation might be able to take care of this: it would depend on the analytical motivation. In Webern this might create issues of statistical power given the small frequencies involved, but in less terse repertoire that should not be an issue. In particular, I have speculated that it is a combination of dispersion and frequency that creates a perception of a repertoire as characterised by a given property (be it an interval, pc set, or other entity). In Webern's music, I have suggested that on the inter-movement level it is low dispersion values combined with high frequencies that seem to be typical of those entities seen as typically

Webernian, as demonstrated both by the semitone and the Viennese Trichord. A valuable next step might be to coordinate such frequency and dispersion analysis with some cognitive research. I deployed Brown's (2020) work for its analytical inspiration, but the majority of her paper is actually cognitive, exploring listeners' perception of repetitive intervals in dodecaphonic music. This intersection of quantitative analysis and cognitive study is ripe for further investigation in a number of areas.

Looking ahead, the most obvious path forward for this sort of analysis is the eternal cry for ever-bigger corpora. I described in Chapter 2 the difficulties attending to the construction of corpora, especially in repertoire like that under discussion here. And yet, I hold out hope that advances in Optical Music Recognition might make this increasingly feasible. In the first instance, I would compile a corpus of seemingly 'similar-sounding' composers: Schoenberg and Berg, obviously, but also Müller-Hermann, Hauer, Schmidt, Zemlinsky. I have tried to avoid complaining about the limitations of this corpus, but it must be acknowledged that at times throughout this thesis the research has been fundamentally limited by the inability to compare it to other music. I have endeavoured to be careful in my phrasing, avoiding comparative claims for which I have no evidence, and therefore not overstepping. Nonetheless, many of the techniques I have used could easily be expanded to a larger corpus and have the potential to provide fascinating results in the process. The most obvious example of this is the application of keyword analysis in Chapter 6. While it is certainly valuable to establish the primary pc sets in Webern's language, it would be stimulating to compare his pc set vocabulary to those of his peers, perhaps most obviously Berg. Likewise, applying some of the dodecaphonic analysis to other composers using the same technique (Stravinsky or Lutyens, for example) would help identify further quite what are the implications of adopting this compositional approach. Yust and Gotham have suggested that there is something of a 'Webern effect' in the harmonic qualities of rows constructed by Boulez, Lutyens, Yun, and others (Gotham and Yust 2021, 39). They find high

$|f_1|$ and low $|f_4|$, $|f_5|$, and $|f_6|$ in hexachordal partitions in the rows and hypothesise that this was a characteristic first developed by Webern and followed by these other composers. It would be fascinating to apply some of the DFT analysis explored in this thesis to a body of dodecaphonic music, not just rows, by these composers, and establish whether, firstly, the same relationship holds between row contents and music, and if so whether that creates similar harmonic effects on multiple structural levels to the music of Webern.

Remaining focussed on Webern, a largely unexplored feature of the corpus is the level of similarity between different genres in his output. Bailey notes that from a topographical perspective the vocal and instrumental music is quite differentiated, and indeed suggested that 'his innovations and experiments take place in the instrumental, and choral and instrumental music; the sets of songs for solo voice are much simpler in conception' (Bailey 1991, 32). Putting aside for the moment whether this can be an accurate claim given the innovation that took place in the mid-period Lieder, the point stands that generic difference is worthy of consideration. An obvious place to start would be by investigating the difference between vocal writing and instrumental writing. My hunch would be that over time they became more similar, at least from the perspective of interval distributions, and indeed that this 'instrumental' approach to the voice, particularly in the dodecaphonic music, contributes to its difficulty and relative lack of performances. Indeed, my suspicion is that the increase in interval size identified by Boulez and explored in Chapter 5 Section 5.4 may be particularly extreme in vocal writing, where these sorts of large intervals are atypical in earlier music, and indeed often deemed unidiomatic. Expanding the very brief reference to textural features that I made in Chapter 6, it could also be worthwhile investigating whether register affects interval size (instinct would suggest that, in line with the harmonic series, lower intervals would tend to be larger), and again how and when this changes.

Overall, this thesis stands as an example of the possibilities in music analysis for quantifying features of interest and exploring them empirically. Through the adaptation of borrowed techniques from other fields I aim to have explored a variety of phenomena many of which are not obviously numerical in origin. As just one example, adapting an economic metric to measure the concentration in intervallic presentation may well not have much of a future elsewhere. Nonetheless, it exemplifies the possibilities in adopting techniques from other disciplines to answer musical questions, an approach that other researchers will surely continue. I discussed the importance of critique in Chapter 2. The context there was the tendency of DH practitioners to lean towards building, excited by the possibilities of creating and eschewing reflective criticism in the process. Despite being firmly digital, this thesis has remained fundamentally critical throughout: although I have developed various tools to probe this repertoire, the goal has been throughout to comprehend better the music of Webern. In the first lecture from the series that became *Der Weg zur Neuen Musik*, he suggested that art is 'based on rules of order … which we … can only aim at proving' (1963, 10). While that aim will remain eternally unfinished, I hope this thesis has helped inch slightly closer, and point the way for future steps.

Appendices

Appendix A

Chronological List of Works

Title	Opus	Movement	Position
Passacaglia	1		1
Entflieht auf leichten Kähnen	2		2
Fünf Lieder aus “Der Siebente Ring”	3	i	3
		ii	4
		iii	5
		iv	6
		v	7
Fünf Lieder nach Gedichten von Stefan George	4	i	8
		ii	9
		iii	10
		iv	11
		v	12
Fünf Sätze für Streichquartett	5	i	13
		ii	14
		iii	15
		iv	16
		v	17
Sechs Stücke für grosses Orchester	6	i	18
		ii	19
		iii	20
		iv	21
		v	22
		vi	23
Vier Stücke für Geige und Klavier	7	i	24
		ii	25
		iii	26
		iv	27
Zwei Lieder nach Gedichten von Rainer Maria Rilke	8	i	28
		ii	29

Fünf Stücke für Orchester	10	i	30
Sechs Bagatellen für Streichquartett	9	iii iv v vi	31 32 33 34
Fünf Stücke für Orchester	10	ii	35
Sechs Bagatellen für Streichquartett	9	i	36
Fünf Stücke für Orchester	10	iv iii	37 38
Sechs Bagatellen für Streichquartett	9	ii	39
Fünf Stücke für Orchester	10	v	40
Vier Lieder für Sopran und Kammerorchester	13	ii	41
Drei Kleine Stücke	11	i ii iii	42 43 44
Vier Lieder	12	i iii iv ii	45 46 47 48
Vier Lieder für Sopran und Kammerorchester	13	ii	49
Sechs Lieder nach Gedichten von Georg Trakl	14	iv	50
Vier Lieder für Sopran und Kammerorchester	13	iii	51
Fünf Geistliche Lieder	15	v	52
Vier Lieder für Sopran und Kammerorchester	13	iv	53
Sechs Lieder nach Gedichten von Georg Trakl	14	iii vi v ii i	54 55 56 57 58
Fünf Geistliche Lieder	15	i iii ii iv	59 60 61 62
Fünf Canons	16	ii iii iv v i	63 64 65 66 67
Drei Volkstexte	17	i iii ii	68 69 70
Drei Lieder für Sopran, Klarinette und Gitarre	18	i ii iii	71 72 73

Zwei Lieder	19	i ii	74 75
Streichtrio	20	ii i	76 77
Symphonie	21	ii i	78 79
Quartett	22	ii i	80 81
Drei Gesänge aus 'Viae inviae'	23	iii ii i	82 83 84
Konzert für neun Instrumente	24	i	85
Drei Lieder nach Gedichten von Hildegard Jone	25	i	86
Konzert für neun Instrumente	24	ii iii	87 88
Drei Lieder nach Gedichten von Hildegard Jone	25	iii ii	89 90
Das Augenlicht	26		91
Variationen für Klavier	27	iii i ii	92 93 94
Streichquartett	28	iii i ii	95 96 97
Kantate I	29	ii i iii	92 93 94
Variationen für Orchester	30		101
Kantate II	31	iv v vi i ii iii	102 103 104 105 106 107

Appendix B

Pitch Class Distributions

Movt.	0	1	2	3	4	5	6	7	8	9	10	11
1	5.92	10.04	12.23	7.25	7.96	9.38	8.47	6.10	5.81	11.33	11.93	3.58
2	7.31	4.93	13.74	6.06	8.55	1.00	15.15	10.09	6.48	8.48	9.51	8.70
3/i	3.69	8.39	13.06	11.12	11.93	5.74	5.30	4.78	10.70	3.74	13.27	8.29
3/ii	5.81	8.44	8.74	9.24	11.55	11.26	8.70	6.44	10.37	8.85	4.77	5.84
3/iii	6.89	8.52	5.72	12.10	10.03	9.38	8.05	3.63	9.07	9.65	7.39	9.57
3/iv	7.58	6.62	5.93	6.49	6.18	8.57	10.06	11.10	11.34	5.88	9.75	10.51
3/v	6.88	9.93	12.17	10.75	14.88	10.23	4.31	4.84	3.76	7.14	9.49	5.62
4/i	5.84	9.77	9.70	8.85	14.77	10.79	5.99	6.90	6.48	3.73	6.86	10.31
4/ii	10.05	12.02	9.73	8.81	6.11	8.32	8.70	7.73	9.72	6.81	4.53	7.45
4/iii	7.61	7.38	6.19	13.09	6.66	12.09	7.05	8.17	8.40	7.05	6.90	9.41
4/iv	8.55	6.20	4.75	4.98	12.68	11.30	8.67	9.23	12.25	6.44	2.56	12.38
4/v	7.60	8.81	10.48	4.68	4.61	3.94	6.88	10.29	13.34	9.62	10.55	9.21
5/i	7.95	10.25	7.25	7.12	10.36	7.50	8.78	10.07	9.45	6.97	6.53	7.76

5/ii	11.63	6.44	5.37	11.09	10.38	11.99	8.59	6.98	3.94	12.52	3.04	8.05
5/iii	7.49	18.11	3.98	8.49	8.17	4.37	5.96	9.86	8.18	6.91	9.17	9.33
5/iv	15.39	7.84	6.42	4.88	14.62	7.94	15.43	4.01	2.03	0.91	3.60	16.94
5/v	15.74	16.54	2.91	4.67	7.48	10.40	5.90	8.25	8.41	2.33	9.76	7.60
6/i	5.77	6.63	6.52	8.23	7.67	8.93	6.21	3.80	7.89	10.35	14.00	13.98
6/ii	8.32	9.30	6.97	6.98	6.32	10.48	10.15	8.55	9.37	6.47	8.32	8.76
6/iii	6.92	4.24	9.15	4.46	12.05	6.62	17.18	8.37	10.93	10.26	7.92	1.90
6/iv	10.17	5.97	5.16	11.89	10.08	12.55	4.93	9.23	10.39	4.33	8.48	6.82
6/v	10.05	9.85	5.23	6.61	8.71	8.79	6.78	7.87	9.81	10.19	8.47	7.64
6/vi	8.56	8.13	6.06	6.38	10.52	11.86	9.94	5.98	4.51	3.42	14.26	10.35
7/i	5.44	9.62	5.44	17.44	8.61	9.29	6.12	4.08	6.34	12.91	7.70	7.02
7/ii	12.26	9.90	7.09	10.03	7.66	6.55	5.06	7.91	6.13	11.12	7.73	8.56
7/iii	13.57	7.51	2.59	8.33	7.02	8.79	7.99	5.75	6.24	12.02	11.88	8.30
7/iv	6.13	12.90	9.64	11.52	5.05	4.97	5.62	8.20	15.29	5.13	4.74	10.80
8/i	11.23	9.07	8.74	7.66	6.42	6.18	7.85	6.97	9.31	8.09	9.31	9.17
8/ii	8.73	8.51	5.46	7.85	9.45	11.46	7.92	9.92	7.66	6.38	7.32	9.36
10/i	7.11	5.96	4.70	6.02	1.61	6.77	2.29	7.11	19.95	22.82	7.11	8.55
9/iii	7.05	9.02	8.67	3.98	6.10	6.55	12.88	15.11	4.18	9.86	10.74	5.86
9/iv	5.01	6.85	6.12	4.45	11.88	6.68	16.70	13.36	6.12	5.57	12.25	5.01
9/v	8.93	4.96	3.97	6.94	13.88	8.59	9.92	8.10	9.58	4.63	5.95	14.55
9/vi	10.53	6.85	6.85	10.66	7.99	3.81	7.61	11.17	13.45	8.88	6.73	5.46
10/ii	8.27	9.44	9.54	9.15	5.72	4.31	8.89	12.11	9.07	6.01	7.07	10.41
9/i	9.27	12.57	9.50	6.65	5.68	6.43	12.39	8.00	5.62	7.39	8.62	7.87
10/iv	9.93	11.35	3.50	3.03	4.68	7.33	9.22	2.36	10.12	15.60	13.90	8.98
10/iii	6.96	12.30	11.76	3.94	34.05	7.53	1.09	1.09	6.54	6.23	6.85	1.66
9/ii	14.02	5.84	8.46	12.44	8.51	8.73	5.68	6.06	4.75	6.77	7.42	11.35
10/v	8.63	8.34	11.97	6.16	7.01	7.39	7.01	10.52	6.38	11.66	9.64	5.28

13/ii	7.37	9.29	8.94	9.88	8.81	7.57	7.37	6.15	6.76	7.81	9.99	10.06
11/i	6.47	6.97	14.18	5.97	10.28	7.96	7.21	7.80	8.78	9.45	5.97	8.96
11/ii	6.10	9.61	9.72	6.32	8.13	8.36	8.59	9.04	8.36	7.46	13.56	4.75
11/iii	12.46	5.19	7.27	11.07	6.92	15.57	11.07	8.30	1.38	1.04	10.73	9.00
12/i	7.72	8.25	6.96	7.01	6.64	10.08	11.28	7.25	9.04	9.95	7.10	8.72
12/iii	7.66	7.78	6.64	8.54	10.90	9.15	7.91	7.54	9.26	7.49	6.68	10.43
12/iv	9.46	7.30	9.45	8.07	8.11	7.01	7.81	7.51	9.15	8.75	10.52	6.86
12/ii	6.29	9.03	9.78	10.57	6.28	9.69	8.14	6.85	7.04	9.95	8.74	7.62
13/i	8.76	10.01	9.74	6.96	7.53	8.56	8.06	7.63	7.86	8.29	8.85	7.76
14/iv	10.61	8.64	8.06	8.36	7.62	9.65	9.81	6.95	7.45	6.83	7.34	8.68
13/iii	7.83	9.42	8.71	7.39	7.52	10.13	7.83	8.71	9.49	7.64	6.82	8.51
15/v	9.59	8.10	9.12	7.19	7.57	9.94	8.54	7.22	7.74	8.62	8.45	7.92
13/iv	8.25	8.82	9.81	8.69	6.22	8.38	8.22	9.05	7.11	8.67	8.77	8.02
14/iii	8.61	9.39	7.32	8.58	8.79	8.34	9.99	7.62	6.75	7.59	8.34	8.73
14/vi	8.25	9.40	5.99	5.90	7.85	7.26	8.74	8.19	10.73	9.82	9.18	8.69
14/v	10.37	8.54	8.56	6.11	6.83	6.61	8.07	9.13	10.05	8.75	8.35	8.63
14/ii	7.82	7.23	8.50	7.13	10.42	8.42	8.67	8.52	9.62	8.72	5.88	9.06
14/i	10.75	10.13	8.90	9.03	7.52	7.44	10.15	8.20	7.25	5.48	6.15	8.98
15/i	6.42	8.77	8.25	9.72	6.03	8.63	7.63	8.93	8.55	9.89	8.05	9.13
15/iii	8.16	8.23	7.41	8.16	7.31	9.01	8.95	7.44	9.34	10.03	6.13	9.83
15/ii	8.50	8.24	7.28	7.17	10.73	9.33	7.57	7.09	8.23	7.91	7.64	10.31
15/iv	7.38	6.67	7.26	7.86	9.29	6.31	10.24	10.36	8.10	8.81	11.43	6.31
16/ii	8.99	11.06	8.99	7.37	8.29	7.14	7.83	9.68	7.83	7.14	8.29	7.37
16/iii	8.15	10.48	7.13	8.73	6.84	6.84	9.75	9.32	9.17	8.73	7.13	7.71
16/iv	12.83	6.96	8.94	8.55	6.96	5.70	7.62	8.39	5.76	7.40	8.06	12.83
16/v	9.30	9.55	7.77	7.22	6.13	9.86	8.93	7.04	8.00	8.05	10.70	7.45
16/i	7.89	12.77	10.59	8.41	5.92	4.67	5.61	7.17	11.21	10.07	8.93	6.75

17/i	6.23	6.63	7.50	9.44	10.42	6.76	8.29	9.92	10.39	8.81	7.26	8.34
17/iii	8.78	12.99	9.56	9.40	6.01	6.48	8.06	7.69	8.33	9.15	6.72	6.83
17/ii	6.98	9.77	8.68	7.87	7.07	9.76	9.74	6.13	6.18	8.70	9.82	9.30
18/i	7.29	7.56	8.25	8.59	8.66	7.97	8.73	7.97	9.62	8.11	9.62	7.63
18/ii	9.04	7.35	8.57	7.58	7.37	7.28	8.14	7.84	9.55	9.06	8.96	9.26
18/iii	9.24	7.52	7.75	8.17	8.95	8.68	8.40	8.43	7.76	8.87	7.81	8.41
19/i	7.83	8.76	8.12	9.44	8.93	8.18	9.90	7.50	7.85	8.07	7.68	7.72
19/ii	7.47	7.19	8.04	8.27	6.67	8.04	9.07	10.44	8.73	8.78	9.53	7.76
20/ii	7.31	8.04	7.13	10.01	9.02	8.15	8.40	8.58	8.91	7.88	9.11	7.44
20/i	12.83	9.36	6.43	8.18	8.44	8.93	8.46	8.34	6.33	8.50	7.41	6.80
21/ii	8.79	9.36	9.13	6.76	7.35	8.09	9.41	8.21	8.92	7.33	7.15	9.49
21/i	9.80	7.75	8.28	8.28	10.86	7.30	7.93	8.73	9.44	6.95	8.73	5.97
22/ii	6.77	8.88	8.13	9.58	9.59	7.04	7.65	8.81	8.82	7.57	8.26	8.90
22/i	7.42	9.24	7.73	7.73	8.18	8.18	8.48	8.48	10.00	7.58	7.88	9.09
23/iii	8.21	8.00	7.98	8.55	8.83	7.45	9.31	11.66	7.71	7.73	7.13	7.45
23/ii	8.46	8.61	9.07	7.45	6.76	8.30	7.22	9.07	7.84	9.45	9.53	8.23
23/i	7.65	8.43	8.12	9.92	8.88	7.20	8.43	7.44	9.28	8.17	8.52	7.98
24/i	8.42	7.24	8.15	9.65	7.63	7.64	8.60	9.16	8.33	8.56	8.54	8.07
25/i	8.34	8.35	8.20	5.60	7.80	8.41	9.40	7.72	10.52	8.67	8.20	8.79
24/ii	8.25	7.44	6.04	5.73	7.14	11.07	9.15	8.15	8.35	10.97	9.05	8.65
24/iii	8.33	7.90	7.73	8.61	7.65	9.00	8.81	7.73	8.93	8.73	8.69	7.89
25/iii	7.61	6.80	8.47	10.12	9.11	5.75	9.69	10.13	7.43	8.91	8.03	7.94
25/ii	8.20	9.15	7.29	8.98	9.90	8.54	8.17	8.76	8.13	6.90	7.93	8.05
26	8.58	10.38	6.67	8.15	8.03	7.03	8.34	7.61	8.48	8.93	9.66	8.13
27/iii	8.10	10.40	9.91	9.78	7.43	7.99	8.12	8.83	8.11	5.79	7.55	7.99
27/i	8.58	9.49	8.76	7.85	8.76	8.94	8.76	7.85	8.76	8.03	7.85	6.39
27/ii	12.05	8.43	8.43	2.41	8.43	8.43	12.05	8.43	6.63	9.64	6.63	8.43

28/iii	9.04	7.84	8.27	8.08	7.74	9.19	8.75	6.78	7.94	8.95	8.71	8.71
28/i	8.50	7.87	8.12	7.93	8.14	7.88	8.11	7.90	9.42	9.15	7.50	9.48
28/ii	7.32	7.81	7.87	8.98	8.18	8.07	9.19	8.25	8.64	8.46	8.67	8.56
29/ii	8.47	7.82	8.19	8.06	7.96	9.84	7.82	8.23	7.94	8.39	8.57	8.71
29/i	8.97	10.63	9.84	7.93	8.62	7.14	8.01	7.75	7.93	6.71	7.84	8.62
29/iii	8.05	8.30	8.22	7.14	8.07	8.78	8.70	8.80	8.45	7.25	9.05	9.20
30	8.22	9.56	8.50	8.48	7.80	8.51	9.20	7.40	7.48	8.50	8.67	7.68
31/iv	6.86	10.29	9.80	9.31	6.37	7.84	6.37	9.31	8.33	8.33	8.82	8.33
31/v	8.24	9.67	7.60	8.59	9.64	8.09	7.90	8.86	9.08	7.53	6.57	8.24
31/vi	5.63	10.74	6.14	10.49	5.88	12.53	6.65	9.21	5.12	9.21	6.65	11.76
31/i	8.52	5.78	10.23	8.69	6.40	9.56	6.31	7.04	6.80	10.10	11.76	8.81
31/ii	7.52	8.87	8.72	9.77	8.72	9.02	6.92	7.07	8.57	7.97	8.27	8.57
31/iii	8.57	7.43	8.79	8.96	8.09	8.66	8.18	8.18	9.01	8.18	7.87	8.09

Appendix C

Pitch Class Distribution DFT Magnitudes

Movt.	$\lvert f_1 \rvert$	$\lvert f_2 \rvert$	$\lvert f_3 \rvert$	$\lvert f_4 \rvert$	$\lvert f_5 \rvert$	$\lvert f_6 \rvert$
1	9.65	75.44	358.07	19.28	20.02	21.58
2	13.88	43.15	366.62	37.81	240.02	461.63
3/i	108.70	284.87	68.05	202.56	75.43	252.23
3/ii	174.73	28.33	79.79	19.98	17.21	0.01
3/iii	17.55	120.87	28.40	38.57	102.57	32.46
3/iv	139.09	49.88	49.90	30.22	28.27	2.75
3/v	437.05	201.07	37.39	81.19	25.55	8.81
4/i	270.40	73.89	23.79	179.50	28.84	0.53
4/ii	47.92	156.76	18.41	19.89	16.03	5.29
4/iii	35.59	19.33	23.77	41.39	89.55	207.17
4/iv	114.29	116.99	313.19	124.69	85.87	1.13
4/v	284.69	170.20	8.83	52.12	1.50	47.74

5/i	14.26	31.01	27.16	34.80	19.50	0.42
5/ii	76.09	90.28	103.33	252.53	69.33	199.72
5/iii	130.86	79.08	25.17	328.69	192.29	199.49
5/iv	276.97	1292.80	126.83	32.11	352.17	224.34
5/v	169.95	473.73	246.98	170.75	226.01	0.17
6/i	176.24	355.21	29.12	37.71	34.69	14.81
6/ii	12.51	65.29	5.86	11.71	32.24	1.23
6/iii	448.28	13.25	59.85	79.73	87.61	800.92
6/iv	67.37	39.51	171.79	10.12	234.97	2.50
6/v	35.67	1.54	110.59	9.03	3.06	3.68
6/vi	11.73	457.32	44.90	84.80	45.22	59.89
7/i	72.99	355.89	12.07	167.95	90.66	429.31
7/ii	125.12	2.98	39.13	77.20	21.58	66.16
7/iii	152.04	151.51	54.46	194.90	48.85	1.98
7/iv	45.51	328.64	98.33	126.68	243.21	49.82
8/i	94.80	11.41	1.33	7.05	4.51	32.67
8/ii	22.30	77.82	27.05	14.82	6.37	47.98
10/i	1313.49	621.98	404.58	302.18	0.45	209.04
9/iii	126.81	135.45	224.35	185.04	92.42	0.57
9/iv	360.26	97.83	159.16	318.03	29.93	261.12
9/v	28.48	276.25	288.15	30.04	122.09	19.85
9/vi	44.93	181.96	176.28	50.26	13.30	40.26
10/ii	14.65	139.80	147.64	3.07	7.46	8.16
9/i	33.00	144.75	113.76	35.03	24.09	4.72
10/iv	637.91	43.25	399.69	48.70	75.00	7.23
10/iii	1178.67	671.69	1150.40	1053.05	571.23	1189.74
9/ii	208.34	92.96	105.11	92.03	57.40	5.49

10/v	12.73	62.14	73.14	15.13	154.75	1.68
13/ii	53.67	34.35	13.31	2.46	12.57	2.29
11/i	16.10	45.94	6.08	99.14	164.07	33.63
11/ii	9.57	12.42	114.26	115.95	32.60	79.82
11/iii	188.62	601.41	112.19	15.21	313.05	0.12
12/i	37.59	19.42	31.87	38.88	16.58	6.26
12/iii	9.17	26.46	47.74	11.59	25.55	3.57
12/iv	13.71	15.27	1.52	1.99	12.31	80.88
12/ii	9.66	29.53	62.89	13.73	6.72	55.46
13/i	14.03	7.20	26.56	3.64	0.92	2.57
14/iv	15.76	52.03	1.50	19.92	4.53	3.15
13/iii	3.06	15.86	8.89	30.05	0.98	12.99
15/v	1.63	8.24	20.29	9.51	10.26	4.10
13/iv	4.70	8.75	27.35	0.72	10.07	10.55
14/iii	6.87	22.26	2.40	11.28	10.80	0.20
14/vi	86.36	2.68	22.23	3.68	21.62	2.22
14/v	55.81	35.63	5.15	0.75	2.92	19.86
14/ii	18.11	0.01	23.21	11.65	40.14	3.43
14/i	54.60	92.93	10.07	9.99	6.40	2.14
15/i	6.26	16.06	8.78	6.03	2.68	102.74
15/iii	7.35	1.18	8.78	40.29	15.46	29.25
15/ii	2.04	40.95	25.49	12.32	14.90	0.01
15/iv	58.03	5.80	24.86	73.51	1.04	54.48
16/ii	13.02	41.62	0.85	35.89	0.15	0.21
16/iii	4.01	49.09	0.11	15.70	22.65	13.24
16/iv	152.65	52.52	95.04	27.30	30.10	0.11
16/v	13.28	15.45	48.79	0.14	39.55	2.71

16/i	111.95	237.80	26.97	5.92	20.66	0.10
17/i	45.67	34.49	46.15	1.86	17.14	0.04
17/iii	53.48	118.01	23.64	13.22	15.41	25.80
17/ii	7.75	23.29	88.44	0.96	10.43	9.39
18/i	7.80	10.39	1.37	0.95	7.09	18.75
18/ii	30.57	0.84	1.08	7.94	0.26	10.68
18/iii	0.58	5.26	3.96	4.11	5.59	0.03
19/i	16.03	0.06	1.29	8.98	11.38	0.41
19/ii	38.39	4.39	20.19	1.34	7.64	0.94
20/ii	9.49	10.76	4.20	7.47	15.47	0.05
20/i	13.62	50.34	40.25	67.79	23.79	0.04
21/ii	2.64	38.80	0.49	12.01	9.03	2.26
21/i	8.76	3.01	27.63	19.66	5.41	101.25
22/ii	2.86	11.55	15.64	12.19	13.08	2.44
22/i	3.48	4.84	2.39	11.23	17.55	0.37
23/iii	25.28	22.50	20.13	19.47	9.12	2.75
23/ii	20.11	2.89	10.28	1.47	14.42	4.98
23/i	1.34	15.42	2.90	1.89	15.90	3.05
24/i	2.28	2.88	12.68	8.19	2.90	0.42
25/i	29.41	9.45	11.75	15.11	8.78	24.33
24/ii	79.07	35.84	48.44	5.35	3.06	16.19
24/iii	3.90	0.87	2.06	4.84	6.20	0.08
25/iii	9.69	11.63	49.54	35.33	12.64	0.50
25/ii	14.40	2.13	9.41	13.11	4.06	0.56
26	20.85	0.06	6.25	22.60	19.42	0.22
27/iii	44.34	32.71	9.54	5.42	11.21	2.46
27/i	9.37	6.93	19.71	0.93	1.29	8.52

27/ii	16.80	191.97	52.26	17.78	107.31	71.13
28/iii	2.47	9.73	6.80	11.89	0.39	0.84
28/i	7.36	0.32	5.57	9.52	5.90	0.18
28/ii	4.95	2.20	4.61	0.89	5.51	0.08
29/ii	0.15	6.97	1.12	4.11	5.98	4.48
29/i	46.16	16.85	0.06	7.88	5.23	5.95
29/iii	2.74	10.31	2.66	10.95	0.26	1.14
30	3.82	0.69	17.36	3.77	3.19	0.07
31/iv	14.01	40.61	12.01	13.70	5.69	47.10
31/v	8.91	6.31	24.12	4.86	7.63	3.78
31/vi	7.32	31.66	8.96	11.25	5.50	777.14
31/i	34.74	69.02	43.92	14.84	69.61	0.00
31/ii	14.56	14.95	1.02	5.86	6.33	6.54
31/iii	1.41	1.56	1.62	7.32	2.21	1.01

Appendix D

Interval Distributions

Movt.	0	1	2	3	4	5	6	7	8	9	10	11
1	14.19	27.00	15.57	9.86	12.71	4.11	3.23	2.11	3.28	3.68	1.21	3.05
2	12.57	34.69	23.36	10.62	6.19	5.49	2.30	3.01	1.06	0.71	0.00	0.00
3/i	4.55	29.87	13.64	15.58	7.14	7.79	5.84	1.95	3.90	2.60	1.30	5.84
3/ii	2.14	25.21	14.10	2.56	7.69	12.39	12.82	8.12	3.85	5.13	2.99	2.99
3/iii	1.25	25.63	22.50	5.63	7.50	7.50	8.75	3.75	3.13	1.25	5.63	7.50
3/iv	3.98	21.89	15.42	8.96	17.91	5.97	6.47	4.98	3.48	6.47	1.49	2.99
3/v	6.77	46.24	4.89	8.27	10.90	3.76	6.02	3.38	4.14	2.26	1.50	1.88
4/i	13.54	31.83	11.29	7.22	8.80	5.87	7.45	3.84	4.51	3.39	1.58	0.68
4/ii	1.27	39.68	13.33	4.76	6.67	6.35	7.30	2.22	2.86	3.81	4.13	7.62
4/iii	3.30	28.57	20.51	5.86	11.72	8.06	7.69	4.40	4.03	0.73	0.73	4.40
4/iv	12.36	34.83	17.98	12.36	5.62	7.87	0.00	3.37	2.25	2.25	0.00	1.12
4/v	7.59	41.96	28.57	3.57	5.80	3.13	2.68	0.45	2.68	3.13	0.00	0.45
5/i	22.35	18.27	10.69	8.94	6.32	5.34	4.28	3.11	8.94	2.62	1.85	7.29

5/ii	6.25	28.13	25.00	3.13	10.42	9.38	4.17	1.04	2.08	6.25	0.00	4.17
5/iii	8.16	25.00	7.11	15.26	13.68	2.89	7.63	0.79	12.63	1.32	2.11	3.42
5/iv	1.14	13.64	11.36	7.95	26.14	15.91	10.23	3.41	6.82	1.14	1.14	1.14
5/v	16.16	7.86	12.66	14.41	16.16	11.35	4.80	2.18	7.86	2.18	0.44	3.93
6/i	4.49	27.56	19.55	15.38	14.10	6.41	3.53	0.32	7.05	0.96	0.32	0.32
6/ii	38.48	24.61	14.31	6.01	5.29	1.72	3.00	2.29	1.43	0.86	1.14	0.86
6/iii	38.75	8.75	13.75	12.50	8.75	8.75	2.50	0.00	2.50	3.75	0.00	0.00
6/iv	46.93	31.84	10.06	4.47	3.07	1.12	1.96	0.28	0.28	0.00	0.00	0.00
6/v	14.60	21.53	23.72	17.52	4.74	13.14	1.46	0.36	1.82	1.09	0.00	0.00
6/vi	23.91	38.70	22.17	3.91	2.17	1.30	0.87	0.43	4.35	0.43	0.43	1.30
7/i	0.00	16.67	51.67	10.00	3.33	1.67	3.33	0.00	3.33	5.00	0.00	5.00
7/ii	25.17	19.43	11.32	8.61	8.28	3.72	4.56	3.04	7.09	3.04	2.70	3.04
7/iii	7.84	29.41	7.84	1.96	5.88	21.57	7.84	5.88	7.84	0.00	0.00	3.92
7/iv	18.03	25.41	11.48	4.92	9.84	7.38	7.38	5.74	2.46	3.28	1.64	2.46
8/i	19.76	16.77	15.57	10.18	5.99	6.59	5.39	1.20	7.19	1.80	0.60	8.98
8/ii	6.57	23.36	13.87	8.03	6.57	8.03	9.49	1.46	6.57	4.38	2.92	8.76
10/i	8.51	23.40	23.40	6.38	6.38	2.13	4.26	2.13	10.64	0.00	2.13	10.64
9/iii	21.21	9.09	19.19	6.06	7.07	6.06	15.15	7.07	1.01	4.04	1.01	3.03
9/iv	50.57	19.54	0.00	2.30	4.60	2.30	9.20	2.30	2.30	2.30	1.15	3.45
9/v	21.05	5.26	21.05	5.26	10.53	10.53	5.26	5.26	0.00	0.00	5.26	10.53
9/vi	6.58	5.26	13.16	7.89	10.53	6.58	9.21	3.95	11.84	9.21	3.95	11.84
10/ii	15.52	19.83	13.79	11.21	10.34	4.31	4.31	2.59	4.31	5.17	0.00	8.62
9/i	5.56	9.72	26.39	4.17	8.33	5.56	12.50	4.17	8.33	6.94	0.00	8.33
10/iv	56.67	6.67	10.00	0.00	3.33	0.00	10.00	0.00	0.00	0.00	0.00	13.33
10/iii	72.12	17.26	0.88	0.44	1.77	0.44	1.33	1.77	0.88	1.77	0.44	0.88
9/ii	19.79	12.50	9.38	8.33	9.38	11.46	6.25	4.17	3.13	5.21	3.13	7.29
10/v	23.08	21.30	8.28	5.33	6.51	7.69	6.51	5.33	6.51	1.78	1.78	5.92

13/ii	8.67	21.39	10.98	11.56	11.56	5.20	7.51	1.73	4.05	2.89	3.47	10.98
11/i	3.57	14.29	0.00	25.00	14.29	7.14	10.71	0.00	7.14	0.00	3.57	14.29
11/ii	3.17	17.46	15.87	7.94	7.94	11.11	6.35	3.17	6.35	4.76	4.76	11.11
11/iii	0.00	22.22	0.00	0.00	11.11	0.00	0.00	11.11	11.11	11.11	11.11	22.22
12/i	2.47	10.49	20.99	13.58	12.35	8.02	9.26	4.32	8.02	4.94	1.85	3.70
12/iii	1.75	20.41	9.91	10.20	12.24	8.75	8.16	4.96	6.41	5.54	5.25	6.41
12/iv	1.63	28.80	10.33	8.15	9.78	5.43	10.87	3.80	7.61	3.80	1.63	8.15
12/ii	1.90	32.70	9.21	10.16	6.03	6.98	9.21	3.49	5.40	4.13	3.81	6.98
13/i	7.95	31.80	7.53	7.53	9.83	6.49	8.37	2.93	4.81	2.72	2.30	7.74
14/iv	0.78	36.24	9.88	9.88	7.17	6.59	6.20	2.52	3.88	1.74	1.74	13.37
13/iii	15.74	37.31	6.35	7.87	4.82	5.84	4.82	2.79	5.08	2.03	0.51	6.85
15/v	10.03	27.51	21.04	9.71	11.00	5.50	5.83	1.29	1.62	0.65	0.97	4.85
13/iv	0.92	21.54	10.46	14.77	9.85	7.69	5.85	2.15	6.77	4.31	2.77	12.92
14/iii	10.56	22.22	3.89	9.44	8.89	9.17	6.39	4.17	6.39	3.33	2.50	13.06
14/vi	11.14	32.26	3.52	10.26	6.45	5.28	5.57	3.81	6.45	2.93	1.76	10.56
14/v	7.20	20.80	5.20	9.60	11.20	7.60	5.60	4.00	7.60	3.60	3.20	14.40
14/ii	12.04	16.99	3.44	10.54	8.82	8.60	6.45	3.44	7.53	4.30	2.80	15.05
14/i	5.43	18.09	5.68	11.89	8.01	8.79	8.53	3.36	6.46	5.17	1.81	16.80
15/i	22.74	21.19	6.72	8.53	7.75	2.84	5.68	3.36	5.94	4.39	2.58	8.27
15/iii	8.70	20.95	5.14	10.28	10.67	5.53	6.72	3.56	3.95	4.74	4.35	15.42
15/ii	2.43	20.49	4.86	11.11	11.81	5.21	5.21	3.82	5.56	5.21	3.13	21.18
15/iv	2.60	16.88	3.90	12.99	12.34	9.09	6.49	0.65	5.84	5.84	4.55	18.83
16/ii	0.00	17.78	11.11	22.22	13.33	6.67	0.00	0.00	11.11	0.00	2.22	15.56
16/iii	0.00	23.26	4.65	13.95	9.30	13.95	4.65	0.00	9.30	6.98	0.00	13.95
16/iv	0.00	14.58	2.08	18.75	12.50	8.33	6.25	0.00	10.42	6.25	2.08	18.75
16/v	0.00	28.00	5.33	12.00	0.00	13.33	6.00	4.00	4.00	2.00	2.00	23.33
16/i	2.45	22.70	5.52	13.50	14.72	1.84	4.29	3.68	6.75	3.68	0.00	20.86

17/i	30.41	20.00	1.10	4.66	4.66	5.21	4.38	2.47	3.84	3.29	1.64	18.36
17/iii	25.34	18.24	2.70	6.76	5.74	3.04	4.39	0.68	5.41	6.42	0.68	20.61
17/ii	14.11	12.88	3.07	9.82	5.52	8.90	8.90	2.45	5.52	6.13	4.91	17.79
18/i	7.07	12.54	9.32	10.29	9.97	9.32	4.18	2.57	7.72	8.04	4.82	14.15
18/ii	2.97	13.04	8.92	9.38	8.24	12.36	6.86	4.12	8.24	6.18	4.35	15.33
18/iii	11.71	18.87	3.25	8.68	11.28	4.12	4.56	2.17	9.98	4.56	2.60	18.22
19/i	29.38	9.79	7.44	10.08	6.31	7.34	4.90	3.11	7.06	4.71	2.73	7.16
19/ii	34.51	8.27	7.70	8.65	7.89	5.70	4.56	1.90	5.42	3.52	3.23	8.65
20/ii	8.65	19.64	6.98	6.93	5.57	9.11	4.96	2.88	8.40	3.95	4.10	18.83
20/i	18.62	18.62	3.95	7.74	4.28	7.58	4.45	1.15	7.58	4.12	3.62	18.29
21/ii	46.24	12.05	2.78	5.46	4.12	3.40	4.12	2.47	3.19	3.81	0.82	11.53
21/i	4.15	28.11	1.84	13.82	2.76	6.68	2.30	2.76	2.76	4.84	2.76	27.19
22/ii	4.48	17.06	11.05	11.33	7.13	5.45	7.41	2.10	5.03	5.31	4.90	18.74
22/i	3.49	13.55	2.67	12.94	4.72	11.50	8.21	1.64	9.45	3.29	4.31	24.23
23/iii	3.17	14.70	8.36	12.39	13.83	7.20	6.34	2.59	10.37	5.76	5.19	10.09
23/ii	2.01	12.08	10.07	15.77	10.74	7.05	7.38	4.70	9.73	9.06	5.03	6.38
23/i	3.41	13.45	9.28	12.88	12.12	8.52	7.39	3.41	10.23	6.25	5.11	7.95
24/i	1.55	15.89	3.31	7.73	18.32	3.31	2.43	2.87	18.10	4.42	3.97	18.10
25/i	2.96	12.59	5.93	25.19	5.19	5.93	1.48	2.22	3.70	10.37	5.93	18.52
24/ii	0.00	16.81	5.31	12.39	30.53	6.19	3.98	0.00	11.50	0.88	3.54	8.85
24/iii	3.23	21.70	11.14	3.52	13.49	2.35	4.99	4.40	20.23	0.59	2.35	12.02
25/iii	2.20	13.22	9.69	17.18	8.37	14.54	4.85	3.96	3.96	7.05	4.41	10.57
25/ii	6.15	15.08	11.38	15.38	6.46	6.77	3.38	2.46	7.38	11.69	4.31	9.54
26	7.69	23.70	4.34	17.37	11.54	1.24	0.25	0.50	10.67	5.96	1.24	15.51
27/iii	2.41	12.03	13.37	11.50	6.95	11.76	5.61	4.81	7.49	6.68	5.61	11.76
27/i	3.20	11.47	11.47	12.80	10.93	8.00	8.00	3.73	9.87	2.93	5.07	12.53
27/ii	10.00	6.88	15.63	11.25	12.50	11.25	8.75	0.00	10.00	2.50	7.50	3.75

28/iii	6.72	22.58	1.88	15.86	6.45	0.54	0.27	0.00	7.80	12.10	0.27	25.54
28/i	1.27	26.14	1.52	11.17	6.85	3.55	1.52	2.03	5.84	9.39	1.02	29.70
28/ii	0.00	25.34	1.57	19.28	12.11	0.00	0.00	0.90	9.19	8.74	0.00	22.87
29/ii	2.90	25.84	2.45	10.24	12.47	1.56	2.90	1.56	8.46	5.79	0.67	25.17
29/i	1.69	21.94	3.38	16.46	23.21	0.00	0.00	0.00	16.88	7.59	0.42	8.44
29/iii	5.07	25.91	3.26	18.48	11.23	1.09	0.54	0.18	11.05	8.70	1.27	13.22
30	26.13	14.41	0.90	17.12	0.64	5.92	0.39	1.29	0.90	9.27	1.03	22.01
31/iv	0.00	23.53	0.00	7.84	25.49	13.73	0.00	0.00	15.69	0.00	0.00	13.73
31/v	2.66	29.71	1.21	5.80	23.19	6.04	0.72	0.48	14.98	4.59	0.00	10.63
31/vi	2.10	20.17	0.00	6.30	30.67	10.08	0.00	0.00	18.07	2.94	0.42	9.24
31/i	4.17	20.00	0.00	2.50	13.33	8.33	1.67	0.00	25.83	1.67	0.00	22.50
31/ii	2.89	14.08	0.00	5.78	16.61	5.78	0.36	1.44	20.94	2.53	0.36	29.24
31/iii	0.16	24.24	0.16	6.38	24.56	8.29	0.32	1.59	20.73	1.59	0.00	11.96

Table D.1: Linear simple interval distributions.

Movt.	0	1	2	3	4	5	6	7	8	9	10	11
1	24.54	3.95	4.30	7.70	14.25	5.35	6.97	6.82	8.31	6.86	7.17	3.77
2	4.73	2.96	6.76	11.24	16.65	6.38	8.23	9.22	15.05	7.46	6.11	5.23
3/i	6.21	7.04	14.90	7.17	3.62	7.22	13.43	6.28	8.44	6.97	4.87	13.84
3/ii	4.76	9.91	8.08	9.01	11.32	8.34	5.30	3.91	11.16	10.94	9.23	8.02
3/iii	2.21	12.92	8.38	12.60	11.54	2.57	7.27	3.36	6.08	10.43	6.95	15.69
3/iv	2.41	13.35	7.14	6.71	6.71	8.58	7.14	11.17	6.84	13.01	6.17	10.76
3/v	6.62	4.89	11.03	10.26	8.35	6.91	5.61	7.63	10.22	6.62	6.24	15.64
4/i	9.55	10.76	8.26	9.36	8.82	5.00	8.44	7.30	4.93	10.35	8.32	8.91
4/ii	3.76	13.75	9.09	4.87	7.85	7.69	10.10	12.90	8.69	4.91	3.13	13.26
4/iii	2.59	12.24	4.95	4.89	10.70	12.34	11.86	3.29	8.97	11.34	8.79	8.04

4/iv	1.49	13.46	6.61	7.28	12.64	8.64	4.93	10.94	10.23	13.47	3.28	7.00
4/v	1.91	13.95	8.45	12.29	11.39	13.44	10.68	3.04	6.54	4.14	8.08	6.10
5/i	4.24	12.86	7.04	7.98	9.82	6.55	7.24	6.04	10.47	7.70	7.62	12.44
5/ii	5.48	10.96	3.56	4.93	13.42	9.04	10.14	0.00	6.85	17.53	10.96	7.12
5/iii	7.47	10.76	6.58	8.89	7.98	6.11	12.43	6.36	7.27	5.28	9.81	11.05
5/iv	5.78	13.48	3.53	6.90	2.89	25.20	14.77	4.17	4.82	2.89	1.28	14.29
5/v	3.98	8.77	11.84	5.38	2.97	8.33	11.28	9.26	11.16	9.75	6.04	11.25
6/i	7.65	12.63	8.40	5.32	8.74	9.76	7.17	5.53	8.19	9.56	8.12	8.94
6/ii	9.15	11.78	7.22	7.62	7.70	6.76	6.43	8.26	8.69	4.63	5.19	16.56
6/iii	1.12	4.84	17.50	13.41	5.59	7.45	14.15	1.12	18.25	6.70	2.23	7.64
6/iv	2.51	7.43	11.75	16.54	11.39	4.51	6.61	5.92	3.74	7.70	2.28	19.64
6/v	1.18	6.16	7.23	9.99	9.04	6.36	9.61	11.33	9.38	8.10	8.94	12.68
6/vi	0.47	13.03	6.42	12.23	12.14	7.03	10.31	13.17	8.29	5.62	3.80	7.50
7/i	6.59	4.40	6.28	11.15	8.95	5.49	14.29	7.38	4.40	9.26	6.12	15.70
7/ii	7.35	7.22	8.45	10.68	6.84	4.80	10.17	8.07	6.93	9.30	7.41	12.77
7/iii	1.11	29.36	13.02	9.42	11.63	0.33	1.44	2.77	2.49	11.63	10.91	5.87
7/iv	0.78	12.67	5.88	8.09	8.04	12.22	8.11	9.66	10.23	7.90	7.01	9.43
8/i	0.00	9.65	10.36	11.18	8.80	9.12	6.17	8.27	8.23	8.09	7.10	13.02
8/ii	0.00	15.70	7.58	9.42	6.39	7.46	6.93	5.36	13.58	8.38	5.94	13.27
10/i	4.08	18.66	8.75	13.99	2.92	4.08	3.50	6.71	6.41	9.33	13.41	8.16
9/iii	5.42	9.87	4.57	6.99	6.73	9.62	6.77	1.97	15.21	10.77	6.28	15.81
9/iv	0.00	11.48	4.92	0.00	4.26	13.44	11.80	14.43	4.92	2.95	2.95	28.85
9/v	0.00	33.51	13.62	6.54	9.81	9.81	4.90	9.81	8.17	0.00	1.63	2.18
9/vi	0.00	28.80	6.81	8.64	2.62	5.76	10.99	10.73	3.14	2.36	7.85	12.30
10/ii	2.56	12.39	7.05	13.28	7.87	8.10	9.57	6.78	10.73	7.25	4.88	9.56
9/i	0.00	18.19	4.28	10.17	6.59	5.46	6.49	8.90	7.91	5.54	2.90	23.58
10/iv	0.00	35.90	5.13	2.05	4.62	7.90	9.74	2.05	11.28	4.10	4.51	12.72

10/iii	1.44	14.23	13.61	7.80	16.98	13.06	10.31	2.20	13.20	3.71	2.06	1.38
9/ii	0.00	21.83	3.51	2.73	10.92	6.48	2.00	7.60	9.36	10.53	7.26	17.79
10/v	2.24	9.45	5.92	7.47	9.45	4.96	11.69	3.84	10.09	11.58	10.19	13.13
13/ii	1.06	10.13	7.08	10.35	6.83	7.82	10.29	8.92	10.32	9.39	5.30	12.51
11/i	0.00	9.84	11.42	7.87	12.60	12.20	9.06	3.15	11.02	7.87	4.72	10.24
11/ii	0.00	19.53	3.52	8.98	10.74	12.70	9.77	4.30	8.59	7.81	7.03	7.03
11/iii	0.00	22.86	12.57	11.43	1.71	3.43	2.29	6.29	17.71	8.57	0.00	13.14
12/i	0.26	8.49	7.13	8.81	12.64	10.63	7.84	3.95	8.55	10.89	6.03	14.78
12/iii	0.15	11.85	7.85	10.16	10.70	8.49	8.95	6.82	8.47	7.16	8.08	11.31
12/iv	0.00	6.77	10.64	10.98	5.99	8.91	8.34	1.40	13.95	9.71	8.84	14.46
12/ii	0.48	7.61	5.96	7.51	12.68	11.95	10.23	4.48	8.47	7.81	7.23	15.59
13/i	4.32	10.03	7.86	6.69	7.17	8.50	8.80	8.51	11.54	9.90	7.13	9.54
14/iv	3.60	9.43	7.28	11.79	11.43	9.32	9.31	9.43	8.36	7.09	6.09	6.86
13/iii	0.51	11.57	8.23	8.00	9.67	11.30	9.73	8.23	7.11	5.44	5.28	14.95
15/v	5.42	10.27	9.17	10.18	8.65	7.47	8.61	9.26	8.48	8.96	5.11	8.43
13/iv	2.59	8.77	3.65	9.84	9.71	8.57	14.02	9.58	9.18	5.59	9.28	9.21
14/iii	1.08	10.98	7.79	9.11	8.21	10.51	7.81	8.33	9.87	10.17	4.28	11.86
14/vi	0.75	9.44	5.19	14.84	8.28	11.08	14.47	7.10	10.31	5.21	4.48	8.85
14/v	5.87	10.94	10.07	13.50	7.42	6.79	6.32	6.87	12.39	7.97	4.02	7.85
14/ii	2.42	11.66	8.53	9.33	5.76	10.17	7.70	7.21	11.79	7.53	6.07	11.83
14/i	3.08	12.33	5.41	10.42	9.71	11.55	9.54	7.89	5.18	6.97	5.68	12.24
15/i	0.75	9.74	7.07	12.50	8.92	8.95	8.33	8.73	9.88	9.81	6.45	8.85
15/iii	1.04	7.91	3.75	14.21	6.61	8.64	9.89	10.62	9.94	9.94	3.18	14.26
15/ii	3.92	5.96	4.94	9.83	8.62	10.64	11.46	11.67	9.83	7.50	5.71	9.91
15/iv	2.95	6.82	3.86	14.32	6.14	9.77	4.77	10.00	8.64	13.86	4.55	14.32
16/ii	3.42	8.90	4.11	17.12	6.85	6.16	16.44	13.70	10.96	8.90	0.00	3.42
16/iii	1.85	9.54	1.85	12.00	18.46	7.69	15.08	9.54	6.77	8.92	1.85	6.46

16/iv	0.00	3.77	18.67	11.72	5.86	21.10	6.17	0.84	3.13	5.27	6.70	16.77
16/v	2.16	11.42	3.10	17.87	9.68	8.01	7.19	9.19	6.76	12.48	3.45	8.70
16/i	2.89	15.61	5.20	4.62	5.78	4.05	17.53	5.59	14.45	12.14	1.16	10.98
17/i	4.39	13.30	8.68	9.55	8.72	3.83	6.97	9.11	11.82	8.72	3.27	11.64
17/iii	2.12	11.68	6.64	8.79	8.33	11.29	10.25	11.39	9.50	6.51	3.42	10.09
17/ii	4.99	19.08	6.93	9.60	6.99	7.38	6.93	4.60	7.15	9.48	2.16	14.70
18/i	1.65	5.32	7.07	7.07	4.74	15.68	13.94	8.71	8.03	4.45	4.74	18.59
18/ii	4.30	5.93	4.96	11.47	9.85	6.95	10.26	6.83	9.65	2.60	6.83	20.37
18/iii	3.03	5.08	7.94	8.22	4.71	7.56	10.53	6.63	11.54	2.74	3.59	28.44
19/i	2.41	8.21	8.97	10.86	7.72	9.84	7.58	6.01	8.71	8.10	8.30	13.29
19/ii	1.85	9.65	11.49	7.72	8.89	9.31	9.40	6.38	6.29	5.96	7.72	15.35
20/ii	0.05	8.25	3.50	6.04	8.67	21.56	10.20	4.74	5.49	2.90	6.00	22.60
20/i	0.44	10.51	1.82	3.42	5.95	28.17	13.20	3.19	8.83	1.51	2.77	20.20
21/ii	0.00	10.92	11.56	11.05	5.75	13.96	11.71	5.60	7.50	4.90	6.12	10.93
21/i	0.15	10.31	8.15	7.69	7.85	16.38	6.77	2.92	9.31	7.77	9.15	13.54
22/ii	3.69	14.23	4.75	8.98	8.16	10.85	7.93	5.81	6.67	7.63	3.84	17.46
22/i	3.73	12.69	11.94	14.18	8.96	5.97	4.48	6.34	4.48	10.45	4.10	12.69
23/iii	0.00	5.69	9.15	13.20	11.99	9.15	2.76	0.76	15.83	8.39	7.53	15.55
23/ii	0.00	5.43	7.81	15.11	10.56	8.84	6.07	5.18	13.50	3.66	6.52	17.31
23/i	0.24	5.43	9.31	11.37	7.01	11.02	6.57	3.45	13.71	6.78	6.34	18.78
24/i	2.61	11.19	4.50	8.27	6.57	10.70	5.16	4.02	10.78	5.87	5.54	24.80
25/i	0.00	12.34	15.32	12.77	11.49	6.38	3.83	1.70	6.38	5.11	5.53	19.15
24/ii	0.54	6.31	1.80	9.01	19.82	3.60	2.34	2.70	4.32	2.88	2.34	44.32
24/iii	1.21	9.98	0.39	15.95	7.77	3.70	3.79	3.23	15.52	4.61	3.23	30.64
25/iii	0.00	7.73	7.73	10.62	6.56	13.12	4.28	2.04	9.85	9.53	6.50	22.03
25/ii	0.00	11.61	9.93	4.52	3.46	10.55	6.21	2.39	9.57	12.41	6.03	23.32
26	1.48	13.04	9.09	12.71	14.19	10.54	5.84	2.75	9.25	4.82	4.58	11.72

27/iii	0.00	2.64	10.58	7.90	1.59	5.82	11.60	2.12	8.99	2.12	9.86	36.80
27/i	0.00	9.20	9.96	5.36	3.07	18.39	13.41	0.77	2.30	2.30	1.53	33.72
27/ii	0.00	12.90	0.00	12.90	3.23	12.90	19.35	0.00	0.00	0.00	6.45	32.26
28/iii	0.00	4.15	11.34	11.34	5.53	18.04	5.28	2.26	8.00	8.67	9.72	15.67
28/i	0.00	5.55	6.43	14.17	13.20	11.27	5.01	3.05	14.30	5.60	6.58	14.82
28/ii	0.00	7.04	10.88	11.22	8.04	18.52	9.93	4.61	4.70	7.51	12.74	4.80
29/ii	4.04	7.11	11.52	9.19	8.24	7.99	11.51	6.95	6.15	7.88	9.20	10.22
29/i	3.34	9.07	16.79	12.08	6.93	10.94	3.05	10.59	3.16	8.92	5.79	9.35
29/iii	2.40	6.63	13.07	5.35	9.27	12.38	7.27	6.82	7.01	9.12	6.60	14.08
30	2.64	5.86	11.51	15.09	8.98	7.73	7.25	4.56	10.25	5.60	5.20	15.32
31/iv	0.00	3.70	0.92	4.36	20.21	13.34	1.85	3.70	22.46	8.06	12.29	9.11
31/v	0.11	17.22	3.54	4.94	6.05	6.30	3.29	18.36	8.80	18.32	2.90	10.16
31/vi	4.97	4.97	13.81	11.60	4.97	15.47	6.63	9.39	6.63	4.97	10.50	6.08
31/i	3.54	16.47	13.72	8.50	10.12	11.52	2.81	7.51	7.48	2.92	2.60	12.83
31/ii	5.01	6.49	7.67	11.80	10.03	10.62	6.49	4.13	5.60	4.72	12.68	14.75
31/iii	3.88	6.44	12.46	8.72	7.65	10.87	5.28	10.02	6.89	10.18	8.00	9.61

Table D.2: Vertical simple interval distributions.

Appendix E

Row Interval Classes

Movt.	1	2	3	4	5	6
17/i	6	1	3	1	1	0
17/ii	7	0	2	1	1	1
17/iii	8	0	2	1	0	1
18/i	3	1	2	1	1	4
18/ii	1	0	5	5	0	1
18/iii	7	0	1	3	0	1
19	3	1	2	2	3	1
20	6	0	1	2	3	0
21	6	0	2	2	0	2
22	5	1	2	1	1	2
23	1	1	4	4	1	1
24	5	2	0	4	1	0
25	4	1	5	1	1	0
26	5	1	3	3	0	0
27	4	2	1	3	1	1
28	6	0	4	2	0	0
29	4	0	3	4	1	0
30	8	0	4	0	0	0
31	4	0	1	5	1	1

Appendix F

Pitch-Class Set DFT Magnitudes

Movt.	$\lvert f_1 \rvert$	$\lvert f_2 \rvert$	$\lvert f_3 \rvert$	$\lvert f_4 \rvert$	$\lvert f_5 \rvert$	$\lvert f_6 \rvert$
1	0.31	0.35	0.31	0.31	0.31	0.25
2	2.44	2.52	3.00	4.46	3.66	2.52
3/i	3.07	2.81	3.49	3.14	2.77	2.79
3/ii	1.70	3.15	3.65	3.15	2.10	1.03
3/iii	3.43	2.37	4.31	4.77	2.33	0.92
3/iv	2.48	2.48	2.45	2.94	2.94	2.33
3/v	3.26	2.17	2.54	4.35	3.12	2.17
4/i	1.28	1.38	1.53	2.08	1.03	1.26
4/ii	2.32	2.47	3.10	2.66	1.59	0.94
4/iii	1.04	2.42	1.59	1.75	1.20	1.46
4/iv	3.16	2.41	2.15	3.38	2.91	2.16
4/v	3.04	2.43	4.42	2.43	2.34	1.71
5/i	0.73	1.21	1.39	1.22	0.93	0.56
5/ii	3.09	4.12	6.19	4.12	3.09	2.06
5/iii	2.51	2.23	2.78	2.37	1.47	2.37
5/iv	2.93	8.78	3.89	3.36	2.93	3.42
5/v	2.58	5.03	2.58	2.15	2.15	1.70
6/i	1.07	2.12	1.82	1.34	1.07	0.89
6/ii	1.38	1.35	1.54	1.38	1.07	1.13
6/iii	4.91	7.89	5.26	6.80	3.98	3.95
6/iv	3.45	2.35	2.05	3.37	1.60	1.97
6/v	2.89	1.45	3.61	2.17	1.80	1.45
6/vi	2.28	3.21	3.70	3.50	2.53	1.75
7/i	2.53	4.27	4.88	2.85	3.26	2.44
7/ii	1.45	1.23	1.90	1.45	1.16	1.01
7/iii	3.41	2.98	4.21	2.98	3.41	1.72
7/iv	3.16	3.15	5.42	4.34	3.13	2.17
8/i	2.69	3.08	2.69	3.30	1.50	2.06

8/ii	1.67	1.48	1.97	1.72	1.97	0.98
10/i	9.04	5.23	3.25	2.26	3.54	2.20
9/iii	3.00	6.01	5.44	5.44	2.72	2.91
9/iv	6.40	9.88	6.83	3.21	6.89	1.50
9/v	13.36	25.63	13.03	3.15	7.39	5.04
9/vi	4.18	5.59	5.28	3.26	2.92	2.64
10/ii	2.31	3.64	4.01	2.35	2.41	1.46
9/i	6.98	4.40	5.49	3.40	2.90	1.28
10/iv	7.78	14.58	5.73	2.08	5.00	0.00
10/iii	5.88	2.36	4.72	1.89	2.35	2.83
9/ii	4.52	6.02	7.53	3.01	3.50	1.51
10/v	2.24	2.24	2.99	2.62	2.00	1.50
13/ii	1.37	1.67	1.31	1.43	0.95	0.83
11/i	7.43	8.11	10.14	8.11	4.78	5.41
11/ii	3.49	7.24	9.65	4.32	2.15	4.27
11/iii	25.03	11.76	13.24	11.76	7.66	25.00
12/i	2.14	2.49	2.49	3.20	2.50	1.42
12/iii	1.78	1.67	1.67	1.67	1.25	1.15
12/iv	3.35	2.69	2.69	3.59	2.00	1.57
12/ii	1.64	2.12	1.67	2.12	1.51	1.41
13/i	0.95	0.97	0.99	1.27	1.01	0.59
14/iv	3.70	3.32	3.06	3.44	2.64	1.40
13/iii	1.82	1.46	1.54	1.30	1.47	0.65
15/v	3.49	2.52	4.65	4.65	3.88	3.10
13/iv	1.40	1.50	1.69	1.35	1.41	1.12
14/iii	1.79	1.90	1.51	2.42	1.56	1.12
14/vi	1.67	1.34	1.79	1.34	1.28	0.89
14/v	3.19	2.79	3.71	2.79	2.24	2.23
14/ii	1.45	2.04	1.42	1.36	1.59	0.91
14/i	1.73	1.74	1.69	1.33	1.49	0.92
15/i	1.67	1.67	2.08	1.46	1.67	0.83
15/iii	3.17	2.23	2.98	2.05	2.23	0.74
15/ii	1.55	2.13	1.94	1.55	1.31	1.55
15/iv	5.89	6.85	7.23	8.76	7.34	5.71
16/ii	13.33	18.95	13.33	16.84	13.33	21.05
16/iii	4.89	5.04	2.52	7.56	5.66	2.24
16/iv	21.75	15.92	21.75	15.92	21.75	15.92
16/v	5.96	5.81	5.88	9.15	9.37	4.90
16/i	5.36	4.17	3.57	3.57	4.17	5.36
17/i	3.08	2.67	2.25	2.25	2.25	1.83
17/iii	2.14	2.90	2.22	2.22	2.77	2.22
17/ii	3.36	3.21	3.36	2.69	1.99	1.68
18/i	1.83	4.34	1.37	3.65	2.05	1.37

18/ii	2.27	1.65	3.01	2.24	1.57	0.92
18/iii	2.24	2.99	2.19	2.02	1.62	0.73
19/i	1.12	1.06	1.18	0.94	1.21	0.47
19/ii	2.18	1.80	1.65	1.50	1.20	0.75
20/ii	0.60	1.01	1.31	0.42	0.45	0.26
20/i	2.99	3.25	1.66	2.32	2.01	1.00
21/ii	0.98	2.25	1.15	2.56	1.16	1.13
21/i	1.94	2.64	2.54	2.64	1.82	1.93
22/ii	1.47	1.29	0.93	0.98	1.07	0.77
22/i	3.84	2.57	2.83	3.60	2.95	1.03
23/iii	1.78	0.99	2.22	1.97	1.23	1.50
23/ii	1.74	1.25	2.30	1.61	0.90	1.07
23/i	1.77	2.02	2.27	1.81	1.21	1.21
24/i	1.27	1.01	1.55	1.16	1.44	0.89
25/i	8.94	4.92	5.53	3.90	2.20	1.03
24/ii	12.79	17.06	15.44	5.29	5.51	6.59
24/iii	11.33	3.83	5.49	2.76	3.06	0.43
25/iii	3.08	2.48	5.67	3.30	1.42	0.83
25/ii	2.83	2.28	1.52	2.03	1.49	0.38
26	0.87	0.93	0.71	0.70	0.71	0.47
27/iii	3.15	2.80	2.75	4.08	1.61	2.80
27/i	5.57	8.36	5.57	9.06	5.04	8.36
27/ii	12.50	12.50	12.50	12.50	9.38	12.50
28/iii	1.90	5.14	2.67	5.34	4.69	2.47
28/i	4.26	4.54	2.27	3.13	4.24	2.55
28/ii	6.32	3.87	4.31	8.04	7.47	3.87
29/ii	2.51	3.30	1.99	2.11	2.54	1.36
29/i	3.21	3.21	1.07	3.21	2.51	1.07
29/iii	2.16	1.89	3.02	1.51	2.26	1.51
30	1.17	0.97	1.11	1.13	0.99	0.56
31/iv	4.44	3.45	9.20	2.30	3.91	1.15
31/v	1.18	1.03	2.22	0.96	0.89	0.74
31/vi	7.04	4.23	4.23	8.45	7.04	5.63
31/i	8.71	1.32	1.66	2.26	1.57	0.65
31/ii	1.54	2.00	2.00	4.15	2.30	1.38
31/iii	2.21	1.58	1.97	1.18	1.47	0.79

Table F.1: Vertical pc sets.

Movt.	$\|f_1\|$	$\|f_2\|$	$\|f_3\|$	$\|f_4\|$	$\|f_5\|$	$\|f_6\|$
1	0.55	0.73	0.64	0.67	0.60	0.43
2	0.95	0.74	0.78	0.54	0.64	0.46
3/i	0.97	0.87	0.78	0.78	0.88	0.58

3/ii	0.62	0.62	0.59	0.51	0.63	0.34
3/iii	0.90	1.08	0.90	0.82	0.77	0.56
3/iv	0.80	0.80	0.86	0.80	0.70	0.48
3/v	1.05	0.94	1.03	1.03	0.60	0.38
4/i	0.97	0.92	1.15	0.81	0.80	1.04
4/ii	0.60	0.56	0.43	0.52	0.59	0.34
4/iii	0.71	0.67	0.76	0.76	0.71	0.38
4/iv	1.84	2.15	1.53	1.23	0.82	1.23
4/v	1.37	1.81	1.20	0.75	0.75	0.75
5/i	0.40	0.54	0.42	0.45	0.41	0.33
5/ii	1.94	5.83	4.37	3.88	2.07	5.34
5/iii	0.30	0.37	0.37	0.37	0.28	0.20
5/iv	1.44	5.56	2.14	2.78	3.19	1.28
5/v	3.41	2.51	2.28	2.05	2.07	1.83
6/i	0.65	0.35	0.46	0.69	0.35	0.29
6/ii	1.41	0.97	0.97	1.56	0.86	0.49
6/iii	6.72	4.95	6.31	5.86	1.38	2.25
6/iv	2.17	1.32	1.25	0.93	0.91	0.47
6/v	1.88	2.50	2.81	1.25	1.56	4.38
6/vi	1.24	0.69	0.92	0.92	0.61	0.23
7/i	16.88	4.84	8.06	1.61	12.90	4.84
7/ii	0.60	0.66	0.62	0.53	0.59	0.46
7/iii	2.61	3.96	3.96	3.96	2.97	0.99
7/iv	1.53	1.43	2.46	1.23	1.23	0.61
8/i	1.47	1.15	1.15	0.86	0.86	1.00
8/ii	2.02	2.12	1.06	1.23	0.80	0.71
10/i	5.00	6.88	5.00	1.25	5.00	1.88
9/iii	0.94	1.14	0.98	0.65	0.82	0.65
9/iv	4.25	8.79	4.95	2.75	2.35	3.30
9/v	168.30	100.00	50.00	150.00	81.70	250.00
9/vi	1.61	2.81	2.41	1.61	1.61	1.61
10/ii	2.24	2.34	3.13	1.56	1.17	1.56
9/i	2.09	1.78	2.37	2.37	2.37	2.37
10/iv	15.79	36.84	21.05	5.26	5.26	5.26
10/iii	11.66	18.75	6.25	9.38	6.25	6.25
9/ii	1.45	1.48	1.98	1.48	1.19	0.99
10/v	1.58	1.06	2.11	1.58	1.48	1.06
13/ii	1.13	1.06	1.06	0.93	0.80	0.53
11/i	12.04	12.90	9.68	12.90	6.45	9.68
11/ii	2.67	2.67	2.22	1.78	1.43	1.78
11/iii	74.64	60.00	80.00	20.00	40.00	20.00
12/i	1.42	1.29	1.29	1.11	1.29	0.92
12/iii	0.75	0.71	0.85	0.62	0.71	0.57

12/iv	0.84	0.64	0.79	0.64	0.64	0.57
12/ii	0.76	0.63	0.67	0.60	0.75	0.53
13/i	0.79	0.63	0.77	0.63	0.79	0.49
14/iv	0.46	0.62	0.40	0.47	0.33	0.29
13/iii	0.81	0.76	0.65	0.65	0.46	0.49
15/v	0.79	0.74	0.85	0.74	0.74	0.64
13/iv	1.04	0.83	0.74	0.83	0.69	0.56
14/iii	0.58	0.69	0.73	0.58	0.54	0.40
14/vi	0.51	0.67	0.77	0.57	0.45	0.29
14/v	0.58	0.73	0.62	0.58	0.57	0.44
14/ii	0.60	0.61	0.63	0.61	0.58	0.39
14/i	0.62	0.58	0.64	0.59	0.51	0.29
15/i	0.51	0.72	0.63	0.54	0.51	0.36
15/iii	0.92	0.74	0.99	0.86	0.56	0.55
15/ii	0.70	0.76	0.76	0.82	0.47	0.47
15/iv	1.13	1.07	0.95	0.89	0.71	0.48
16/ii	2.81	1.96	1.96	1.96	1.96	0.98
16/iii	3.46	2.08	2.31	1.39	1.46	1.62
16/iv	0.73	1.08	0.90	0.72	0.72	0.54
16/v	1.18	1.08	1.00	0.76	0.61	0.47
16/i	0.83	0.77	0.87	0.77	0.54	0.58
17/i	0.64	0.62	0.64	0.64	0.52	0.32
17/iii	0.59	0.62	0.68	0.53	0.47	0.20
17/ii	0.69	0.59	0.69	0.69	0.55	0.41
18/i	0.68	0.72	0.72	0.76	0.67	0.40
18/ii	0.62	0.65	0.73	0.63	0.63	0.42
18/iii	0.51	0.49	0.55	0.55	0.60	0.41
19/i	0.74	0.68	0.84	0.66	0.72	0.46
19/ii	0.67	0.66	0.70	0.59	0.66	0.42
20/ii	0.50	0.48	0.57	0.58	0.49	0.36
20/i	0.57	0.51	0.44	0.44	0.34	0.22
21/ii	0.40	0.48	0.48	0.48	0.37	0.21
21/i	3.57	2.68	2.68	2.34	2.01	1.00
22/ii	0.66	0.60	0.66	0.66	0.62	0.44
22/i	0.99	0.99	0.99	0.99	0.62	0.37
23/iii	0.66	0.80	0.71	0.71	0.62	0.53
23/ii	0.66	0.66	0.66	0.82	0.66	0.44
23/i	0.65	0.60	0.54	0.71	0.61	0.49
24/i	0.61	0.85	0.80	0.62	0.64	0.62
25/i	0.78	0.91	1.04	1.04	0.70	0.39
24/ii	1.50	1.50	1.88	1.31	1.16	1.69
24/iii	1.52	1.23	0.95	0.76	0.88	0.76
25/iii	0.83	0.68	0.83	0.73	0.74	0.39

25/ii	0.89	0.89	0.89	0.59	0.89	0.50
26	0.49	0.49	0.31	0.37	0.29	0.21
27/iii	0.57	0.63	0.63	0.63	0.50	0.50
27/i	0.60	0.60	0.80	0.60	0.60	0.36
27/ii	0.92	0.73	0.92	0.73	0.79	0.73
28/iii	1.81	0.52	0.45	0.52	0.52	0.19
28/i	1.40	1.08	1.61	1.29	0.80	0.43
28/ii	0.99	1.03	0.53	0.61	0.68	0.30
29/ii	1.32	0.85	0.94	0.85	0.64	0.75
29/i	6.13	2.04	2.45	3.27	1.23	3.27
29/iii	0.70	0.48	0.75	0.70	0.48	0.35
30	1.30	0.82	1.52	1.06	0.69	0.29
31/iv	8.43	4.52	11.30	3.95	2.11	2.26
31/v	3.11	1.27	1.63	1.09	0.65	0.45
31/vi	4.89	2.19	4.90	2.19	0.89	1.28
31/i	3.91	2.73	4.88	2.34	1.31	1.17
31/ii	3.66	2.02	2.16	1.15	0.81	0.58
31/iii	0.27	0.33	0.28	0.33	0.23	0.09

Table F.2: Linear pc sets.

Appendix G

Code & Data

Other sets of results are simply too unwieldy to be printed, and in any case are unlikely to be of much value to a reader without some method of computational sorting. As such, they, along with spreadsheets of the data presented in the other appendices, are provided in a Github repository.

I also include in this repository some of the code I used to collect the data, presented in Jupyter Notebooks.

The items in the repository are as follows:

Code

- Interval Distributions (Linear and Vertical)
- PC Circulation
- PC Distributions.
- PC Set Distributions (Linear and Vertical)

Data

- PC Distribution DFT Magnitudes
- PC Distributions

- PC Set DFT Magnitudes (Linear and Vertical)
- PC Set Distributions (Linear and Vertical)
- Row IC Counts
- Simple Interval Distributions (Linear and Vertical)

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